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1	A novel analytical model for the transit time distributions in urban
2	groundwater systems
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17 Highlights:

- 18 1. An analytical model for urban groundwater TTDs is developed.
- 19 2. The imperviousness significantly affects groundwater TTDs.
- 20 3. Depth of the underground structure strongly controls groundwater TTDs.

22 Abstract

With the ongoing rapid urbanization across the globe, its interference with groundwater 23 resources is critical to freshwater sustainability. The groundwater transit time distribution (TTD) 24 lumps the flow and transport processes of a regional groundwater system and therefore 25 characterizes the aquifer's resilience to nonpoint-source contamination. The influence of large-26 scale urban areas on the regional groundwater TTD is not clear. This study proposed a novel 27 analytical model for groundwater TTDs accounting for the effects of impervious urban 28 29 structures. After the verification against results from particle tracking, we apply this analytical 30 expression to investigate how the position and spatial extent of the urban area change the TTDs from the pre-urban ones. The sensitivity analysis suggests that urban areas tend to increase both 31 32 the mean and the variance of groundwater transit times. For aquifers intersected by a local urban 33 area, mean transit time (MTT) is dominated by the horizontal extent of the urban area, whereas 34 for aquifers intersected by a regional urban area, MTT is strongly controlled by the vertical 35 extent of the urban area in addition to its horizontal size. Modeling results highlight the importance of the spatial relationship between the urban area and the aquifer in determining the 36 37 urban groundwater TTDs. Being computationally efficient, the proposed analytical model can aid decision-making in urban freshwater resources management and urban planning. 38

39

40 **1** Introduction

Rapid urbanization threatens the safety and sustainability of freshwater resources, 41 especially in developing countries. In 2014, around 3.9 billion people lived in cities, and this 42 number is expected to rise to two-thirds of the global population in 2050 (Connor, 2015). 43 Urbanization and the improvement of living standards lead to increasing demand for water in 44 45 cities (Connor, 2015). Besides, rapid urbanization modifies the water cycle in several ways. Cities extract substantial amounts of water from streams and groundwater and challenge the 46 47 sustainability of regional water resources (Larsen et al., 2016; Ostad-Ali-Askari & Shayannejad, 48 2021). Besides, urbanization also extends the area of impervious surface and prevents 49 groundwater recharge, therefore causing higher flood risks (Golian et al., 2020; Vázquez-Suñé et 50 al., 2005). Finally, urbanization may threaten the downstream surface water body by discharging 51 pollutants from cities.

In this context, urban aquifers play a critical part in sustaining the urban water supply and 52 must be integrated into the strategies of water resources management. On one hand, the 53 54 expansion of impervious areas may reduce groundwater recharge and further reduce baseflows, 55 although the actual baseflow in urban watersheds can be increased by anthropogenic influences (Bonneau et al., 2018; Lerner, 2002; Ostad-Ali-Askari et al., 2019; Vázquez-Suñé et al., 2010). 56 On the other hand, modern construction technologies tend to extend not only the area but also the 57 58 depth of the urban impact region, which typically cuts the aquifer and alters the local and regional groundwater flow field (Attard et al., 2017; Font-Capo et al., 2015; Ostad-Ali-Askari et 59 al., 2020; Pujades et al., 2012). 60

Most of the previous works have investigated the influence of urbanization on 61 groundwater quantity and flux-related aspects (Attard, Winiarski, et al., 2016). Underground 62 63 structures intersecting the confined aquifer may exert a barrier effect on groundwater heads, which can be expressed as the increase in head difference across the underground structure after 64 the construction (Pujades et al., 2012, 2016). This barrier effect may cause many environmental 65 problems such as ground settlements (Xu et al., 2019) and contaminants mobilization (Font-Capo 66 et al., 2015; Jurado et al., 2012). Many other studies have focused on the modification in water 67 budget partitioning and baseflow generation (Bhaskar et al., 2016; Bonneau et al., 2017; Hamel 68 & Fletcher, 2014; Janke et al., 2014; Schwartz & Smith, 2014). Despite these works, few have 69 70 focused on the modification in water transit times in urban aquifers.

71 For a local/regional groundwater system, transit/travel time is defined as the time spent 72 by a water parcel from its entrance as recharge till its discharge into the surface water body (i.e., 73 streams, lakes, or seas). Transit/travel time distribution (TTD) is the distribution of transit times, 74 and it provides a lumped description of the transport and mixing processes in a regional aquifer system. Groundwater TTDs have important implications for the interpretation of tracer data 75 (Benettin et al., 2015; Kuppel et al., 2020; Małoszewski & Zuber, 1982; Stewart et al., 2010; 76 77 Zuber, 1986), biogeochemical processes (Van Der Velde et al., 2010), groundwater vulnerability to nonpoint-source contamination (Basu et al., 2012; Jing et al., 2019; Kumar et al., 2020; Ostad-78 79 Ali-Askar et al., 2018), and groundwater response to external changes including climate change (Engdahl & Maxwell, 2015; Havril et al., 2018; Jing et al., 2020). The mean transit time (MTT) 80 of groundwater in an aquifer system ranges typically from years to decades, which is highly 81 82 relevant to the lag in streamwater response to nonpoint-source pollutants. Employing simple lumped parameter models (Ginn et al., 2009; Małloszewski & Zuber, 1992; McCallum et al., 83

2015), transient travel time models (Asadollahi et al., 2020; Benettin et al., 2017; Heße et al.,
2017), or particle-tracking models (Danesh-Yazdi et al., 2018; Eberts et al., 2012; Jing et al.,
2021; Remondi et al., 2019), many studies have assessed the responses of streamwater to
agricultural nonpoint-source contamination through deducing TTDs.

The climate forcing, topography, geometry, and hydraulic properties of aquifers cause 88 89 TTDs to present unique shapes that reflect the transport and mixing processes within the aquifer 90 (Abrams & Haitjema, 2018; Jing et al., 2019; Leray et al., 2016; Talebmorad et al., 2021). Many 91 analytical solutions have been reported for catchments or aquifers, among which the exponential 92 model is one of the most popular models (Eriksson, 1958; Haitjema, 1995; Luo & Cirpka, 2008; Małoszewski & Zuber, 1982). Analytical models bridge the gap between observations and the 93 94 systems' intrinsic properties, which helps to understand why the real-world aquifer systems 95 deviate from the reference ones (Leray et al., 2016). Unfortunately, most analytical models about 96 groundwater TTDs assume natural conditions, with few considering the disturbance of 97 urbanization on their shape and breadth.

98 Despite some previous studies that have employed the tracer data to infer the transit times 99 in urban catchments and aquifers, these tracer-based studies often apply presumed lumped parameter models that typically fit the natural aquifers rather than urban aquifers (Hrachowitz & 100 101 Clark, 2017; Kuhlemann et al., 2020; McCance et al., 2018; Soulsby et al., 2015). Moreover, 102 presuming simple travel-time models without considering the impact of impervious urban zones may bias the interpretation of tracer data and cause unrealistic results (Jing et al., 2019; 103 McCallum et al., 2015). With the development of modern construction techniques such as the cut 104 and cover method, the urban structure often intersects with the water table and obstructs the 105 106 natural groundwater flow, which could also strongly regulate the transport process and the transit

107	times (Attard, Rossier, et al., 2016; Font-Capo et al., 2015). Consequently, it is useful to derive
108	explicit expressions for groundwater TTDs in urban aquifers accounting for the influence of such
109	impervious urban structures.

In this paper, we aim to quantify the influence of impervious urban construction on the 110 shape and breadth of groundwater TTDs by deriving novel analytical solutions. In doing so, we 111 112 expect to answer the following scientific questions: (1) How does the impervious urban area affect the regional-scale groundwater TTDs? (2) What is the most important factor that causes 113 114 the deviation of groundwater transit times from the pre-urban ones? (3) How can we take 115 adaptive strategies to reduce environmental risks caused by the modification in groundwater 116 transit times? To answer these questions, we start by deriving novel analytical solutions for an 117 idealized aquifer intersected by an urban sealing area. We then verify the analytical solutions by 118 comparing them with numerical simulations and conduct a comprehensive parametric study 119 under different scenarios considering a range of parameters of analytical expressions. Finally, we 120 discuss the results and their implications for urban water resources management and urban planning. 121

122 2 Model and Methods

123 2.1 Problem description

This study focuses on the saturated groundwater system in an unconfined aquifer intersected by an impervious urban area. Compared to deep confined aquifers, shallow unconfined aquifers are more vulnerable to urban constructions and typically interact more actively with surface water, which is of great concern from an environmental perspective. The

geometry and boundary conditions (BCs) of real-world watersheds are typically very complex, 128 which hinders the derivation of analytical solutions (Fig. 1a). For the sake of simplification, we 129 consider an idealized rectangular aquifer bounded by an upgradient natural groundwater divide 130 and a downgradient water body (Fig. 1b and c). The upgradient and downgradient boundaries are 131 assigned with a no-flux BC and a fixed-head BC, respectively. This aquifer is embedded in an 132 133 aquitard and therefore, the lateral boundaries are also impervious. The discharge zone is at the downstream limit, which is normally a body of impounded surface water such as a river, a lake, 134 or a sea. The groundwater flow is at steady state, meaning that the outflow equals the total 135 amount of recharge. We only consider the groundwater flow in the saturated zone, which is 136 assumed to be purely advective. 137

(a) A real-world watershed







Fig. 1. Conceptualization of a regional aquifer system intersected by an urban area. A real-world
watershed (a) is conceptualized into two scenarios: aquifers intersected by a local urban area (b)
and aquifers intersected by a regional urban area (c).

The groundwater flow is horizontal based on the Dupuit-Forchheimer assumption 142 143 (Dupuit, 1863; Forchheimer, 1886). This can be justified since the thickness of the saturated aquifer is typically small relative to its horizontal length, and the vertical component of flow 144 velocity is neglectable relative to the horizontal one. Therefore, the groundwater head and 145 velocity do not change with depth. The upper surface of the unconfined aquifer receives a 146 uniform diffuse recharge, R [LT⁻¹], except for the impervious urban area (Fig. 1b and Fig. 1c). 147 148 An impervious urban area seals the ground and prevents infiltration, and therefore no recharge occurs in this area. This is an idealized case because, in reality, leakages from water mains or 149 sewage systems often significantly recharge groundwater (Vázquez-Suñé et al., 2005). 150 Horizontally, the urban area is rectangular with a length of $2w_A$ at the flow direction (A-A' in 151 Fig. 1) and a length of $2w_B$ perpendicular to the flow direction (B-B' in Fig. 1). The bottom of 152 153 the urban structure can be either higher or lower than the water table under natural conditions.

Depending on the spatial extent of urban areas interacting with groundwater flows, we 154 155 categorize them into local urban areas and regional urban areas. In cases that the urban structure is above the natural water table, it is defined as a local urban area (Fig. 1b). The local urban area 156 does not confine the aquifer section since the phreatic surface exists alongside the local urban 157 area. This urban scenario is named Urban Scenario I. If the urban structure is deep enough to 158 intersect with the water table and large enough to fully penetrate the horizontal boundaries of the 159 aquifer, it is defined as a regional urban area (Fig. 1c), and this scenario is named Urban 160 Scenario II. In this scenario, the urban structure partially confines the aquifer (Fig. 1c). Because 161

this study aims to unveil the alteration of urban construction on regional groundwater TTDs, we
 first introduce the analytical solution for the pre-urban scenario and then derive the analytical
 solutions for the urban scenarios.

165 2.2 Solutions in the pre-urban scenario

We consider a 2-D cross-sectional model for the derivation of analytical solutions in the pre-urban scenario (Fig. 2a). The water table (*h*) against distance (*x*) at this cross-section can be expressed using the Dupuit-Forchheimer ellipse (Dupuit, 1863):

$$h(x) = \sqrt{\frac{R}{K}(L^2 - x^2) + h_L^2}$$
(1)

where *L* is the length of the aquifer [L], *R* is the recharge rate $[LT^{-1}]$, *K* is the saturated hydraulic conductivity $[LT^{-1}]$, and h_L is the fixed head at the downstream boundary.

Given that the groundwater mounding is small to moderate, the groundwater TTDs can be modeled as an exponential function (Haitjema, 1995; Leray et al., 2016; Raats, 1977):

$$\rho(a) = \frac{1}{\tau_p} \exp\left(-\frac{1}{\tau_p}\right) \tag{2}$$

173 where τ_p is the MTT, which is controlled by the system's properties:

$$\tau_p = \frac{\theta \overline{H}}{R} \tag{3}$$

- where θ is the effective porosity of aquifer [-], \overline{H} is the mean saturated thickness [L], and *R* is
- the recharge rate [LT⁻¹]. The mean saturated thickness for the unconfined aquifer, \overline{H} , is expressed
- by Eq. (36) in Appendix A.



(b) Urban Scenario I: local urban area



(c) Urban Scenario II: regional urban area



Fig. 2. Cross-sectional models (A-A' in Fig. 1) of the regional aquifer system in pre-urban and
 urban scenarios.

180 2.3 Solutions in the urban scenarios

In the urban scenario, the urban area divides the A-A' cross-section of the unconfined aquifer into three compartments: the upgradient zone, the urban area, and the downgradient zone (Fig. 2). The upgradient and downgradient zones have a recharge rate of R, whereas the urban area has a reduced recharge rate, R_c (Fig. 2b), which is calculated by:

$$R_c = (1 - 2w_B^*)R (4)$$

185 where $w_B^* = \frac{w_B}{W}$, w_B is half of the length of the urban area perpendicular to the flow direction [L], 186 and *W* is the aquifer length perpendicular to the flow direction [L] (Fig. 1b). Given that the 187 groundwater flow is purely advective, the groundwater TTD can be expressed by the flux-188 weighted average of three sub-systems (Leray et al., 2016):

$$\rho_T(a) = \sum_{i=0}^2 \lambda_i \rho_i(a) \tag{5}$$

189 where λ_i is the weight of the recharge in the *i* th sub-system [-], and $\rho_i(a)$ is the TTD in the *i* th 190 sub-system.

In the following subsections, we derive the explicit expressions of groundwater TTDs in unconfined aquifers intersected by a local urban area (Urban Scenario I) and by a regional urban area (Urban Scenario II), respectively.

194 2.3.1 Urban Scenario I: aquifer intersected by a local urban area

This subsection provides the explicit expressions of groundwater TTDs for three subsystems in Urban Scenario I. In the downgradient zone, the aquifer is recharged by the diffuse recharge on the upper boundary. Besides, it is also recharged by the upgradient section of the aquifer. The total inflow for the downgradient zone, Q_{in} , can be expressed as:

$$Q_{in}(\zeta) = R(L - l + w_A - 4w_A w_B^* + \zeta)$$
(6)

where ζ is the distance to the downgradient end of the urban area [L] and *l* is the distance from the center of the urban area to the downgradient boundary [L] (Fig. 2b). The fluid velocity, $v(\zeta)$, is only dependent on the horizontal position ζ , and it can be expressed as:

$$v(\zeta) = \frac{R(L-l+w_A - 4w_A w_B^* + \zeta)}{\theta h(\zeta)} \tag{7}$$

Because the mounding of the water table is small compared to the length of the aquifer, the mean thickness of the downgradient saturated aquifer, \overline{H}_d , can be used to approximate $h(\zeta)$: $h(\zeta) \approx \overline{H}_d$, and \overline{H}_d can be calculated using Eq. (37) in Appendix A.

205 The transit time of water parcel recharged at the position ζ can be determined through 206 integrating the inverse of $v(\zeta)$:

$$a(\zeta) = \int_{\zeta}^{l-w_A} \left[\frac{R(L-l+w_A-4w_Aw_B^*+\zeta)}{\theta \overline{H}_d} \right]^{-1} d\zeta$$

$$= \tau_d \ln \frac{L-4w_Aw_B^*}{L-l+w_A-4w_Aw_B^*+\zeta}$$
(8)

207 where τ_d is expressed as:

$$\tau_d = \frac{\theta \bar{H}_d}{R} \tag{9}$$

Then, the inverse function of $a(\zeta)$, $\zeta(a)$, is expressed as:

$$\zeta(a) = (L - 4w_A w_B^*) \exp\left(-\frac{a}{\tau_d}\right) - (L - l + w_A - 4w_A w_B^*), \qquad a < a_0 \tag{10}$$

where

208

$$a_0 = \tau_d \ln \frac{L - 4w_A w_B^*}{L - l + w_A - 4w_A w_B^*}$$
(11)

Eq. (10) describes the relationship between the recharge position and its corresponding transit time. Hence, the outflux with an age inferior or equal to a, $Q_{out}(a)$, can be expressed as:

$$Q_{out}(a) = R[l - w_A - \zeta(a)] \tag{12}$$

The cumulative TTD for water parcels recharged from the downgradient zone, $P_d(a)$, equals to the mass fraction of outflux with a transit time inferior or equal to *a* (Etcheverry & Perrochet, 2000):

$$P_d(a) = \frac{Q_{out}(a)}{Q_{out}} \tag{13}$$

where Q_{out} is the total outflux [L²T⁻¹]. The probability density function (pdf) of TTD for

downgradient recharge, $\rho_0(a)$, is then expressed as:

$$\rho_0(a) = \frac{dP_d(a)}{da} = \frac{1}{Q_{out}} \frac{dQ_{out}(a)}{da}$$
(14)

217 Combining Eq. (10), Eq. (12), and Eq. (14), the TTD of water parcels recharged from the 218 downgradient zone can be expressed as:

$$\rho_0(a) = \begin{cases} \frac{L - 4w_A w_B^*}{\tau_d (l - w_A)} \exp\left(-\frac{a}{\tau_d}\right), & a < a_0 \\ 0, & a \ge a_0 \end{cases}$$
(15)

The MTT for water mass recharged from the downgradient zone, τ_0 , is determined by:

$$\tau_0 = \left(1 + \frac{L - l + w_A - 4w_A w_B^*}{l - w_A} \ln \frac{L - l + w_A - 4w_A w_B^*}{L - 4w_A w_B^*}\right) \tau_d \tag{16}$$

In the urban area, the recharge rate is reduced by the impervious urban area as shown in Eq. (4). The derivation of TTDs for water parcels recharged in this zone is similar to that in the downgradient zone, except for the fact that it experiences a lag to reach the discharge area (Małoszewski & Zuber, 1982). The TTD for water recharged from the urban area, $\rho_1(a)$, is expressed as:

$$\rho_1(a) = \frac{L - l + w_A - 4w_A w_B^*}{2w_A \tau_c (1 - 2w_B^*)} \exp\left(-\frac{a - a_0}{\tau_c}\right), \qquad a_0 \le a < a_1 \tag{17}$$

225 where a_0 is determined using Eq. (11), and

219

$$\tau_c = \frac{\theta_0 \bar{H}_c}{R(1 - 2w_B^*)} \tag{18}$$

$$a_1 = a_0 + \Delta a_1 \tag{19}$$

$$\Delta a_1 = \tau_c \ln \frac{L - l + w_A - 4w_A w_B^*}{L - l - w_A}$$
(20)

226 \overline{H}_c is the mean saturated thickness in the urban area [L], which can be expressed as Eq. (38) in 227 Appendix A. The MTT for water parcels recharged from the urban area, τ_1 , is given by:

$$\tau_1 = \left[1 + \frac{L - l - w_A}{2w_A(1 - 2w_B^*)} \ln \frac{L - l - w_A}{L - l + w_A - 4w_A w_B^*}\right] \tau_c + a_0$$
(21)

The derivation of groundwater TTD for the upgradient zone is similar to that in the urban area, which means it also has an exponential form with a shift a_1 :

$$\rho_2(a) = \begin{cases} 0, & a < a_1 \\ \frac{1}{\tau_u} \left(-\frac{a - a_1}{\tau_u} \right), & a \ge a_1 \end{cases}$$
(22)

where

$$\tau_u = \frac{\theta \bar{H}_u}{R} \tag{23}$$

The mean saturated thickness for the upgradient zone, \overline{H}_u , is expressed by Eq. (39) in Appendix A.

Similarly, the MTT for water parcels recharged from the upgradient zone, τ_2 , is expressed as:

$$\tau_2 = \tau_u + a_1 \tag{24}$$

Having derived individual TTDs for three independent sub-systems, we can now assemble them to estimate the overall TTD. The groundwater TTDs for the whole aquifer, $\rho_T(a)$, can be simply expressed as a mass-weighted average of three individual TTDs:

$$\rho_{T}(a) = \begin{cases}
\frac{1}{\tau_{d}} \exp\left(-\frac{a}{\tau_{d}}\right), & a < a_{0} \\
\frac{L - l + w_{A} - 4w_{A}w_{B}^{*}}{\tau_{c}(L - 4w_{A}w_{B}^{*})} \exp\left(-\frac{a - a_{0}}{\tau_{c}}\right), & a_{0} \le a < a_{1} \\
\frac{L - l - w_{A}}{\tau_{u}(L - 4w_{A}w_{B}^{*})} \left(-\frac{a - a_{1}}{\tau_{u}}\right), & a \ge a_{1}
\end{cases}$$
(25)

Eq. (25) can be expressed using a dimensionless form. To that end, we define the following dimensionless parameters:

$$a^* = \frac{a}{\tau_p}, \qquad \tau^* = \frac{\tau}{\tau_p}, \qquad l^* = \frac{l}{L}, \qquad w_A^* = \frac{w_A}{L}, \qquad d^* = \frac{d}{H_L}$$
 (26)

where d is the depth of the urban area below the downstream fixed head [L]. With these

dimensionless parameters, Eq. (25) can be transformed into the following dimensionless form:

$$\rho_{T}(a^{*}) = \begin{cases}
\frac{1}{\tau_{d}} \exp\left(-\frac{\tau_{p}a^{*}}{\tau_{d}}\right), & a < a_{0}^{*} \\
\frac{1 - l^{*} + w_{A}^{*} - 4w_{A}^{*}w_{B}^{*}}{\tau_{c}(1 - 4w_{A}^{*}w_{B}^{*})} \exp\left[-\frac{\tau_{p}(a^{*} - a_{0}^{*})}{\tau_{c}}\right], & a_{0}^{*} \le a < a_{1}^{*} \\
\frac{1 - l^{*} - w_{A}^{*}}{\tau_{u}(1 - 4w_{A}^{*}w_{B}^{*})} \exp\left[-\frac{\tau_{p}(a^{*} - a_{1}^{*})}{\tau_{u}}\right], & a \ge a_{1}^{*}
\end{cases}$$
(27)

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Similarly, the MTT for the whole aquifer can also be expressed by calculating the massweighted average of three individual MTTs:

$$\tau^* = \frac{1}{\tau_p} \left(\frac{l^* - w_A^*}{1 - 4w_A^* w_B^*} \tau_0 + \frac{2w_A^* - 4w_A^* w_B^*}{1 - 4w_A^* w_B^*} \tau_1 + \frac{1 - l^* - w_A^*}{1 - 4w_A^* w_B^*} \tau_2 \right)$$
(28)

where τ_0 , τ_1 , and τ_2 can be determined using Eq. (16), Eq. (21), and Eq. (24), respectively.

247 2.3.2 Urban Scenario II: aquifer intersected by a regional urban area

Unlike the above scenario of a local urban area, the scenario of a regional urban area is defined such that it fully penetrates the horizontal boundaries of the aquifer. Hence w_B^* is a constant of 0.5 rather than a variable. Since the bottom of the urban area is beneath the water table, the aquifer section beneath the regional urban area is essentially confined. The derivation of the analytical solution for Urban Scenario II is similar to that for Urban Scenario I, except for the fact that (1) the groundwater recharge in the urban area is completely suppressed by the impervious urban area and (2) no phreatic surface of groundwater exists in the urban area.

Accordingly, the dimensionless form of groundwater TTDs for Urban Scenario II is expressed as:

$$\rho_{T}(a^{*}) = \begin{cases}
\frac{1}{\tau_{d}} \exp\left(-\frac{\tau_{p}a^{*}}{\tau_{d}}\right), & a < a_{0}^{*} \\
0, & a_{0}^{*} \le a < a_{1}^{*} \\
\frac{1-l^{*}-w_{A}^{*}}{\tau_{u}(1-2w_{A}^{*})} \exp\left[-\frac{\tau_{p}(a^{*}-a_{1}^{*})}{\tau_{u}}\right], & a \ge a_{1}^{*}
\end{cases}$$
(29)

where τ_p , τ_d , and τ_u can be determined using Eq. (3), Eq. (9), and Eq. (23).

$$a_0^* = \frac{\tau_d}{\tau_p} \ln \frac{1 - 2w_A^*}{1 - l^* - w_A^*}$$
(30)

$$a_1^* = a_0^* + \Delta a_1^*, \qquad \Delta a_1^* = \frac{2w_A^*(1-d^*)}{1-l^* - w_A^*}$$
(31)

The MTT for the whole groundwater system, τ^* , is also calculated using the mass-

259 weighted average of two subsystems:

$$\tau^* = \frac{l^* - w_A^*}{1 - 2w_A^*} \tau_0^* + \frac{1 - l^* - w_A^*}{1 - 2w_A^*} \tau_2^*$$
(32)

260 where

$$\tau_0^* = \frac{\tau_d}{\tau_p} \left(1 + \frac{1 - l^* - w_A^*}{l^* - w_A^*} \ln \frac{1 - l^* - w_A^*}{1 - 2w_A^*} \right)$$
(33)

$$\tau_2^* = \frac{\tau_u}{\tau_p} + a_1^* \tag{34}$$

261 2.4 Numerical simulation

We set up numerical models to test the reliability and accuracy of analytical solutions. 262 The numerical codes used in this study are MODFLOW-2005 for the simulation of groundwater 263 flow and MODPATH Version 6 for particle tracking (Harbaugh, 2005; Pollock, 2012). All 264 geometric and hydrogeologic configurations of the numerical model are consistent with the 265 analytical model. Specifically, the test aquifer has a length of 500 m (L=500) with a no-flux base 266 and upstream boundary and a fixed head of 10 m ($h_L = 10$) at the downstream boundary. The 267 aquifer is discretized into a 500×50 mesh in the horizontal and vertical directions. A large 268 269 number (2000 ~5000, depending on the scale of the urban area) of particles are released from the phreatic surface of groundwater. The transit time pdfs are estimated using the histogram of 270 271 transit times of the whole set of particles.

To quantify the relative change in TTDs before and after urbanization, we use the normalized form of the mean transit time, τ^* , and the normalized form of variance of transit times, σ^{2*} , as two summary statistics. σ^{2*} is expressed by:

$$\tau^* = \frac{\tau}{\tau_p}, \qquad \sigma^{2*} = \frac{\sigma^2}{\tau_p^2} \tag{35}$$

where σ^2 is the variance of transit times in the urban scenario, and τ_p^2 is the expected variance in the pre-urban scenario where the variance is equal to the square of the mean. These two summary statistics serve as performance metrics for the parametric and sensitivity study.

We applied two approaches to analyze the parametric sensitivity. First, we conducted a local sensitivity analysis focusing on every single parameter; and evaluated their perspective effects on water tables and TTDs. This helps to better understand how the spatial extent and location of the urban area alter the groundwater transit times. Second, we conducted a global sensitivity analysis (GSA) in the full parameter space as well as considering their interactions using Sobol' variance-based method. GSA can quantify the interactive contributions among parameters to the total variance in TTDs.

As a variance-based approach, Sobol' method has become one of the most popular GSA 286 methods in environmental modeling (Rosolem et al., 2012; Saltelli et al., 1999; Sobol, 2001). 287 Sobol' method relates the fraction of the variability in the entity (i.e., the performance metrics) to 288 the variance in the values of various parameters. It typically produces two indices: the first-order 289 sensitivity indices, S_i , and the total-order sensitivity indices, S_{Ti} . The S_i index specifies the 290 individual contribution of factor *i* to the total variance in performance metrics, whereas the S_{Ti} 291 index quantifies the contribution from both individual parameters and the interactions among 292 them to the total variability in performance metrics (Rosolem et al., 2012). 293

The adjustable ranges of parameters are shown in Table 1. Parameter ranges are set broadly to incorporate potential urban conditions based on the problem conceptualization in Fig. 1 and Fig. 2. Since transit times are independent of d^* , there are five parameters to be evaluated for Urban Scenario I (l^* , w_A^* , w_B^* , R, and K). With w_B^* being fixed, the total number of adjustable parameters for Urban Scenario II is also five (l^* , w_A^* , d^* , R, and K).

Parameter	Expression	Description	Lower bound	Upper bound	Default value
l* (-)	$\frac{l}{L}$	Distance from the center of the urban area to the downstream boundary (normalized)	0.35	0.65	0.50
<i>w</i> [*] _A (-)	$\frac{W_A}{L}$	Half of the urban area length in the flow direction (normalized)	0.05	0.25	0.20
<i>w</i> [*] _B (-)	$\frac{W_B}{W}$	Half of the urban area length perpendicular to the flow direction (normalized)	0.00	0.50	0.30
<i>d</i> * (-)	$rac{d}{h_L}$	Depth of urban structures below the downstream fixed head (normalized)	0.00	0.90	0.50
<i>R</i> (m/yr)	-	Recharge rate	0.10	0.30	0.15
<i>K</i> (m/s)	-	Saturated hydraulic conductivity	1.0×10^{-5}	6.0×10^{-5}	2.0×10^{-5}

299	Table 1 Adjustable ranges and default values of parameters (See Fig. 2 for more schematic
300	representation).

301 **3 Results**

In this section, we first display the detailed comparison between the TTDs derived from analytical and numerical solutions. We also show the results of the sensitivity analysis, including the sensitivity analysis for individual parameters and the GSA using Sobol' method.

305 3.1 Verification of the analytical solution

We first set up six scenarios, ranging from Val1 to Val6, for the verification of analytical solutions. The parameter values for these scenarios are shown in Table 2. Note that Val1 represents the pre-urban scenario, whereas Val2 to Val4 represent Urban Scenario 1, and Val5 to Val6 belong to Urban Scenario 2. As shown in Table 1, the parameter settings in Val5 and Val6 are identical to Val3 and Val 4, respectively, except for the fixed value of w_B^* .

311

 Table 2 Parameter settings for six verification scenarios.

Parameter	Val1	Val2	Val3	Val4	Val5	Val6
l* (-)	0.00	0.50	0.55	0.40	0.55	0.40
$w_{A}^{*}(-)$	0.00	0.20	0.15	0.10	0.15	0.10
$w_{B}^{*}(-)$	0.00	0.10	0.30	0.40	0.50	0.50
d* (-)	-	-	-	-	0.70	0.20
<i>R</i> (m/yr)	0.20	0.10	0.15	0.25	0.15	0.25
<i>K</i> (m/s)	$2.0 imes 10^{-5}$	2.0×10^{-5}	$6.0 imes 10^{-5}$	2.0×10^{-5}	$6.0 imes 10^{-5}$	2.0×10^{-5}

The verification results of analytical solutions for six verification scenarios are shown in 312 Fig. 3. As can be seen from Fig. 3, the derived TTDs using analytical solutions correspond well 313 with those using numerical methods for all six verification scenarios. Some minor discrepancies 314 between analytical and numerical results can be found in Val1, wherein the analytical method 315 seems to slightly underestimate the frequency of early breakthrough and overestimate the 316 317 frequency of the late breakthrough of water parcels. This minor discrepancy could be due to the simplification of using the average value to represent the sloping water table. This discrepancy is 318 less pronounced for urban scenarios since the water table mounding in subsystems is smaller 319 320 than that in the entire aquifer system, which can be further evidenced by the good correspondence between analytical and numerical results from Fig. 3b to f. For both Urban 321 Scenario I and II, the results from analytical solutions fit well with those estimated from 322 numerical models. The urban areas significantly modify the shapes of TTDs, leading to abrupt 323 324 changes in TTDs across the urban area (Fig. 3).



Fig. 3. Verification and comparisons of analytical solutions for groundwater TTDs in six
 verification scenarios.

The simulated flow pathlines marked by their transit times in six verification scenarios are shown in Fig. 4. From this figure, we can see different organizations of flow pathlines in Urban Scenario I (Val2 to Val4) and Urban Scenario II (Val5 and Val6). Phreatic surfaces can be observed in urban areas in Urban Scenario I, whereas in Urban Scenario II, the corresponding aquifer section is confined by the impervious urban area (Fig. 4). Besides, a small portion of flow pathlines with extremely old ages can be observed at the bottom of the aquifer (denoted by the red color), which corresponds with the long tails of TTDs (Fig. 3). Overall, through the

comparisons with the numerical simulations, the analytical solution seems accurate in



336 characterizing groundwater TTDs.



340 3.2 Influence on the water table

337

Groundwater TTDs are closely related to the water table, as higher water tables lead to longer transit times as long as the recharge rate does not change (indicated by Eq. (3)). It is, therefore, necessary to evaluate the influence of the impervious urban area on the water table before evaluating the influence on TTDs. Fig. 5 reveals the influence of urbanization on water tables in two urban scenarios. In Urban Scenario I, the impervious urban area tends to drop the water table compared to the pre-urban scenario. Specifically, the drop of the water table in the upgradient zone is greater than that in the downgradient zone. The position of the urban area, l^* , has a marginal impact on the water table, whereas the lengths in the horizontal directions, w_A^* and w_B^* , strongly control the drop in the water table.

These above-mentioned correlations also hold for Urban Scenario II, except for the depth of the urban area, d^* (Fig. 5). In Urban Scenario I, d^* does not impact the water table because the transmissivity in the urban area depends on the height of the phreatic surface. However, in Urban Scenario II, the upgradient water table increases with d^* , which is because the urban structure diminishes the transmissivity by confining the aquifer and reducing the saturated aquifer thickness.



Fig. 5. Responses of the water table to model parameters in two urban scenarios. Overall, in Urban Scenario I, the urbanization decreases the water table on both the upgradient and the downgradient zones, whereas, in Urban Scenario II, the upgradient water table can be either higher or lower than the pre-urban one, depending on the horizontal extent and depth of the urban area. The distance from the urban area to the downstream boundary appears to have a minor influence on the water table.

This subsection presents a sensitivity analysis to investigate the sensitivity of the groundwater TTDs to individual parameters. Here we present results on the sensitivity for two summary statistics to model parameters (see Appendix B for the sensitivity of the entire pdfs of TTDs).

Sensitivities of the normalized MTT, τ^* , to six model parameters in two scenarios are 369 shown in Fig. 6. Those six model parameters can be classified into construction-related parameters 370 $(l^*, w_A^*, w_B^*, and d^*)$ and aquifer-related parameters (R and K). Our results indicate that l^* has only 371 a minor influence on τ^* in Urban Scenario I, which is consistent with its marginal influence on the 372 water table. In Urban Scenario II, τ^* is more sensitive to l^* than in Urban Scenario I. There is also 373 a positive correlation between τ^* and w_A^* for both two scenarios, which is not surprising since the 374 sealing of urban areas reduces the recharge and slows down the movement of groundwater. What 375 stands out from Fig. 6c is the abrupt drop when the urban area transits from local to regional, i.e., 376 377 when the urban structure interferes with the natural groundwater flow. This is because the confinement of the aquifer section reduces the water transit times across the urban area due to the 378 reduction in aquifer transmissivity. τ^* has a non-monotonous correlation with d^* in Urban 379 Scenario II (Fig. 6c). This is because the regional urban area impacts τ^* in two ways. First, external 380 381 disturbance of the aquifer section through the introduction of urban areas reduces the total pore space and increases the fluid velocity. Accordingly, transit times across the urban area are reduced. 382 383 Second, the urban structure reduces the aquifer transmissivity and lifts the water table in the upgradient zone, therefore increases the transit times of water recharged upgradient. Depending 384

on the interplay between these two cases, τ^* experiences an early drop followed by a rise with d^* , and the minimum τ^* occurs when d^* equals 0.71 (Fig. 6d).





388

Fig. 6. Sensitivities of τ^* to model parameters in two urban scenarios.

Except for those construction-related parameters, TTDs are not independent of the aquiferrelated parameters (*R* and *K*). Specifically, τ^* and σ^{2*} correlate negatively with *R*, whereas they correlate positively with *K*. This can be attributed to the different degrees of changes in water tables with varying *R* and *K* as indicated in Fig. 5g and i.

393	In addition to τ^* , we also investigated the sensitivities of the normalized variance of
394	transit times, σ^{2*} , to model parameters in two scenarios (Fig. 7). As shown in Fig. 7, σ^{2*} is
395	positively correlated with l^* and w_A^* in both scenarios. σ^{2*} also increases with w_B^* in Urban
396	Scenario I (Fig. 7c). Similar to the correlation between τ^* and d^* , σ^{2*} has a non-monotonous
397	correlation with d^* , which is also attributed to the interplay between the increase in transit times
398	across the upgradient zone and the decrease in transit times across the urban area. Two aquifer-
399	related parameters, <i>R</i> and <i>K</i> , however, appear to have minor effects on the total variance in σ^{2*}
400	as indicated by the small fluctuations in τ^* (Fig. 7e and f).



401

Fig. 7. Sensitivities of σ^{2*} to model parameters in two urban scenarios.

403 3.4 Sobol' sensitivity indices for model parameters

The above local sensitivity analysis reveals the relationship between summary statistics and individual parameters. The GSA in this subsection focuses more on disentangling the influences of individual parameters and interactions among them on the summary statistics. Fig. 8 displays Sobol' sensitivity indices for τ^* and σ^{2*} in two urban scenarios. In Urban Scenario I, the variance in τ^* is strongly controlled by the horizontal lengths of the urban area, and it is not sensitive to *R* and *K*. In contrast, σ^{2*} is not only sensitive to horizontal lengths but also the

position of the urban area. In Urban Scenario II, τ^* is most sensitive to w_A^* and d^* with $S_{\tau i}$ of 410 0.67 and 0.38, and σ^{2*} is dominantly controlled by w_A^* , l^* , and d^* . The contributions from 411 aquifer-related parameters, i.e., R and K, to the variances in τ^* and σ^{2*} are quite limited 412 413 compared to those construction-related parameters. In both scenarios, the variances in τ^* are mainly controlled by the direct influence of parameters. By contrast, the indirect influence (i.e., 414 the interaction among parameters) significantly contributes to the variance in σ^{2*} . This indicates 415 416 that the urban area has a potentially greater disturbance on the variance of transit times than the MTT. 417

418 Overall, Sobol' sensitivity analysis reveals that the parameter sensitivity depends on the 419 chosen summary metrics. Despite the urban scenario, τ^* and σ^{2*} appear to be most sensitive to 420 the horizontal size of the urban area. They are also strongly controlled by d^* in urban scenario II.



Fig. 8. Sobol' sensitivity analysis for two summary statistics in two urban scenarios. S_{Ti} denotes total-order sensitivity indices, and S_i denotes first-order sensitivity indices.

424 **4 Discussion**

425 4.1 Implications for urban water resources management

426 Pujades et al. (2012) have defined the barrier effect of underground construction as the

427 *"increase in head loss across the construction area relative to the natural head loss before*

428 *construction*". They found that the underground structure tends to increase the head difference

429 when intersecting the confined aquifers. However, Pujades et al. (2012) have assumed that the

total flux through the aquifer remains unchanged after the construction. Font-Capo et al. (2015)

further developed a method to identify the barrier effect based on the response time of 431 piezometers in pumping tests. Our study suggests that the head loss across the urban area can be 432 433 either higher or smaller than that in the pre-urban case depending on the spatial extent and depth of the urban area (Fig. 5). This does not contradict the findings from Pujades et al. (2012) 434 because their solutions are built on the assumptions of a confined aquifer and an unchanged total 435 436 recharge. By contrast, our study aims to reveal the regional behavior of groundwater transit times affected by a large-scale urban area, wherein the modification of recharge by the impervious area 437 cannot be neglected (Lerner, 2002; Voisin et al., 2018). 438

439 In a rapidly urbanizing world, many agricultural catchments are influenced by the 440 expansion of urban areas. Urbanization introduces additional organic contaminants into the 441 regional groundwater system that threatens the surface water and groundwater quality (Jurado et 442 al., 2012; Larsen et al., 2016). Through the derived analytical solutions, our study further reveals 443 the potentially prolonged MTT in both urban settings compared to pre-urban conditions. This prolonged MTT is mainly due to the prevention of recharge by the impervious zones in Urban 444 Scenario II. In contrast, it may also be induced from the rise in the upgradient water table as 445 indicated by Eq. (23). Meanwhile, the increase in MTT may be partially compensated by the 446 decrease in aquifer transmissivity beneath the constructed zone. The increase in groundwater 447 transit times may cause the lag in the response of stream water to the nonpoint-source 448 contamination (e.g., nitrogen and phosphorus). This lag could amplify the response time of the 449 450 stream water to nonpoint-source contamination, which could further endanger the goal of maintaining good surface water quality (Chen et al., 2014; Jing et al., 2021; Van Meter et al., 451 452 2017).

The increased σ^{2*} in the urban aquifer system indicates the potential divergence in the transit times of water parcels recharged upgradient and downgradient. This implies the fates of nonpoint-source contaminants are also influenced by the impervious urban zone (Parajulee et al., 2019; Soulsby et al., 2015). While the contaminants recharged from the downgradient zone mainly flow in the shallow part of the aquifer and have short transit times, those recharged upgradient may have deep flow paths and tailing behaviors.

459

4.2 Implications for urban planning

A major goal of urban water resources management is to restore the flow regime and 460 freshwater quality to the natural condition, which has been widely accepted as a hydrological 461 objective for urban water resources management (Bonneau et al., 2017; Fatahi Nafchi et al., 462 2021; Jefferson et al., 2017; Poff et al., 1997). As a key component of the water cycle, the 463 restoration of groundwater quantity and quality is critical for achieving this goal. This study 464 shows that urbanization shifts the groundwater system onto a new trajectory by altering the water 465 table and transit times, which may not be recovered to the pre-development condition. With the 466 biased mean and variance of transit times from the natural ones, it is only possible to minimize 467 the urban disturbance to groundwater transit times. This finding is in line with the results of 468 Jefferson et al. 2017, wherein they suggested that the restoration of pre-urban hydrology and 469 water quality is rarely achieved in practice. 470

Our in-depth analysis of the parametric uncertainty with different sensitivity approaches has great implications for the planning of large-scale urban construction projects. It shows that (1) the expansion of urban areas will inevitably exacerbate the deviation of groundwater TTDs, and (2) the interplay between urban area and aquifer must be considered when assessing the

environmental risk of urban construction. When the urban area is large enough to fully penetrate 475 the horizontal boundaries and deep enough to intersect with the water table, there exists an 476 477 optimal depth of construction whereby the disturbance of construction to the MTT is minimal (Fig. 6d). This depth of the urban construction has great implications for urban planning as it 478 suggests the reference depth of construction that disturbs the groundwater transit times the least. 479 480 The sensitivity analysis further highlights that the spatial relationship between the urban area and the aquifer should be primarily accounted for when planning the urban construction project. 481 Accordingly, the suggested analytical model benefits urban planning by providing decision-482 making support for the environmental risk assessment. 483

484 4.3 Advantages and limitations of the analytical model

The proposed analytical model has all distinguishing attributes of close-formed 485 equations. An obvious advantage lies in the fact that all parameters of our analytical model can 486 be derived from hydrogeological and geographic information. That being said, TTDs can be 487 488 directly approximated employing the proposed analytical model without the use of environmental tracers. Another advantage of the analytical solution lies in its robustness in the 489 490 characterization of the discontinuous TTDs, which is often the case in urban groundwater systems. Deriving TTDs by direct sampling of particles suffer from numerical errors (Yang et 491 al., 2021). If the number of particles or the discretization of the time step is insufficient, the 492 493 probability distributions derived from numerical tracking may deviate from reality. In this regard, the analytical approach is more promising in characterizing the discontinuous TTDs. 494 Besides, the high computational efficiency of the proposed analytical model permits the easy 495 implementation of the computationally expensive GSA. 496

As a simplified model relying on certain prior assumptions, the analytical model 497 inevitably has some limitations. The analytical solution is only valid for the homogeneous nature 498 499 of a porous medium and does not apply to highly heterogeneous media wherein the preferential flow in fractures may dominate the groundwater circulation pattern. The analytical model also 500 implicitly assumes that the free surface of the unconfined aquifer can be represented by the water 501 502 table. However, Cheng et al. (2021) found that the water table equation is not self-consistent. Nevertheless, the discrepancy between the free surface and water table is small relative to the 503 504 total saturated thickness and therefore has a marginal influence on the regional groundwater TTDs. Another limitation is that the groundwater mounding has to be small to moderate, 505 otherwise the derived groundwater TTDs would significantly deviate from the reality (Abrams & 506 Haitjema, 2018; Haitjema, 1995). This implies that the groundwater head difference between the 507 upgradient and downgradient boundaries should be primarily evaluated before applying the 508 proposed analytical model. Besides, the analytical model does not account for any leakages from 509 510 water mains or sewage pipes, which may underestimate the total recharge and overestimate the MTT. 511

512 **5 Conclusions**

We have developed an analytical model for groundwater TTDs in urban unconfined aquifers intersected by an impervious urban area. The solution has been derived for a purely advective flow system in a homogeneous aquifer under steady-state conditions. Following the Dupuit-Forchheimer assumption and assuming a small to moderate groundwater mounding, we have derived the explicit expressions for the water tables, the pdfs of TTDs, and the MTTs.

The analytical solution seems correct through the comparisons with numerical solutions 518 using particle tracking. A comprehensive analysis of parametric uncertainty shows that the local 519 impervious urban area lowers down the water table, while the regional impervious urban area 520 can either increase or decrease the upgradient water table. The urban area tends to increase the 521 groundwater MTT despite the urban scenario. Groundwater TTDs show varying degrees of 522 523 sensitivities to model parameters in two urban scenarios. While the groundwater MTT is only sensitive to the horizontal size of the urban area in Urban Scenario I, it is sensitive to both the 524 horizontal size and the vertical extension of the urban area in Urban Scenario II. Furthermore, 525 there appears to be an optimal depth of the impervious urban structure that minimizes the 526 disturbance of urbanization to the groundwater transit times in Urban Scenario II. Accordingly, 527 the spatial relationship between the urban area and the aquifer should be considered in priority 528 when inferring the groundwater TTDs. 529

As a first-order approximation of the complex real-world groundwater system, the developed analytical model complements the existing analytical transit time models and provides additional information on influences from imperviousness and interference with natural groundwater flow. Being computationally efficient and easy to implement, it provides decisionmaking support for regional water resources management and urban planning.

535 Appendix A: Solutions for the mean saturated thickness

536 In the pre-urban scenario, the mean saturated thickness of the whole domain, \overline{H} , can be 537 calculated by integrating Eq. (1):

$$\overline{H} = \frac{h_L}{2} + \sqrt{\frac{R}{K} \left(\frac{L}{2} + \frac{Kh_L^2}{2RL}\right)} \arcsin\frac{L}{\sqrt{L^2 + \frac{Kh_L^2}{R}}}$$
(36)

The derivation of mean saturated thickness in the urban scenario is similar to that in the pre-urban scenario. In this scenario, the mean thickness of the downgradient saturated aquifer, \overline{H}_d , can be expressed by:

$$\bar{H}_{d} = \frac{\sqrt{\frac{R}{K}}}{l - w_{A}} \left\{ \frac{L - 4w_{A}w_{B}^{*}}{2} \sqrt{\psi^{2} - (L - 4w_{A}w_{B}^{*})^{2}} + \frac{\psi^{2}}{2} \arcsin\frac{L - 4w_{A}w_{B}^{*}}{\psi} - \frac{L - l + w_{A} - 4w_{A}w_{B}^{*}}{2} \sqrt{\psi^{2} - [L - l + w_{A}(1 - 4w_{B}^{*})]^{2}} - \frac{\psi^{2}}{2} \arcsin\frac{L - l + w_{A} - 4w_{A}w_{B}^{*}}{\psi} \right\}$$

$$(37)$$

541 where
$$\psi = \sqrt{(L - 4w_A w_B^*)^2 + \frac{\kappa}{R} h_L^2}$$
.

542

The mean saturated thickness in the urban area, \overline{H}_c , can be expressed as:

$$\overline{H}_{c} = \frac{\sqrt{\frac{R}{K}}}{2w_{A}(1-2w_{B}^{*})} \left[\frac{L-l+w_{A}-4w_{A}w_{B}^{*}}{2} \sqrt{\psi - (L-l+w_{A}-4w_{A}w_{B}^{*})^{2}} + \frac{\psi^{2}}{2} \arcsin \frac{L-l+w_{A}-4w_{A}w_{B}^{*}}{\psi} - \frac{L-l-w_{A}}{2} \sqrt{\psi^{2} - (L-l-w_{A})^{2}} - \frac{\psi^{2}}{2} \arcsin \frac{L-l-w_{A}}{\psi} \right]$$
(38)

543 where
$$\psi = \sqrt{(L - 4w_A w_B^*)^2 + \frac{\kappa}{R} h_L^2}$$
.

The mean saturated thickness for the upgradient zone, \overline{H}_u , is expressed as:

$$\overline{H}_{u} = \frac{h_{u}}{2} + \sqrt{\frac{R}{K}} \left[\frac{L - l - w_{A}}{2} + \frac{Kh_{u}^{2}}{2R(L - l - w_{A})} \right] \arcsin \frac{L - l - w_{A}}{\sqrt{(L - l - w_{A})^{2} + \frac{Kh_{u}^{2}}{R}}}$$
(39)

545 where h_u is the water table at the downgradient end of the upgradient zone [L], which is given 546 by:

$$h_u = \sqrt{\frac{4Rw_A}{K}(L - l - 2w_A w_B^*) + h_d^2}$$
(40)



547 Appendix B: Full transit time pdfs in two urban scenarios



Figure A1. Transit time pdfs using sampled parameter values in Urban Scenario I.



Figure A2. Transit time pdfs using sampled parameter values in Urban Scenario II.

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