Uncertainty and Sustainability in the Management of Rangelands

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Abstract. We analyze a dynamic and stochastic ecological-economic model of grazing management in semi-arid rangelands. The ecosystem is driven by stochastic precipitation. A risk averse farmer chooses a grazing management strategy under uncertainty such as to maximize expected utility from farming income. Grazing management strategies are rules about which share of the rangeland is given rest depending on the actual rainfall in that year. In a first step we determine a myopic farmer's optimal grazing management strategy and show that a risk averse farmer chooses a strategy such as to obtain insurance from the ecosystem: the optimal strategy reduces income variability, but yields less mean income than possible. In a second step we analyze the long-run ecological and economic impact of different strategies. We conclude that a myopic farmer, if he is sufficiently risk-averse, will choose a sustainable grazing management strategy, even if he does not take into account long-term ecological and economic benefits of conservative strategies.

JEL-Classification: Q57, Q12, Q24

Keywords: Ecological-economic model, semi-arid rangeland, grazing management, risk aversion, uncertainty, sustainability

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[*]The contributions of all authors are equally important. We thank Hans-Peter Weikard, Ralph Winkler, Christian Wissel, participants of the 8th Biennial Conference of the International Society for Ecological Economics (Montreal, June 2004) as well as the ESF-Workshop *Ecological-economic modelling for biodiversity conservation* (Leipzig, September 2004), and three anonymous reviewers for helpful discussion and suggestions. Maximilian Mihm has helped improve language and style. Financial support from the Volkswagen Foundation under grant II/79 628 and from the German Academic Exchange Service (DAAD) is gratefully acknowledged.

1 Introduction

There is a widely held belief that individual myopic optimization is at odds with long-term sustainability of an ecological-economic system. In this paper, we want to take a fresh look at this position. We show that for typical ecosystems and under plausible and standard assumptions about individual decision making, myopic optimization may lead to sustainable outcomes. In particular, in order to explain the sustainable use of ecosystems, it is not necessary to assume preferences for sustainability – or any special concern for the distant future – on the part of the decision maker; it suffices to assume that a myopic decision maker is sufficiently risk averse.

The ecological-economic system under study here is grazing in semi-arid rangelands. Semi-arid regions cover one third of the Earth's land surface. They are characterized by low and highly variable precipitation. Their utilization in livestock farming provides the livelihood for a large part of the local populations. Yet, over-utilization and non-adapted grazing strategies lead to environmental problems such as desertification.

Grazing in semi-arid rangelands is a prime object of study for ecological economics, as the ecological and economic systems are tightly coupled (e.g. Beukes et al. 2002, Heady 1999, Janssen et al. 2004, Perrings 1997, Perrings and Walker 1997, 2004, Westoby et al. 1989). The grass biomass is directly used as forage for livestock, which is the main source of income; and the grazing pressure from livestock farming directly influences the ecological dynamics. The crucial link is the grazing management.

The ecological dynamics, and thus, a farmer's income, essentially depend on the low and highly variable rainfall. The choice of a properly adapted grazing management strategy is crucial in two respects: first, to maintain the rangeland system as an income base, that is, to prevent desertification; and second, to smooth out income fluctuations, in particular, to avoid high losses in the face of droughts.

Assuming that the farmer is non-satiated in income and risk averse, we analyze the choice of a grazing management strategy from two perspectives. In a first step we determine a myopic farmer's optimal grazing management strategy. We show that a risk averse farmer chooses a strategy in order to obtain 'insurance' from the ecosystem (Baumgärtner and Quaas 2005). That is, the optimal strategy reduces income variability, but yields less mean income than possible. In a second step we analyze the long-term ecological and economic impact of different strategies. We conclude that the more risk averse a myopic farmer is, the more conservative is his optimal grazing management strategy. If he is sufficiently risk averse, the optimal strategy is conservative enough to be sustainable.

Following the literature on grazing management under uncertainty, we analyze the choice of a *stocking rate* of livestock, as this is the most important aspect of rangeland management (e.g. Hein and Weikard 2004, Karp and Pope 1984, McArthur and Dillon 1971, Perrings 1997, Rodriguez and Taylor 1988, Torell et al. 1991, Westoby et al. 1989). The innovative analytical approach taken here is to consider the choice of a *grazing management strategy*, which is a *rule* about the stocking rate to apply in any given year depending on the rainfall in that year. This is inspired by empirical observations in Southern Africa. Rule-based grazing management has the twofold advantage that a farmer has to make a choice (concerning the rule) only once, and yet, keeps a certain flexibility and scope for adaptive management (concerning the stocking rate). The flexibility thus obtained is the decisive advantage of choosing a constant rule over choosing a constant stocking rate.

The paper is organized as follows. In Section 2, we discuss grazing management in semi-arid rangelands in more detail and describe one particular 'good practice'-example: the Gamis Farm, Namibia. In Section 3, we develop a dynamic and stochastic ecological-economic model, which captures the essential aspects and principles of grazing management in semi-arid rangelands, and features the key aspect of the Gamis-strategy. Our results are presented in Section 4, with all derivations and proofs given in the Appendix. Section 5 concludes.

2 Grazing management in semi-arid rangelands: The Gamis Farm, Namibia

The dynamics of ecosystems in semi-arid regions are essentially driven by low and highly variable precipitation (Behnke et al. 1993, Sullivan and Rhode 2002, Westoby et al. 1989).[1] Sustainable economic use of these ecosystems requires an adequate adaption to this environment. The only sensible economic use, which is indeed predominant (Mendelsohn et al. 2002), is by extensive livestock farming. However, over-utilization and inadequate management lead to pasture degradation and desertification. Rangeland scientists have proposed different types of grazing management strategies in order to solve these problems. A low constant stocking rate was recommended by Lamprey (1983) and Dean and Mac Donald (1994), who assumed that grazing pressure is the main driving force for vegetation change and that rangeland systems reach an equilibrium state. Other authors considered the highly variable rainfall to be the major driving force and claimed that grazing has only marginal influence on vegetation dynamics (Behnke et al. 1993, Scoones 1994, Sandford 1994, Westoby et al. 1989). They recommend an 'opportunistic' strategy which matches the stocking rate with the available forage in every year. Thus, the stocking rate should be high in years with sufficient rainfall, and low when there is little forage in dry years (Beukes et al. 2002: 238). Recent studies have shown that both grazing and variable rainfall are essential for the vegetation dynamics on different temporal and spatial scales (Cowling 2000, Briske et al. 2003, Fuhlendorf and Engle 2001, Illius and O'Connor 1999, 2000, Vetter 2005).

One example of a sophisticated and particularly successful grazing management system has been employed for forty years at the Gamis Farm, Namibia (Müller et al. forthcoming, Stephan et al. 1996, 1998a, 1998b). The Gamis Farm is located 250 km southwest of Windhoek in Namibia (2405′S 1630′E) close to the Naukluft mountains at an altitude of 1,250 m. The climate of this arid region is characterized by low mean annual precipitation (177 mm/y) and high variability (variation coefficient: 56 %). The vegetation type is dwarf shrub savanna (Giess 1998); the grass layer is dominated by the perennial grasses *Stipagrostis uniplumis*, *Eragrostis nindensis* and *Triraphis ramosissima* (Maurer 1995).

Karakul sheep (race Swakara) are bred on an area of 30,000 hectares. The primary source of revenue is from the sale of lamb pelts. Additionally, the wool of the sheep is sold. In good years, up to 3,000 sheep are kept on the farm. An adaptive grazing management strategy is employed to cope with the variability in forage. The basis of the strategy is a rotational grazing scheme: the pasture land is divided into 98 paddocks, each of which is

[1] Another important driver of ecological dynamics in semi-arid rangelands is the stochastic occurrence of fire (Janssen et al. 2004, Perrings and Walker 1997, 2004). In our case, fire plays only a minor role, but the stochasticity of rainfall is crucial (Müller et al. forthcoming).

grazed for a short period (about 14 days) until the palatable biomass on that paddock is used up completely, and then is rested for a minimum of two months. This system puts high pressure on the vegetation for a short time to prevent selective grazing (Batabyal and Beladi 2002, Batabyal et al. 2001, Heady 1999). While such a rotational grazing scheme is fairly standard throughout semi-arid regions, the farmer on the Gamis Farm has introduced an additional resting: in years with sufficient precipitation one third of the paddocks are given a rest during the growth period (September – May). In years with insufficient rainfall this rest period is reduced or completely omitted. Once a year, at the end of the rainy season (April), the farmer determines – based on actual rainfall and available forage – how many paddocks will be rested and, thus, how many lambs can be reared. This strategy is a particular example of what has been called 'rotational resting' (Heady 1970, 1999, Stuth and Maraschin 2000, Quirk 2002).

The grazing management system employed at the Gamis Farm has been successful over decades, both in ecological and economic terms. It, therefore, represents a model for commercial farming in semi-arid rangelands.

3 The model

Our analysis is based on an integrated dynamic and stochastic ecological-economic model, which captures essential aspects and principles of grazing management in semi-arid regions. It represents a dynamic ecosystem, which is driven by stochastic precipitation, and a risk averse farmer, who rationally chooses a grazing management strategy under uncertainty.

3.1 Precipitation

Uncertainty is introduced into the model by the stochasticity of rainfall, which is assumed to be an independent and identically distributed (iid) random variable. For semi-arid areas, a log-normal distribution of rainfall $r(t)$ is an adequate description (Sandford 1982).[2] The log-normal distribution, with probability density function $f(r)$ (Equation A.25), is determined by the mean μ_r and standard deviation σ_r of precipitation. Here, we measure precipitation in terms of 'ecologically effective rain events', i.e. the number of rain events during rainy season with a sufficient amount of rainfall to be ecologically productive (Müller et al. forthcoming).

3.2 Grazing management strategies

The farm is divided into a number $I \in \mathbb{N}$ of identical paddocks, numbered by $i \in \{1, \ldots, I\}$. In modeling grazing management strategies, we focus on the aspect of additional resting during the growth period, which is the innovative element in the Gamis grazing system. That is, we analyze rotational resting of paddocks from year to year, but do not explicitly consider rotational grazing during the year (cf. Section 2). The strategy is applied in each year, after observing the actual rainfall at the end of the rainy season. Its key feature is that in dry years all paddocks are used, while in years with sufficient

[2]While the distribution of rainfall $r(t)$ is exogenous, all other random variables in the model follow an induced distribution.

rainfall a pre-specified fraction of paddocks is rested. Whether resting takes place, and to what extent, are the defining elements of what we call the farmer's grazing management strategy:

Definition 1
A *grazing management strategy* $(\alpha, \underline{r})$ is a rule of how many paddocks are not grazed in a particular year given the actual rainfall in that year, where $\alpha \in [0, 1]$ is the fraction of paddocks rested if rainfall exceeds the threshold value $\underline{r} \in [0, \infty)$.[3]

Thus, when deciding on the grazing management strategy, the farmer decides on two variables: the rain threshold $\underline{r}$ and the fraction α of rested paddocks. While the rule is constant (i.e. $\alpha =$ const., $\underline{r} =$ const.) its application may yield a different stocking with livestock in any given year depending on actual rainfall in that year.

In the resource economics literature, this type of strategy is called 'proportional threshold harvesting' (Lande et al. 2003). This is a form of adaptive management: the (constant) rule adapts the fraction of fallow paddocks and the number of livestock kept on the farm as actual rainfall changes. Note that the 'opportunistic' strategy (e.g. Beukes et al. 2002: 238) is the special case without resting, i.e. $\alpha = 0$.

3.3 Ecosystem dynamics

Both the stochastic rainfall and grazing pressure are major determinants of the ecological dynamics. Following Stephan et al. (1998a), we consider two quantities to describe the state of the vegetation in each paddock i at time t: the green biomass $G^i(t)$ and the reserve biomass $R^i(t)$ of a representative grass species,[4] both of which are random variables, since they depend on the random variable rainfall. The green biomass captures all photosynthetic ('green') parts of the plants, while the reserve biomass captures the non-photosynthetic reserve organs ('brown' parts) of the plants below or above ground (Noy-Meir 1982). The green biomass grows during the growing season in each year and dies almost completely in the course of the dry season. The amount $G^i(t)$ of green biomass available on paddock i in year t after the end of the growing season depends on rainfall $r(t)$ in the current year, on the reserve biomass $R^i(t)$ on that paddock, and on a growth parameter w_G:

$$G^i(t) = w_G \cdot r(t) \cdot R^i(t). \tag{1}$$

As the green biomass in the current year does not directly depend on the green biomass in past years, it is a flow variable rather than a stock.

In contrast, the reserve biomass $R^i(t)$ on paddock i in year t is a stock variable. That is, the reserve biomass parts of the grass survive several years ('perennial grass'). Thereby, the dynamics of the vegetation is not only influenced by the current precipitation, but also depends on the precipitation of preceding years (O'Connor and Everson 1998). Growth of the reserve biomass from the current year to the next one is

$$R^i(t+1) - R^i(t) = -d \cdot R^i(t) \cdot \left(1 + \frac{R^i(t)}{K}\right) + w_R \cdot (1 - c \cdot x^i(t)) \cdot G^i(t) \cdot \left(1 - \frac{R^i(t)}{K}\right), \tag{2}$$

[3] We assume that the number I of paddocks is so large that we can treat α as a real number.

[4] We assume that selective grazing is completely prevented, i.e. there is no competitive disadvantage for more palatable grasses (see e.g. Beukes et al. 2002). Hence, we consider a single, representative species of grass.

where w_R is a growth parameter and d is a constant death rate of the reserve biomass, which we assume to be sufficiently small, i.e. $d < w_R\, w_G\, \mu_r$. A density dependence of reserve biomass growth is captured by the factors containing the capacity limits K: The higher the reserve biomass on paddock i, the slower it grows. The status variable x^i captures the impact of grazing on the reserve biomass of paddock i. If paddock i is grazed in year t, we set $x^i(t) = 1$, if it is rested, we set $x^i(t) = 0$. The parameter c (with $0 \leq c \leq 1$) describes the amount by which reserve biomass growth is reduced due to grazing pressure. For simplicity, we assume that the initial ($t = 1$) stock of reserve biomass of all paddocks is equal,

$$R^i(1) = R \quad \text{for all } i = 1, \ldots, I. \tag{3}$$

3.4 Livestock and income

As for the dynamics of livestock, the herd size $S(t)$, that can be kept on the farm at time t, is limited by total available forage. We normalize the units of green biomass in such a way that one unit of green biomass equals the need of one livestock unit per year. Thus, total available green biomass on the farm, $\sum_{i=1}^{I} G^i(t)$, determines the 'carrying capacity', i.e. the maximum number of livestock that can be held on the farm in the period under consideration.[5] In general, the farmer will not stock up to this carrying capacity in every year. Rather, the herd size kept on the farm in period t is given by

$$S(t) = \sum_{i=1}^{I} x^i(t) \cdot G^i(t) \ . \tag{4}$$

That is, the herd size in year t is determined by the total green biomass of the paddocks used for grazing (i.e., not rested) in that year. For the sake of the analysis, we assume that the farmer annually rents his livestock on a perfect rental market for livestock.[6] This allows the farmer to exactly adapt the actual herd size to the available forage and to his chosen grazing management strategy.[7]

The herd size $S(t)$ kept on the farm in year t determines the farmer's income $y(t)$. We assume that the quantity of marketable products from livestock, e.g. lamb furs and wool, is proportional to the herd size. Normalizing product units in an appropriate way, the numerical value of output equals livestock $S(t)$. The farmer sells his products on a world market at a given price and takes the annual rental rate of livestock as given. The difference between the two is the net revenue per livestock unit, p. Assuming that farming is profitable, i.e. $p > 0$, the farmer's income $y(t)$ is

$$y(t) = p \cdot S(t). \tag{5}$$

[5] In contrast to the capacity limit K of reserve biomass, the carrying capacity of livestock is not a constant, but it depends on rainfall and the stock of reserve biomass (cf. Equation 1), and, therefore, will change over time.

[6] If the farmer owns a constant herd of size S_0, he would rent a number $S(t) - S_0$ if $S(t) > S_0$ or rent out a number $S_0 - S(t)$ if $S(t) < S_0$. Without loss of generality, we set $S_0 = 0$.

[7] Hence, the herd size $S(t)$ does not follow its own dynamics, but it is determined by precipitation and the chosen strategy.

Since the herd size $S(t)$ is a random variable, income $y(t)$ is a random variable, too.[8] In order to simplify the notation in the subsequent analysis, we normalize

$$p \equiv (w_G \cdot I \cdot R)^{-1}. \tag{6}$$

This means, from now on we measure net revenue per livestock unit in units of total forage per unit of precipitation. As a result, income is measured in units of precipitation.

For the subsequent analysis of a myopic farmer's decision, first and second year income are of particular interest. Given the actual rainfall $r(1)$ in the first grazing period, the initial reserve biomass (Equation 3) and a grazing management rule $(\alpha, \underline{r})$, the herd size $S(1)$ is determined by Equation (4). Inserting Equation (1) and using Assumption (3), as well as normalization (6), the farmer's income $y(1)$ in the first grazing period is given by Equation (5) as

$$y(1) = \frac{1}{I} \sum_{i=1}^{I} x^i(1) \cdot r(1) = \left\{ \begin{array}{cl} r(1) & \text{if} \quad r(1) \leq \underline{r} \\ (1-\alpha) \cdot r(1) & \text{if} \quad r(1) > \underline{r} \end{array} \right. . \tag{7}$$

Given the probability density distribution $f(r)$ of rainfall, the mean $\mu_{y(1)}(\alpha, \underline{r})$ and the standard deviation $\sigma_{y(1)}(\alpha, \underline{r})$ of the first period's income are (see Appendix A.1)

$$\mu_{y(1)}(\alpha, \underline{r}) = \mu_r - \alpha \int_{\underline{r}}^{\infty} r\, f(r)\, dr \tag{8}$$

$$\sigma_{y(1)}(\alpha, \underline{r}) = \sqrt{\sigma_r^2 + 2\,\alpha\,\mu_r \int_{\underline{r}}^{\infty} r\, f(r)\, dr - \alpha^2 \left[\int_{\underline{r}}^{\infty} r\, f(r)\, dr \right]^2 - \alpha\,(2-\alpha) \int_{\underline{r}}^{\infty} r^2\, f(r)\, dr}, \tag{9}$$

where μ_r and σ_r are the mean and the standard deviation of rainfall.

The model implies that resting in the first period has a positive impact on reserve biomass and, thus, on future income. In particular, if the farmer applies a grazing management strategy $(\alpha, \underline{r})$ with $\alpha > 0$ and $\underline{r} < \infty$, rather than full stocking, he can gain an extra income in the second year. Given the actual rainfall $r(1)$ in the first year, the additional reserve biomass in the second year is (cf. Equations 1, 2 and 3)

$$\Delta R = w_R \cdot w_G \cdot I \cdot R \cdot \left(1 - \frac{R}{K}\right) \cdot r(1) \cdot \left\{ \begin{array}{cl} 0 & \text{if} \quad r(1) \leq \underline{r} \\ \alpha & \text{if} \quad r(1) > \underline{r} \end{array} \right. . \tag{10}$$

This additional reserve biomass gives rise to extra green biomass growth, and, hence, to additional income in the second year (cf. Equations 1, 4, 5 and 10):

$$\Delta y(2) = w_G \cdot r(2) \cdot \left\{ \begin{array}{cl} 1 & \text{if} \quad r(2) \leq \underline{r} \\ 1-\alpha & \text{if} \quad r(2) > \underline{r} \end{array} \right\} \cdot w_R \cdot \left(1 - \frac{R}{K}\right) \cdot r(1) \cdot \left\{ \begin{array}{cl} 0 & \text{if} \quad r(1) \leq \underline{r} \\ \alpha & \text{if} \quad r(1) > \underline{r} \end{array} \right. . \tag{11}$$

This means, the reserve biomass can be used as a buffer: by applying a grazing strategy with resting, the farmer can shift income to the next year. For a risk averse farmer, this extra income is particularly valuable if the second year is a dry year.

[8] In our analysis, we neglect uncertainty of prices. Including a price stochasticity uncorrelated to rainfall would not alter our results. Including a price stochasticity with a negative correlation to rainfall would most likely reinforce our central result that a risk averse farmer chooses a conservative grazing management strategy, since high stocking rates in good rainy years become less valuable (as indicated by Hein and Weikard 2004).

3.5 Farmer's choice of grazing management strategy

We assume that the farmer's utility only depends on income y, and that he is a non-satiated and risk averse expected utility maximizer. Let

$$U = \sum_{t=1}^{\infty} \frac{\mathcal{E}_t\, u(y(t))}{(1+\delta)^{t-1}} \tag{12}$$

be his von Neumann-Morgenstern intertemporal expected utility function, where δ is the discount rate, the Bernoulli utility function $u(\cdot)$ is a strictly concave function of income y, and $\mathcal{E}_t$ is the expectancy operator at time t. In particular, we employ a utility function with constant relative risk aversion,

$$u(y) = \frac{y^{1-\rho} - 1}{1-\rho}, \tag{13}$$

where $\rho > 0$ is the constant parameter which measures the degree of relative risk aversion (Gollier 2001).

The farmer will choose the grazing management strategy which maximizes his von Neumann-Morgenstern intertemporal expected utility function (12). The basic idea is to regard the choice of a grazing management strategy as the choice of a 'lottery' (Baumgärtner and Quaas 2005). Each possible lottery is characterized by the probability distribution of pay-off, where the pay-off is given by the farmer's income. Given the ecological dynamics, both the mean income and the standard deviation solely depend on the grazing management strategy applied. Thus, choosing a grazing management strategy implies choosing a particular distribution of income.

We assume that the farmer initially, i.e. at $t = 0$ prior to the first grazing period, chooses a grazing management strategy $(\alpha, \underline{r})$, which is then applied in all subsequent years. When choosing the strategy, the farmer does not know which amount of rainfall will actually occur, but he knows the probability distribution of rainfall. As a result, he knows the probability distribution of his income for any possible grazing management strategy. A far-sighted farmer would choose the grazing management strategy that maximizes his intertemporal utility (12), taking into account the effect of the strategy on the ecosystem dynamics, as given by Equations (1) and (2). In particular, he would account for the effect that resting improves the reserve biomass in the long run, compared to a strategy with full stocking. However, our aim is to show that a sufficiently risk averse farmer will choose a conservative strategy, even if he does not consider the long-term benefits. For this sake, we assume that the farmer is myopic in the following sense (Kurz 1987):

Definition 2
A *myopic farmer* neglects the long-term effects of his grazing management strategy on the ecosystem: (i) He assumes that reserve biomass remains constant at the initial level R on all paddocks, irrespective of the chosen strategy, with the exception that (ii) he takes into account the extra income Δy (Equation 11) in a year after resting.

This means, a myopic farmer bases his decision on a very limited consideration of ecosystem dynamics: he only takes into account the short-term buffering function of reserve biomass, while neglecting all long-term ecological impact of the grazing management strategy chosen. Such a myopic farmer considers his income in year $t \geq 2$ to be

$$y(t) = r(t) \cdot \left\{ \begin{array}{cc} 1 & \text{if} \quad r(t) \leq \underline{r} \\ 1-\alpha & \text{if} \quad r(t) > \underline{r} \end{array} \right\} \cdot \\ \cdot \left[1 + w_R \cdot w_G \cdot \left(1 - \frac{R}{K}\right) \cdot r(t-1) \cdot \left\{ \begin{array}{cc} 0 & \text{if} \quad r(t-1) \leq \underline{r} \\ \alpha & \text{if} \quad r(t-1) > \underline{r} \end{array} \right\} \right] . \quad (14)$$

Since the myopic farmer neglects the long-term ecological impact of his grazing strategy, the functional form of how annual income $y(t)$ (Equation 14) depends on actual rainfall and on the chosen strategy, remains constant over time. Furthermore, since precipitation is independent and identically distributed in each year, and the strategy is constant, the mean $\mu_{y(t)}$ and standard deviation $\sigma_{y(t)}$ of the annual income $y(t)$ for $t \geq 2$ are also constant over time.

In order to be able to express the expected instantaneous utility in any year t in terms of the mean and the standard deviation of that year's income, we approximate the probability density function of annual income by a log-normal distribution with the same mean and standard deviation. Using the specification (13) of the Bernoulli utility function $u(y)$, expected instantaneous utility is given by the following explicit expression (see Appendix A.2):

$$\mathcal{E}\, u(y(t)) = \frac{\mu_{y(t)}^{1-\rho} \left(1 + \sigma_{y(t)}^2 / \mu_{y(t)}^2\right)^{-\rho\,(1-\rho)/2} - 1}{1-\rho}. \quad (15)$$

The indifference curves of the farmer's expected instantaneous utility function can be drawn in the mean–standard deviation space. Figure 1 shows such a set of indifference curves for a given degree ρ of relative risk aversion. The indifference curves are increasing

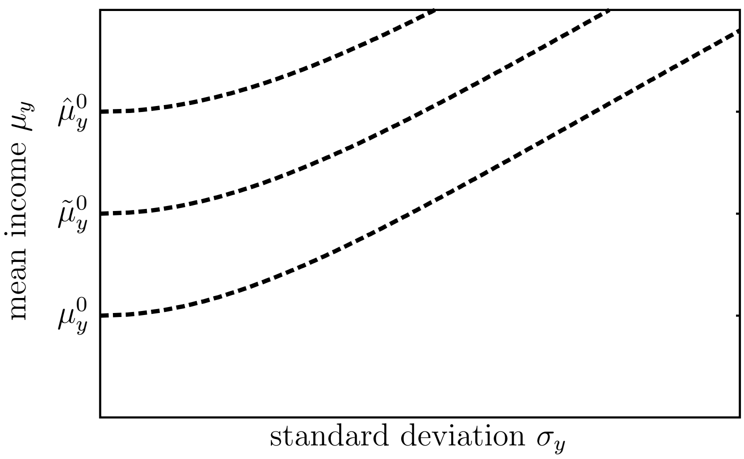

Figure 1: A set of indifference curves of the risk averse farmer in the mean-standard deviation space for log-normally distributed incomes and constant relative risk aversion $\rho = 1$.

and convex if the standard deviation is sufficiently small compared to the mean, i.e. for $(\mu_y/\sigma_y)^2 > 1 + \rho$ (see Appendix A.3). The slope of the indifference curves is increasing in the degree of relative risk aversion ρ (see Appendix A.3). In particular, the indifference curves are horizontal lines for risk-neutral farmers, i.e. for $\rho = 0$.

Formally, the decision problem to be solved by a myopic farmer is to choose a grazing management strategy $(\alpha, \underline{r})$ such as to maximize U (Equation 12) subject to Conditions (7), (14), (15). In the context of semi-arid rangelands, the growth rate of reserve biomass is small, i.e. $w_R \ll 1$. In Appendix A.4 we show that under this condition the farmer's decision problem effectively becomes

$$\max_{(\alpha,\underline{r})} \; \mu_y(\alpha,\underline{r}) \cdot \left(1 + \sigma_y^2(\alpha,\underline{r})/\mu_y^2(\alpha,\underline{r})\right)^{-\rho/2} \; , \tag{16}$$

where the effective mean and standard deviation of income are

$$\mu_y(\alpha,\underline{r}) = \left[\mu_r - \alpha \int_{\underline{r}}^{\infty} r\, f(r)\, dr\right] \cdot \left[1 + \alpha\,\omega \int_{\underline{r}}^{\infty} r\, f(r)\, dr\right] \tag{17}$$

$$\sigma_y(\alpha,\underline{r}) = \sqrt{\sigma_r^2 + 2\,\alpha\,\mu_r \int_{\underline{r}}^{\infty} r\, f(r)\, dr - \alpha^2 \left[\int_{\underline{r}}^{\infty} r\, f(r)\, dr\right]^2 - \alpha\,(2-\alpha) \int_{\underline{r}}^{\infty} r^2\, f(r)\, dr} \cdot \sqrt{1 + 2\,\alpha\,\omega \int_{\underline{r}}^{\infty} r\, f(r)\, dr} \; , \tag{18}$$

with $\omega = w_R \cdot w_G \cdot (1 - R/K)/(1+\delta)$. We analyze this decision problem in the following.

4 Results

The analysis proceeds in three steps (Results 1, 2 and 3 below): First, we analyze the optimization problem of a risk averse myopic farmer who faces a trade-off between strategies which yield a high mean income at a high standard deviation, and strategies which yield a low mean income at a low standard deviation. Second, we analyze the long-term consequences of different grazing management strategies on the ecological-economic system. In particular, we study how the long-term development of the mean reserve biomass and the mean income depend on the strategy. Finally, we put the two parts of the analysis together and derive conclusions about how the long-term sustainability of the short-term optimal strategy depends on the farmer's degree of risk aversion.

4.1 Feasible strategies and income possibility set

To start with, we define the *income possibility set* as the set of all effective mean incomes and standard deviations of income $(\mu_y(\alpha,\underline{r}), \sigma_y(\alpha,\underline{r})) \in (0,\infty) \times [0,\infty)$, which are attainable by applying a feasible management rule $(\alpha,\underline{r}) \in [0,1] \times [0,\infty)$. These are given by Equations (17) and (18). Figure 2 shows the income possibility set for particular parameter values.

The figure provides one important observation: there exist inefficient strategies, i.e. feasible strategies that yield the same mean income, but with a higher standard deviation

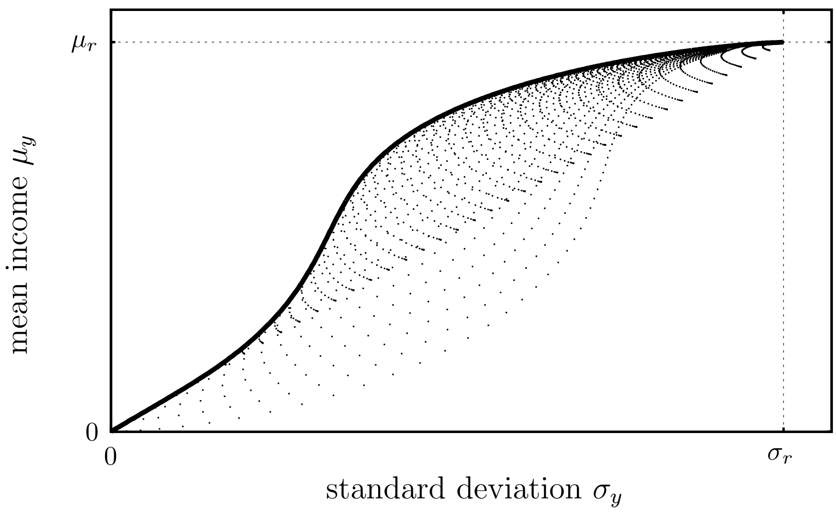

Figure 2: The set of all means μ_y and standard deviations σ_y of the farmer's income y, each point denoting a separate strategy, as well as the income possibility frontier (thick line). Parameter values are $\mu_r = 1.2$, $\sigma_r = 0.7$ and $\omega = 0.14$.

(or: the same standard deviation, but with a lower mean) than others. These strategies can be excluded from the set of strategies from which the optimum is chosen by a risk averse and non-satiated decision maker. In the following, we thus focus on the efficient strategies, which generate the *income possibility frontier* (Figure 2, thick line):

Definition 3
The *income possibility frontier* is the set of expected values μ_y and standard deviations σ_y of income for which the following conditions hold:

1. (μ_y, σ_y) is in the income possibility set, i.e. it is feasible.

2. There is no $(\mu'_y, \sigma'_y) \neq (\mu_y, \sigma_y)$ in the income possibility set with $\mu'_y \geq \mu_y$ and $\sigma'_y \leq \sigma_y$.

The question at this point is, 'What are the grazing management strategies $(\alpha, \underline{r})$ that generate the income possibility frontier?' We call these strategies *efficient*.

Lemma 1
The set of efficient strategies has the following properties.

- *Each point on the income possibility frontier is generated by exactly one (efficient) strategy.*

- *There exists $\Omega \subseteq [0, \infty)$, such that the set of efficient strategies is given by $(\alpha^*(\underline{r}), \underline{r})$ with*

$$\alpha^*(\underline{r}) = \frac{\int_{\underline{r}}^{\infty} r\,(r - \underline{r})\, f(r)\, dr}{\int_{\underline{r}}^{\infty} r\,(r - \underline{r}/2)\, f(r)\, dr} \quad \textit{for all} \quad \underline{r} \in \Omega. \tag{19}$$

- $\alpha^*(\underline{r})$ *has the following properties:*

$$\alpha^*(0) = 1, \quad \lim_{\underline{r}\to\infty} \alpha^*(\underline{r}) = 0, \quad \textit{and} \quad \frac{d\alpha^*(\underline{r})}{d\underline{r}} < 0 \quad \textit{for all} \quad \underline{r} \in \Omega.$$

Proof: see Appendix A.5.

Figure 3 illustrates the lemma. Whereas the set of feasible strategies is the two-

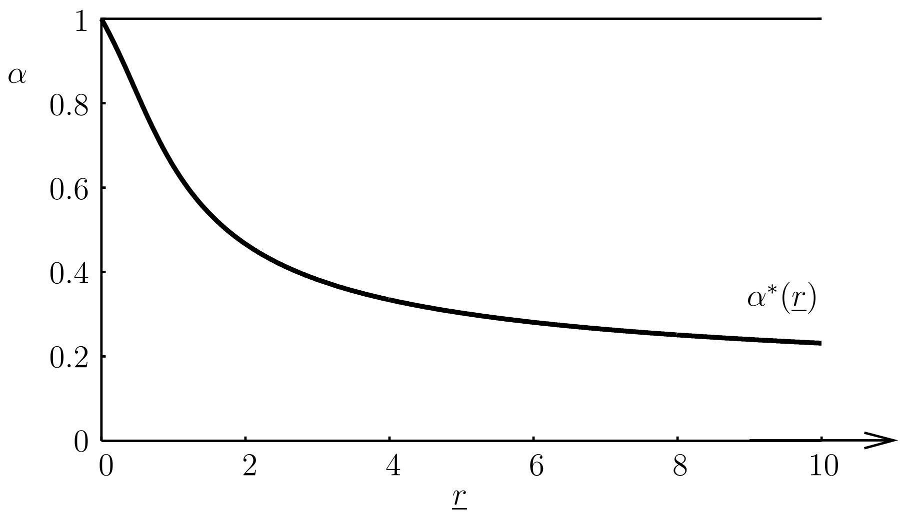

Figure 3: The set of feasible strategies is given by the whole area $\alpha \in [0, 1]$, $\underline{r} \in [0, \infty)$. The set of efficient strategies for parameters $\mu_r = 1.2$ and $\sigma_r = 0.7$ is the curve.

dimensional area bounded by $\underline{r} = 0$, $\alpha = 0$, $\alpha = 1$, the set of efficient strategies, as given by Equation (19), is a one-dimensional curve. Thus, the efficient strategies are described by only one parameter, $\underline{r}$, while the other parameter α is determined by $\alpha = \alpha^*(\underline{r})$ (Equation 19). Alternatively, the inverse function of Equation (19) – which exists by Lemma 1 – specifies the efficient rain threshold $\underline{r}$ as a function of the fraction α of resting. The curve $\alpha^*(\underline{r})$ is downward sloping: With a higher rain threshold $\underline{r}$, i.e. if resting only takes place in years with higher precipitation, the efficient fraction $\alpha^*(\underline{r})$ of rested paddocks is smaller. In other words, for efficient strategies, a higher rain threshold $\underline{r}$ does not only mean that the condition for resting is less likely to be fulfilled, but also that a smaller fraction α^* of paddocks is rested if resting takes place. Hence, if an efficient strategy is characterized by a smaller $\underline{r}$, and, consequently, by a larger $\alpha^*(\underline{r})$, we call it *more conservative.*

Knowledge of the efficient strategies allows us to characterize the income possibility frontier, and to establish a relationship between efficient grazing management strategies and the resulting means and standard deviations of income.

Lemma 2

The farmer's expected income in the first grazing period, $\mu_y(\alpha, \underline{r})$ (Equation 17), is increasing in $\underline{r}$ for all efficient strategies:

$$\frac{d\,\mu_y(\alpha^*(\underline{r}), \underline{r})}{d\underline{r}} > 0 \quad \textit{for all } \underline{r} \in \Omega.$$

The extreme strategies, $\underline{r} = 0$ and $\underline{r} \to \infty$, lead to expected incomes of $\mu_y(\alpha^*(0), 0) = 0$ and $\lim_{\underline{r}\to\infty} \mu_y(\alpha^*(\underline{r}), \underline{r}) = \mu_r$.

Proof: see Appendix A.6.

For all efficient strategies a higher rain threshold $\underline{r}$ for resting, i.e. a less conservative strategy, implies a higher mean income. Whereas no resting, $\underline{r} \to \infty$ (opportunistic strategy), leads to the maximum possible mean income of μ_r, the opposite extreme strategy, $\underline{r} = 0$ (no grazing at all), leads to the minimum possible income of zero. Overall, a change in the grazing management strategy affects both, the mean income and the standard deviation of income.

Lemma 3
The income possibility frontier has the following properties:

- *The income possibility frontier has two corners:*
  - *The southwest corner is at $\sigma_y = 0$ and $\mu_y = 0$. At this point, the income possibility frontier is increasing with slope μ_r/σ_r.*
  - *The northeast corner is at $\sigma_y = \sigma_r$ and $\mu_y = \mu_r$. At this point, the income possibility frontier has a maximum and its slope is zero.*
- *In between the two corners, the income possibility frontier is increasing and located above the straight line from one corner to the other. It is S-shaped, i.e. from southwest to northeast there is first a convex segment and then a concave segment.*

Proof: see Appendix A.7.

Figure 2 illustrates the lemma. With no resting at all (northeast corner of the income possibility frontier), the farmer obtains the highest possible mean income ($\mu_y = \mu_r$), but also faces the full environmental risk ($\sigma_y = \sigma_r$). Conversely, with the most conservative strategy, i.e. no grazing at all (southwest corner of the income possibility frontier), the farmer can completely eliminate his income risk ($\sigma_y = 0$), but also cannot expect any income ($\mu_y = 0$). The property, that the income possibility frontier is increasing, suggests that resting acts like an insurance for the farmer. This means, by choosing a more conservative grazing management strategy, the farmer can continuously decrease his risk (standard deviation) of income, but only at the price of a decreased mean income. Thus, there is an insurance value associated with choosing a more conservative strategy (Baumgärtner and Quaas 2005).

4.2 Optimal myopic strategy

The optimal myopic strategy is obtained by solving Problem (16), and results from both the farmer's preferences (Figure 1) and the income possibility frontier (Figure 2). In mean–standard deviation space, it is determined by the mean μ_y^* and the standard deviation σ_y^*, at which the indifference curve is tangential to the income possibility frontier (Figure 4). It turns out that the optimal strategy is uniquely determined.

Lemma 4
(i) If $(\mu_r/\sigma_r)^2 > 1 + \rho$, the optimum (μ_y^*, σ_y^*) is unique.[9]

[9] This is a sufficient condition which is quite restrictive. A unique optimum exists for a much larger range of parameter values.

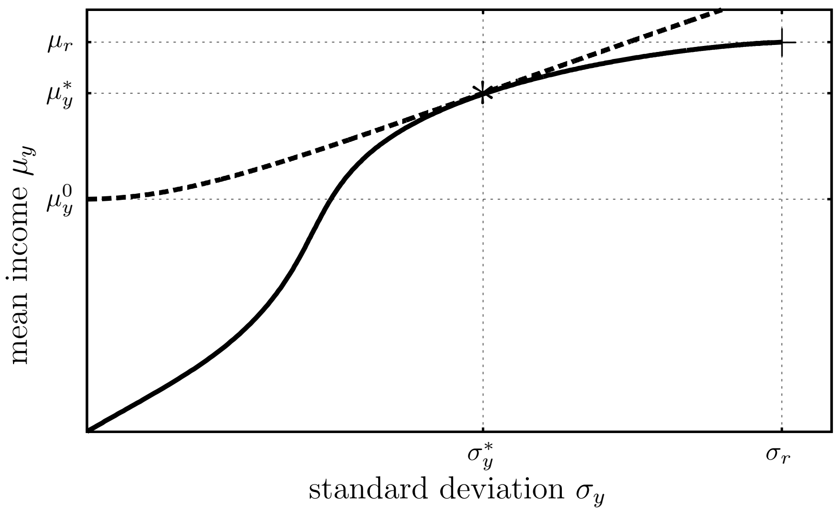

Figure 4: The optimum for a risk averse farmer ($\rho = 5.5$, denoted by $*$) and a risk-neutral farmer ($\rho = 0$, denoted by $+$).

(ii) For $\rho > 0$, the optimum is an interior solution with $0 < \mu_y^* < \mu_r$ and $0 < \sigma_y^* < \sigma_r$. For $\rho = 0$, the optimum is a corner solution with $\mu_y^* = \mu_r$ and $\sigma_y^* = \sigma_r$.

Proof: see Appendix A.8.

The optimal myopic strategy crucially depends on the degree of risk aversion. In the particular case of a risk-neutral farmer ($\rho = 0$), the strategy that yields the maximum mean, irrespective of the standard deviation associated with it, is chosen. The optimal grazing management strategy of such a risk-neutral farmer is the strategy without resting, i.e. with $\underline{r} = \infty$ (and, therefore, $\alpha = 0$). That is, he employs an opportunistic strategy.

If the farmer is risk averse, he faces a trade-off between expected income and variability of the income, because strategies that yield a higher mean income also display a higher variability of income. This leads to the following result, which is illustrated in Figures 4 and 5.

Result 1
A unique interior solution $(\alpha^*(\underline{r}^*), \underline{r}^*)$ to the farmer's decision problem (16), if it exists (see Lemma 4), has the following properties:

(i) The more risk averse the farmer, the smaller are the mean μ_y^* and the standard deviation σ_y^* of his income.

(ii) The more risk averse the farmer, the more conservative is his grazing management strategy:

$$\frac{d\underline{r}^*}{d\rho} < 0 \qquad \text{and} \qquad \frac{d\alpha^*}{d\rho} > 0. \tag{20}$$

Proof: see Appendix A.9.

This means, a risk averse farmer chooses a grazing management strategy such as to obtain insurance from the ecosystem: by choosing a particular grazing management strategy the farmer will reduce his income risk, and carry the associated opportunity

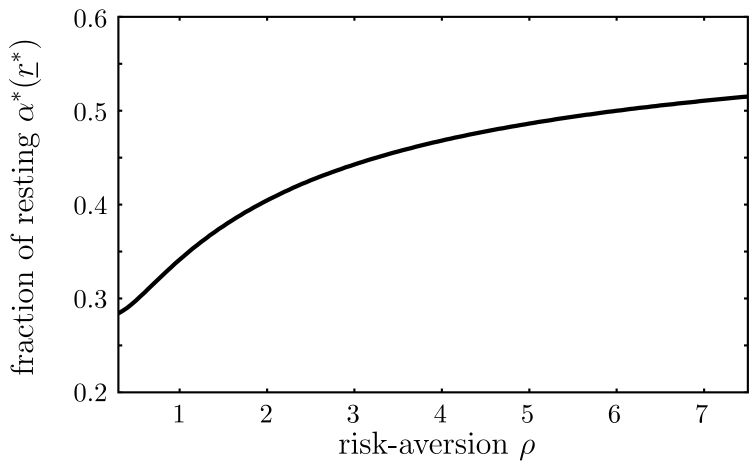

Figure 5: The rain threshold $\underline{r}^*$ of the optimal strategy as a function of the farmer's degree of risk aversion ρ. Parameter values are the same as in Figure 2.

costs in terms of mean income foregone (the 'insurance premium'), to the extent that is optimal according to his degree of risk aversion.

4.3 Long-term impact of grazing management strategies

To study the long-term ecological and economic impact of the grazing management strategy chosen on the basis of myopic optimization (Problem 16), we assume that the farmer continues to apply this strategy in every subsequent period. We compute the resulting probability distribution of income and reserve biomass over several decades in the future. This calculation covers all efficient strategies $(\alpha^*(\underline{r}), \underline{r})$. The results of the numerical computation[10] are shown in Figure 6, which enables the comparison of the long-term impacts, both in ecological and economic terms, of the different strategies that are efficient from the viewpoint of a myopic farmer. In this figure, the mean values $\mu_R(t)$ of reserve biomass and $\mu_y(t)$ of income at different times t are plotted against the efficient fraction $\alpha^\star(\underline{r})$ of resting for different rain thresholds $\underline{r} \in \Omega$. The higher $\alpha^\star(\underline{r})$ is, the more conservative is the respective strategy. Interpreting Figure 6 leads to the following result (see Appendix A.10 for a sensitivity analysis).

Result 2
For parameter values which characterize typical semi-arid rangelands (i.e. w_G, w_R, μ_r are small and c, σ_r are large) the long-term ecological and economic impact of a strategy $(\alpha^\star(\underline{r}), \underline{r})$ is as follows:

(i) The more conservative the strategy, the higher the mean reserve biomass $\mu_R(t)$ in the future:

$$\frac{d\,\mu_R(t)}{d\underline{r}} < 0 \quad \text{and} \quad \frac{d\,\mu_R(t)}{d\alpha} > 0 \quad \text{for all} \quad t > 1.$$

[10]Numerical details are given in Müller et al. (forthcoming).

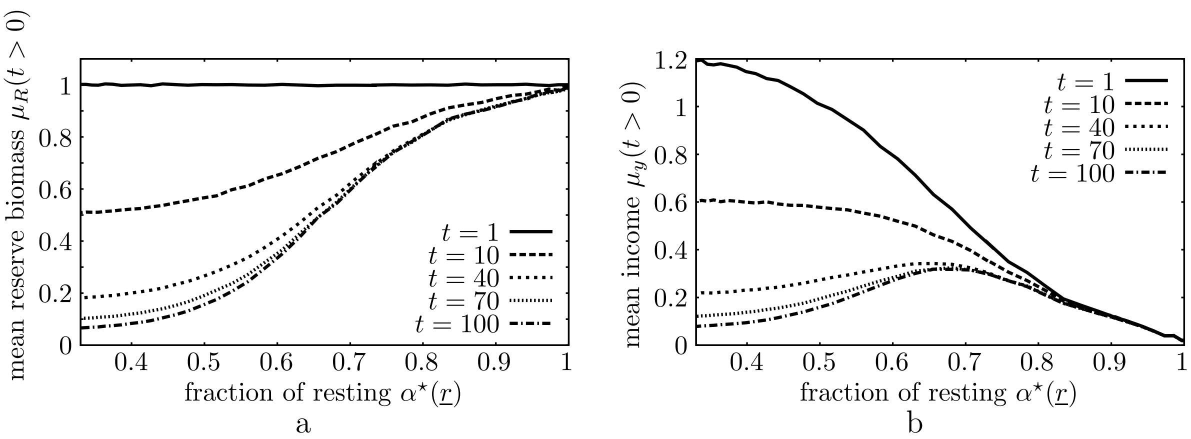

Figure 6: Relation between the grazing management strategy (given by the efficient fraction of resting $\alpha^\star(\underline{r})$) and (a) future mean reserve biomass $\mu_R(t>0)$ (in units of initial reserve biomass), as well as (b) future mean income $\mu_y(t>0)$ for different strategies on the income possibility frontier. Parameter values are $\mu_r = 1.2$, $\sigma_r = 0.7$, $I \cdot K = 8000$, $d = 0.15$, $w_G = 1.2$, $w_R = 0.2$, $c = 0.5$, $I \cdot R = 2400$.

(ii) *For high rain thresholds $\underline{r} > \hat{\underline{r}}$, the following holds: The more conservative the strategy, the higher the mean income $\mu_y(t)$ in the long-term future for $t > \hat{t}$:*

$$\frac{d\,\mu_y(t)}{d\underline{r}} < 0 \quad \textit{and} \quad \frac{d\,\mu_y(t)}{d\alpha} > 0 \quad \textit{for all} \quad t > \hat{t} \quad \textit{and} \quad \underline{r} > \hat{\underline{r}},\ \alpha < \alpha^\star(\hat{\underline{r}}).$$

Result 2 states that the slope of the curves in Figure 6 is positive throughout, as far as reserve biomass is concerned; and is positive for small $\alpha^\star(\underline{r})$, i.e. for $\alpha^\star(\underline{r}) < \alpha^\star(\hat{\underline{r}})$, and $t > \hat{t}$, as far as income is concerned. The higher the fraction $\alpha^\star(\underline{r})$ of paddocks rested, i.e. the more conservative the strategy, the higher is the mean reserve biomass, if the same strategy is applied over the whole period. This effect is in line with intuition: the more conservative the strategy, the better is the state of the rangeland in the future. As far as income is concerned, the argument is less straightforward. In particular, the mean income in the first period is increasing in $\underline{r}$, i.e. decreasing in $\alpha^\star(\underline{r})$ (Lemma 2). A less conservative strategy yields a higher mean income in this period, since more livestock is kept on the rangeland. This holds for several periods in the near future (cf. the line for $t = 10$ in Figure 6b). However, in the long run (for $t > \hat{t} \approx 40$), the strong grazing pressure on the pasture leads to reduced reserve biomass growth and less forage production in the long-term future, compared to a more conservative strategy. As a result, mean income is smaller. This can be seen in Figure 6b: the curves are upward-sloping for sufficiently high $t \geq \hat{t}$ and sufficiently small $\alpha^\star(\underline{r})$. As can be seen in the figure, this effect becomes stronger in the long-term future: the curves are steeper for higher t.

Result 2 holds if the growth rates of the green and reserve biomass are low, the impact of grazing on the growth of the reserve biomass is high, and rainfall is low and highly variable. This is just the range of parameter values which is adequate for semi-arid rangelands, because these are fragile ecosystems which are highly susceptible to degradation if grazing pressure is high. For very robust ecosystems or very low stochasticity of rainfall, however, the result is not valid.

For a large fraction of resting, i.e. $\alpha^\star(\underline{r}) > \alpha^\star(\hat{\underline{r}})$, a more conservative strategy (i.e. a larger $\alpha^\star(\underline{r})$) leads to a lower mean income, not only in the first period (Lemma 2), but also in the future. In this domain of strategies, resting is already so high that the future gains in reserve biomass from additional resting do not outweigh the losses from lower stocking.

While Result 2 describes the dynamic long-term impact of different grazing management strategies, the following lemma analytically extends this result by specifying the steady-state mean values of reserve biomass and income. The steady-state mean value of reserve biomass is determined as the fixed point of the mean vegetation dynamics (according to Equations 1 and 2). The steady-state mean value of reserve biomass, in turn, determines the steady-state mean value of income.[11]

Lemma 5

1. *For an efficient strategy* $(\alpha^\star(\underline{r}), \underline{r})$ *the steady-state mean value of reserve biomass is*

$$\mu_R^{stst} = \max\left\{ K\, \frac{w_G\, w_R\, (\mu_R - c\, \mu_{y(1)}(\alpha^\star(\underline{r}), \underline{r})) - d}{w_G\, w_R\, (\mu_R - c\, \mu_{y(1)}(\alpha^\star(\underline{r}), \underline{r})) + d}, 0 \right\} , \tag{21}$$

and the steady-state mean value of income is

$$\mu_y^{stst} = \frac{\mu_R^{stst}}{R}\, \mu_{y(1)}(\alpha^\star(\underline{r}), \underline{r}) , \tag{22}$$

where $\mu_{y(1)}(\alpha^\star(\underline{r}), \underline{r})$ *is given by Equation* (8), *and* R *is the initial value of reserve biomass.*

2. μ_R^{stst} *is monotonically decreasing in* $\underline{r}$,

$$\frac{d\mu_R^{stst}}{d\underline{r}} < 0 , \tag{23}$$

while μ_y^{stst} *assumes a maximum value at* $\hat{\underline{r}} > 0$, *such that*

$$\frac{d\mu_y^{stst}}{d\underline{r}} < 0 \quad \text{for} \quad \underline{r} > \hat{\underline{r}} . \tag{24}$$

Proof: see Appendix A.11.

For $\underline{r} > \hat{\underline{r}}$, we thus have established the following result: The more conservative the strategy, i.e. the lower $\underline{r}$ and the higher $\alpha^\star(\underline{r})$, the higher the steady-state mean reserve biomass and income in the long run.

As the final step in our analysis, we now relate this insight to the issue of sustainability of grazing management strategies. For the sake of this analysis, we understand *sustainability* in the following way.

Definition 4

A grazing management strategy $(\alpha, \underline{r})$ is called *sustainable*, if and only if it leads to strictly positive steady-state mean values of both reserve biomass and income, $\mu_R^{\rm stst} > 0$ and $\mu_y^{\rm stst} > 0$.

[11] These steady-state mean values represent the trend of the stochastic dynamics, but not the purely random part of the dynamics. The latter could lead, by chance, to irreversible extinction of the reserve biomass in the long-run even when a very conservative strategy is applied.

The notion of *sustainability*, while expressing an idea which seems obvious and clear at first glance, is notoriously difficult to define in an operational way. As a result, there are a multitude of different definitions of 'sustainability', which reveal different aspects and, at bottom, fundamentally different understandings of the term (see e.g. Klauer 1999, Neumayer 2003 and Pezzey 1992 for a detailed discussion). In the framework of our model, Definition 4 captures essential aspects of what has been called 'strong sustainability' (Pearce et al. 1990, Neumayer 2003). It comprises an ecological as well as an economic dimension, with mean reserve biomass as an ecological indicator and mean income as an economic indicator. It expresses the aspect of long-term conservation of an ecological-economic system in the sense that the steady-state mean values of both reserve biomass and income are strictly positive.[12] In contrast, an unsustainable strategy is one that leads to the collapse of the ecological-economic system, in the sense that the steady-state mean value of either reserve biomass or income (or both) is zero. Definition 4 constitutes a rather weak criterion of strong sustainability, by setting the minimum requirements with respect to the steady-state mean values of both reserve biomass and income at zero.[13] Yet, it enables a clear and unambiguous distinction between sustainable and unsustainable strategies in the following manner.

Lemma 6
If $c > 1 - d/(w_G\, w_R\, \mu_r)$, a strategy $(\alpha^\star(\underline{r}'), \underline{r}')$ exists, such that all efficient strategies which are less conservative (i.e. $\underline{r} > \underline{r}'$ and $\alpha^\star(\underline{r}) < \alpha^\star(\underline{r}')$) are unsustainable and all efficient strategies with $\underline{r} > 0$ that are more conservative (i.e. $\underline{r} < \underline{r}'$ and $\alpha^\star(\underline{r}) > \alpha^\star(\underline{r}')$) are sustainable.

Proof: see Appendix A.12

If the impact of grazing on reserve biomass growth is very small, i.e. if $c < 1 - d/(w_G\, w_R\, \mu_r)$, all strategies are sustainable. Long-term degradation of the pasture is only a problem at all when the impact of grazing on the vegetation is high. In this case, there is a clear and unambiguous threshold between strategies that are conservative enough to be sustainable and strategies which are not. From Result 1, we know that the more risk averse a farmer is, the more conservative is his optimal myopic strategy. Combining this result with Lemma 6, we can now make a statement about the relation between a risk averse farmer's myopic decision and its long-term implications in terms of sustainability.

Result 3
If the uncertainty of rainfall, σ_r, is large and the impact of grazing c is not too large, a sufficiently risk averse myopic farmer will choose a sustainable grazing management strategy.

Proof: see Appendix A.13.

Result 3 sheds new light on the question 'How can one explain that people do behave in a sustainable way?' For, Result 3 suggests the following potential explanation. That

[12]Under uncertainty, positive steady-state mean values do not mean that a sustainable strategy will actually yield positive values of reserve biomass and income. For, by chance, a sequence of rain events may occur which drives the reserve biomass to extinction. See Footnote 11.

[13]As an alternative, one could set minimum requirements at strictly positive values, representing e.g. the levels of 'critical natural capital' and 'subsistence income'. We have chosen zero for the sake of analytical clarity.

a farmer A manages an ecosystem in a sustainable manner, while another farmer B does not, may be explained simply by a higher risk aversion of farmer A. In particular, it is not necessary to assume that farmer A has any kind of stronger preferences for future income or sustainability than farmer B. This result holds if (i) uncertainty is large and (ii) the impact of grazing is not too large. If uncertainty were small, it would only play a minor role in the decision making of the farmer. Hence, even a large risk aversion would not induce a myopic farmer to choose a conservative strategy. If, on the other hand, the impact of grazing were very high, the optimal strategy of even a very risk averse myopic farmer would not be conservative enough to ensure sustainability.

For a large standard deviation of rainfall and not too large grazing impacts, the model predicts a critical degree ρ' of risk aversion which separates the myopic farmers choosing a sustainable strategy from those choosing an unsustainable one. This critical degree of risk-aversion characterizes precisely that myopic farmer who chooses the strategy $(\alpha^\star(\underline{r}'), \underline{r}')$, which separates sustainable from unsustainable strategies (Result 1(ii) and Lemma 6). For the parameter values used in our numerical simulations (see the caption of Figure 6), this critical degree of risk aversion is $\rho' = 1.85$, which is well within the range of degrees of risk aversion commonly considered as reasonable (i.e. $\rho \leq 4$; see e.g. Gollier 2001).[14]

5 Conclusions and Discussion

We have developed an integrated dynamic and stochastic ecological-economic model of grazing management in semi-arid rangelands. Within this, we have analyzed the choice of grazing management strategies of a risk averse farmer, and the long-term ecological and economic impact of different strategies. We have shown that a myopic farmer who is sufficiently risk averse will choose a sustainable strategy, although he does not take into account long-term ecological and economic benefits of conservative strategies. The intuition behind this result is that a conservative strategy provides natural insurance for a risk averse farmer. In years with good rainfall the farmer does not fully exploit the carrying capacity of the farm. Due to the buffering function of the reserve biomass of vegetation he thereby can shift income to the next year with possibly worse conditions. The more risk averse the farmer is, the higher is the benefit from this insurance function and the more conservative is his optimal strategy. A sufficiently risk averse farmer chooses a strategy which is conservative enough to be sustainable.

However, one should not conclude from our analysis that risk aversion is sufficient to ensure a sustainable development in semi-arid areas. This issue requires a variety of further considerations. First, one could adopt a more demanding sustainability criterion than we have used (cf. Definition 4). Second, we have focused on environmental risk resulting from the uncertainty of rainfall. Other forms of risk, e.g. uncertainty concerning property-rights, or the stability of social and economic relations in general, might generate a tendency in the opposite direction, and promote a less conservative and less sustainable management of the ecosystem (e.g. Bohn and Deacon 2000). Hence, in the face of different uncertainties, the net effect is not clear and has to be analyzed in detail. Third, additional sources of income (say from tourism) or the availability of financial services (such as savings, credits, or commercial insurance), constitute possibilities for hedging income

[14]If the standard deviation of rainfall is small, or the grazing impact is very large, the threshold value of risk aversion exceeds this range of reasonable degrees of risk aversion.

risk. For farmers, all these are substitutes for obtaining natural insurance by conservative ecosystem management and, thus, may induce farmers to choose less conservative and less sustainable grazing management strategies (Quaas and Baumgärtner 2006). This becomes relevant as farmers in semi-arid regions are more and more embedded in world trade and have better access to global commodity and financial markets.

Our analysis addressed the context of grazing management in semi-arid rangelands. This system is characterized by a strong interrelation between ecology and economic use, which drives the results. While this is a specific ecological-economic system, the underlying principles and mechanisms of ecosystem functioning and economic management are fairly general. Hence, we believe that there are similar types of ecosystems managed for the services they provide, e.g. fisheries or other agro-ecosystems, to which our results should essentially carry over.

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A Appendix

A.1 Mean and standard deviation of the first year's income

The rainfall r is log-normally distributed, i.e. the probability density function is

$$f(r) = \frac{1}{r\sqrt{2\pi s_r^2}} \exp\left(-\frac{(\ln r - m_r)^2}{2s_r^2}\right). \tag{A.25}$$

The two parameters m_r and s_r can be expressed in terms of the mean μ_r and standard deviation σ_r, $m_r = \ln \mu_r - \frac{1}{2} \ln (1 + \sigma_r^2/\mu_r^2)$ and $s_r^2 = \ln (1 + \sigma_r^2/\mu_r^2)$.

Using the probability density function (A.25) of rainfall and Equation (7) for the farmer's first year income, the expected value and the variance of the first year's income are easily calculated. The expected value is

$$\mu_{y(1)}(\alpha, \underline{r}) = \int_0^\infty y(1)\, f(r)\, dr = \int_0^{\underline{r}} r\, f(r)\, dr + (1-\alpha) \int_{\underline{r}}^\infty r\, f(r)\, dr = \mu_r - \alpha \int_{\underline{r}}^\infty r\, f(r)\, dr.$$

The variance is

$$\begin{aligned} \sigma^2_{y(1)}(\alpha, \underline{r}) &= \int_0^\infty \left(y(1) - \mu_{y(1)}\right)^2 f(r)\, dr = -\mu_y^2 + \int_0^{\underline{r}} r^2 f(r)\, dr + (1-\alpha)^2 \int_{\underline{r}}^\infty r^2 f(r)\, dr \\ &= \sigma_r^2 + 2\,\alpha\,\mu_r \int_{\underline{r}}^\infty r\, f(r)\, dr - \alpha^2 \left[\int_{\underline{r}}^\infty r\, f(r)\, dr\right]^2 - \alpha\,(2-\alpha) \int_{\underline{r}}^\infty r^2 f(r)\, dr. \end{aligned}$$

A.2 Expected utility function

With the specification (13) of the farmer's Bernoulli utility function $u(y)$, and the assumption that income is log-normally distributed we get (using the notation $m_y = \ln \mu_y - \frac{1}{2} \ln \left(1 + \sigma_y^2/\mu_y^2\right)$ and $s_y^2 = \ln\left(1 + \sigma_y^2/\mu_y^2\right)$):

$$\begin{aligned} \mathcal{E}\, u(y) &= \int_0^\infty \frac{y^{1-\rho} - 1}{1-\rho} \frac{1}{y\sqrt{2\pi s_y^2)}} \exp\left(-\frac{(\ln y - m_y)^2}{2 s_y^2}\right) dy \\ &\overset{z=\ln y}{=} \frac{1}{1-\rho}\left[\frac{1}{\sqrt{2\pi s_y^2}} \int_{-\infty}^\infty \exp\left((1-\rho)\, z\right) \exp\left(-\frac{(z - m_y)^2}{2 s_y^2}\right) dz - 1\right] \\ &= \frac{\exp\left((1-\rho)\left(m_y + \frac{1-\rho}{2}\, s_y^2\right)\right) - 1}{1-\rho} = \frac{\mu_y^{1-\rho}\left(1 + \sigma_y^2/\mu_y^2\right)^{-\rho\,(1-\rho)/2} - 1}{1-\rho}. \end{aligned}$$

A.3 Properties of the indifference curves

Each indifference curve intersects the μ_y-axis at $\sigma_y = 0$. The point of intersection, μ_0, is the certainty equivalent of all lotteries on that indifference curve. Hence, the indifference curve is the set of all $(\mu_y, \sigma_y) \in \mathbb{R}_+ \times \mathbb{R}_+$ for which

$$\mu_y \left(1 + \sigma_y^2/\mu_y^2\right)^{-\rho/2} = \mu_0. \tag{A.26}$$

The slope of the indifference curve is obtained by differentiating Equation (A.26) with respect to σ_y (considering μ_y as a function of σ_y) and rearranging:

$$\frac{d\mu_y}{d\sigma_y} = \frac{\rho\,\sigma_y\,\mu_y}{(1+\rho)\,\sigma_y^2 + \mu_y^2} > 0. \tag{A.27}$$

The curvature is obtained by differentiating this equation with respect to σ_y, inserting $d\mu_y/d\sigma_y$ again and rearranging

$$\frac{d^2\mu_y}{d\sigma_y^2} = \frac{d}{d\sigma_y}\frac{d\mu_y}{d\sigma_y} = \frac{\rho\,\mu_y\,(\mu_y^2-(1+\rho)\,\sigma_y^2)(\sigma_y^2+\mu_y^2)}{\left((1+\rho)\,\sigma_y^2+\mu_y^2\right)^3}, \tag{A.28}$$

which is positive, if and only if $\mu_y^2 > (1+\rho)\,\sigma_y^2$. Furthermore, the slope of the indifference curves increases with rising risk aversion,

$$\frac{d}{d\rho}\frac{d\mu_y}{d\sigma_y} = \frac{\sigma_y\,\mu_y\,\left(\sigma_y^2+\mu_y^2\right)}{\left((1+\rho)\,\sigma_y^2+\mu_y^2\right)^2} > 0.$$

A.4 Effective decision problem

Using a monotonic transformation of U and employing $\sum_{t=2}^{\infty}(1+\delta)^{1-t} = 1/\delta$, the decision problem becomes

$$\max_{(\alpha,\underline{r})}\ \mu_{y(1)}\ \left(1+\sigma_{y(1)}^2/\mu_{y(1)}^2\right)^{-\rho/2} + \frac{1}{\delta}\,\mu_{y(t)}\ \left(1+\sigma_{y(t)}^2/\mu_{y(t)}^2\right)^{-\rho/2}\ . \tag{A.29}$$

Since rainfall is independent and identically distributed in each year, we find from Equation (14) that mean income is

$$\mu_{y(t)} = \mu_{y(1)}\left[1+\alpha\,w_R\,w_G\,\left(1-\frac{R}{K}\right)\int_{\underline{r}}^{\infty} r\,f(r)\,dr\right]\ . \tag{A.30}$$

When calculating the variance, we neglect terms of second order in w_R, since this is a very small number (see also the specification of parameters in the caption of Figure 6). With this simplification the variance is

$$\sigma_{y(t)} = \sigma_{y(1)}\sqrt{1+2\,\alpha\,w_R\,w_G\,\left(1-\frac{R}{K}\right)\int_{\underline{r}}^{\infty} r\,f(r)\,dr}\ . \tag{A.31}$$

Plugging this into the decision problem (A.29) and again dropping terms of second order in the growth rate of the reserve biomass, we find

$$\begin{aligned}
&\mu_{y(1)}\ \left(1+\sigma_{y(1)}^2/\mu_{y(1)}^2\right)^{-\rho/2} + \frac{1}{\delta}\,\mu_{y(t)}\ \left(1+\sigma_{y(t)}^2/\mu_{y(t)}^2\right)^{-\rho/2} \\
&= \mu_{y(1)}\ \left(1+\sigma_{y(1)}^2/\mu_{y(1)}^2\right)^{-\rho/2}\ \left[1+\frac{1}{\delta}\left[1+\alpha\,w_R\,w_G\,\left(1-\frac{R}{K}\right)\int_{\underline{r}}^{\infty} r\,f(r)\,dr\right]\right] \\
&= \frac{1+\delta}{\delta}\,\mu_{y(1)}\ \left(1+\sigma_{y(1)}^2/\mu_{y(1)}^2\right)^{-\rho/2}\ \left[1+\alpha\,\frac{w_R\,w_G}{1+\delta}\,\left(1-\frac{R}{K}\right)\int_{\underline{r}}^{\infty} r\,f(r)\,dr\right]
\end{aligned} \tag{A.32}$$

Using the abbreviation $\omega = w_R\,w_G\,(1-R/K)/(1+\delta)$, a monotonic transformation of the objective function, i.e. multiplication by $\delta/(1+\delta)$, and, once more, the approximation of dropping second-order terms in ω, one obtains the proposed result.

A.5 Proof of Lemma 1

To find the efficient strategies, we first determine the strategies which minimize the standard deviation of income given the mean income. Out of these strategies those are efficient which maximize the mean income for a given standard deviation. Each point on the income possibility frontier is generated by exactly one efficient strategy, since the solution of the corresponding minimization problem is unique.

Equivalent to minimizing the standard deviation, we minimize the variance for a given mean income,

$$\min_{\alpha,\underline{r}} \sigma_y^2 \quad \text{s.t.} \quad \mu_y \geq \bar{\mu}_y, \ \alpha \in [0,1], \ \underline{r} \in [0,\infty). \tag{A.33}$$

For a more convenient notation, we use the abbreviations

$$R_1(\underline{r}) = \int_{\underline{r}}^{\infty} r\, f(r)\, dr \quad \text{and} \quad R_2(\underline{r}) = \int_{\underline{r}}^{\infty} r^2\, f(r)\, dr. \tag{A.34}$$

The Lagrangian for the minimization problem (A.33) is

$$\begin{aligned}
\mathcal{L} &= \sigma_y^2(\alpha,\underline{r}) + \lambda\,[\mu_y(\alpha,\underline{r}) - \bar{\mu}_y] \\
&= \left[\sigma_r^2 + 2\,\alpha\,\mu_r\,R_1(\underline{r}) - \alpha^2\,R_1^2(\underline{r}) - \alpha\,(2-\alpha)\,R_2(\underline{r})\right] \cdot [1 + 2\,\alpha\,\omega\,R_1(\underline{r})] \\
&\qquad + \lambda\,[[\mu_r - \alpha\,R_1(\underline{r})] \cdot [1 + \alpha\,\omega\,R_1(\underline{r})] - \bar{\mu}_y]\,.
\end{aligned}$$

The first order condition with respect to $\underline{r}$ is

$$\begin{aligned}
&\alpha\,\underline{r}\,f(\underline{r})\;[-2\,(\mu_r - \alpha\,R_1(\underline{r})) + (2-\alpha)\,\underline{r}] \cdot [1 + 2\,\alpha\,\omega\,R_1(\underline{r})] \\
&- \left[\sigma_r^2 + 2\,\alpha\,\mu_r\,R_1(\underline{r}) - \alpha^2\,R_1^2(\underline{r}) - \alpha\,(2-\alpha)\,R_2(\underline{r})\right] \cdot 2\,\omega\,\alpha\,\underline{r}\,f(\underline{r}) \\
&\qquad = -\lambda\,\alpha\,\underline{r}\,f(\underline{r}) \cdot [1 + \alpha\,\omega\,R_1(\underline{r})] + \lambda\;[\mu_r - \alpha\,R_1(\underline{r})]\;\omega\,\alpha\,\underline{r}\,f(\underline{r}).
\end{aligned} \tag{A.35}$$

The first order condition with respect to α is

$$\begin{aligned}
&[2\,R_1(\underline{r})\,(\mu_r - \alpha\,R_1(\underline{r})) - 2\,(1-\alpha)\,R_2(\underline{r})] \cdot [1 + 2\,\alpha\,\omega\,R_2(\underline{r})] \\
&+ \left[\sigma_r^2 + 2\,\alpha\,\mu_r\,R_1(\underline{r}) - \alpha^2\,R_1^2(\underline{r}) - \alpha\,(2-\alpha)\,R_2(\underline{r})\right] \cdot 2\,\omega\,R_1(\underline{r}) \\
&\qquad = \lambda\,R_1(\underline{r}) \cdot [1 + \alpha\,\omega\,R_2(\underline{r})] - \lambda\;[\mu_r - \alpha\,R_1(\underline{r})]\;\omega\,R_1(\underline{r}).
\end{aligned} \tag{A.36}$$

Canceling the common terms $\alpha\,\underline{r}\,f(\underline{r})$ in Equation (A.35), and plugging the result into (A.36) leads, with some rearranging, to

$$R_1(\underline{r})\,(2-\alpha)\,\underline{r} = 2\,(1-\alpha)\,R_2(\underline{r}) \quad \Leftrightarrow \quad \alpha^*(\underline{r}) = \frac{R_2(\underline{r}) - \underline{r}\,R_1(\underline{r})}{R_2(\underline{r}) - \frac{1}{2}\,\underline{r}\,R_1(\underline{r})}.$$

Re-inserting (A.34) leads to (19), which is the unique solution of the first order conditions. $\sigma_y(\alpha^*(\underline{r}),\underline{r})$ is the minimum, since $\sigma_y(\alpha,\underline{r})$ is maximum at the corners $\alpha = 1$ (with $\underline{\rho} > 0$), or $\underline{\rho} = 0$ (with $\alpha < 1$), as can verified easily.

Equation (19) determines the set of strategies, which generate the minimum standard deviation for any given mean income. This set may include different strategies which lead to the same standard deviation, but different mean incomes. In such a case, we drop the strategy associated with the lower mean income, which is determined by $\alpha^*(\underline{r}, \underline{r})$, where $\underline{r}$ is chosen from the appropriate subset $\Omega \subseteq [0, \infty)$ of feasible rain thresholds.

Turning to the properties of $\alpha^*(\underline{r})$, for $\underline{r} = 0$ the numerator and denominator of (19) are equal, hence $\alpha^*(0) = 1$. For $\underline{r} \to \infty$, we have, using L'Hospital's rule repeatedly, $\lim_{\underline{r}\to\infty} \alpha^*(\underline{r}) = 0$. Numerical computations for a wide range of parameters (μ_r, σ_r) resulted in qualitatively the same curves $\alpha^*(\underline{r})$ as shown in Figure 3.

A.6 Proof of Lemma 2

Plugging Equations (19) and (A.34) into (17) and differentiating with respect to $\underline{r}$ yields:

$$\begin{aligned}\frac{d\,\mu_y(\alpha^*(\underline{r}), \underline{r})}{d\underline{r}} &= \left[-\frac{d\alpha^*(\underline{r})}{\underline{r}}\, R_1(\underline{r}) + \alpha^*(\underline{r})\, \underline{r}\, f(\underline{r})\right] \cdot [1 + 2\,\alpha\,\omega\, R_1 - \omega\,\mu_r] \qquad \text{(A.37)}\\ &= \left[\frac{2\, R_1^2(\underline{r})\, R_2(\underline{r})}{(2\, R_2(\underline{r}) - \underline{r}\, R_1(\underline{r}))^2} + \alpha^{*2}(\underline{r})\, \underline{r}\, f(\underline{r})\right] \cdot [1 + 2\,\alpha\,\omega\, R_1 - \omega\,\mu_r] > 0,\end{aligned}$$

since, by assumption, $\omega\,\mu_r < 1$.

For $\underline{r} \to 0$, we have $\lim_{\underline{r}\to 0} R_1(\underline{r}) = \mu_r$, $\lim_{\underline{r}\to 0} R_2(\underline{r}) = \sigma_2^2 + \mu_r^2$, and $\alpha^*(0) = 1$. Inserting into equations (17) and (18) yields $\lim_{\underline{r}\to 0} \mu_y(\alpha^*(\underline{r}), \underline{r}) = 0$ and $\lim_{\underline{r}\to 0} \sigma_y(\alpha^*(\underline{r}), \underline{r}) = 0$.

For $\underline{r} \to \infty$, we have $\lim_{\underline{r}\to\infty} R_1(\underline{r}) = 0$ and $\lim_{\underline{r}\to\infty} R_2(\underline{r}) = 0$, and $\lim_{\underline{r}\to\infty} \alpha^*(\underline{r}) = 0$. Inserting into equations (17) and (18) yields $\lim_{\underline{r}\to\infty} \mu_y(\alpha^*(\underline{r}), \underline{r}) = \mu_r$ and $\lim_{\underline{r}\to\infty} \sigma_y(\alpha^*(\underline{r}), \underline{r}) = \sigma_r$.

A.7 Proof of Lemma 3

As shown in Appendix A.6, $\lim_{\underline{r}\to\infty} \mu_y(\alpha^*(\underline{r}), \underline{r}) = \mu_r$ and $\lim_{\underline{r}\to\infty} \sigma_y(\alpha^*(\underline{r}), \underline{r}) = \sigma_r$. This is the northeast corner of the income possibility frontier, since $\mu_y = \mu_r$ is the maximum possible mean income (cf. Lemma 2). The slope of the income possibility frontier is

$$\frac{d\mu_y^{\text{ipf}}}{d\sigma_y} = \frac{d\mu_y(\alpha^*(\underline{r}), \underline{r})/d\underline{r}}{d\sigma_y(\alpha^*(\underline{r}), \underline{r})/d\underline{r}} = 2\,\sigma_y(\alpha^*(\underline{r}), \underline{r})\, \frac{d\mu_y(\alpha^*(\underline{r}), \underline{r})/d\underline{r}}{d\sigma_y^2(\alpha^*(\underline{r}), \underline{r})/d\underline{r}}.$$

From Appendix A.5 we derive

$$\frac{d\sigma_y^2}{d\underline{r}} = -\lambda\, \frac{d\mu_y}{d\underline{r}} \qquad \text{(A.38)}$$

where λ is the costate-variable of the optimization problem (A.33), which is determined by Equations (A.35) and (A.36),

$$-\lambda = \frac{-2\mu_y(\alpha^*(\underline{r}), \underline{r}) + (2 - \alpha^\star(\underline{r}))\, \underline{r}\, (1 + 2\,\alpha^\star(\underline{r})\,\omega\, R_1(\underline{r})) - \frac{\sigma_y^2(\alpha^*(\underline{r}), \underline{r})\, 2\,\omega}{1 + 2\,\alpha^\star(\underline{r})\,\omega\, R_1(\underline{r})}}{1 + 2\,\alpha^\star(\underline{r})\,\omega\, R_1(\underline{r}) - \omega\,\mu_r}.$$

Thus, we have

$$\frac{d\mu_y^{\text{ipf}}}{d\sigma_y} = \frac{2\,\sigma_y(\alpha^*(\underline{r}), \underline{r})}{-\lambda}. \qquad \text{(A.39)}$$

In particular for $\underline{r} \to \infty$, it is $\lim_{\underline{r}\to\infty} (-\lambda) = \frac{-2\,\mu_r + 2\,\underline{r} - \sigma_r^2\, 2\,\omega}{1-\omega\,\mu_r} = \infty$. Hence,

$$\lim_{\underline{r}\to\infty} \frac{d\mu_y^{\text{ipf}}}{d\sigma_y} = 0.$$

For $\underline{r} \to 0$ both the mean income $\mu_y(\alpha^*(\underline{r}, \underline{r}))$ and the standard deviation of income $\sigma_y(\alpha^*(\underline{r}, \underline{r})$ vanish (cf. Appendix A.6). Since both cannot be negative, this is the southwest corner of the income possibility frontier. At this point, the slope of the income possibility frontier is

$$\lim_{\underline{r}\to 0} \frac{d\mu_y^{\text{ipf}}}{d\sigma_y} = \lim_{\underline{r}\to 0} \frac{\mu_y(\alpha^*(\underline{r}, \underline{r}))}{\sigma_y(\alpha^*(\underline{r}), \underline{r})} = \lim_{\underline{r}\to 0} \sqrt{\frac{(1-\alpha^*(\underline{r}))^2\,\mu_r^2\,(1+\alpha^*(\underline{r})\,\omega\,\mu_r)^2}{\sigma_r^2\,(1-\alpha^*(\underline{r}))^2\,(1+2\,\alpha^*(\underline{r})\,\omega\,\mu_r)}} = \frac{\mu_r}{\sigma_r},$$

neglecting terms of second order in ω. For $\underline{r} = 0$, and any given α, we have

$$\mu_y(\alpha, 0) = [\mu_r - \alpha R_1(0)]\;[1 + \alpha\,\omega\,R_1(0)] = (1-\alpha)\,\mu_r\,(1+\alpha\,\omega\,\mu_r)$$

$$\begin{aligned}\sigma_y^2(\alpha, 0) = \left[\sigma_r^2 + 2\,\alpha\,\mu_r\,R_1(0) - \alpha^2\,R_1^2(0) - \alpha\,(2-\alpha)\,R_2(0)\right]\;[1 + 2\,\alpha\,\omega\,R_1(0)]&\\ = (1-\alpha)^2\,\sigma_r^2\,(1+2\,\alpha\,\omega\,\mu_r),&\end{aligned}$$

i.e., for small ω, the straight line between $(\mu_y, \sigma_y) = (0,0)$ $(\alpha = 1)$ and $(\mu_y, \sigma_y) = (\mu_r, \sigma_r)$ $(\alpha = 0)$ is always within the income possibility set. Since for $\underline{r} = 0$ the standard deviation is maximum for given mean income (cf. Appendix A.5), the income possibility frontier is located above this straight line.

We have numerically determined the income possibility frontier for a large variety of parameters μ_r, σ_r, and ω. The results have provided strong evidence that under any set of parameters the income possibility frontier is divided into two domains: a convex domain for small σ_y and a concave domain for large σ_y. For very small σ_r, these two domains maybe separated by a jump in the income possibility frontier, such taht, in these extreme cases, the left borders of the respective income possibility sets inwardly curved to the right.

A.8 Proof of Lemma 4

To prove part (i), we show that (a) the optimal indifference curve is convex over the whole range $\sigma_y \in [0, \sigma_r]$, and (b) the optimum is within the concave domain of the income possibility frontier.

Ad (a). Rearranging Equation (A.26) yields the following expression for the optimal indifference curve (where μ_0^* is the certainty equivalent for the optimum)

$$\left(\frac{\sigma_y}{\mu_y}\right)^2 = \left(\frac{\mu_y}{\mu_0^*}\right)^{2/\rho} - 1. \tag{A.40}$$

Inserting in the condition for the convexity of the indifference curve yields

$$\left(\frac{\mu_y}{\sigma_y}\right)^2 > 1+\rho \quad \Leftrightarrow \quad \frac{\mu_y}{\mu_0^*} < \left(\frac{2+\rho}{1+\rho}\right)^{2/\rho}. \tag{A.41}$$

By assumption, this condition is fulfilled for $\mu_y = \mu_r$ on the indifference curve which intersects (μ_r, σ_r), i.e. which is below the optimal one. Since $\mu_y \leq \mu_r$ for all efficient strategies, this condition is fulfilled for all μ_y on the optimal indifference curve.

Ad (b). The minimum slope of the income possibility frontier in the convex domain (i.e. at the southwest border) is μ_r/σ_r (Lemma 3). The slope of the indifference curve at the optimum (μ_y^*, σ_y^*), however, is smaller,

$$1+\rho < \left(\frac{\mu_r}{\sigma_r}\right)^2 < \frac{\mu_r}{\sigma_r}\frac{\mu_y^*}{\sigma_y^*} \quad \Rightarrow \quad \frac{\rho}{1+(1+\rho)\,\frac{\sigma_y^{*2}}{\mu_y^{*2}}} < \frac{\mu_r}{\sigma_r}\frac{\mu_y^*}{\sigma_y^*} \quad \Leftrightarrow \quad \frac{\rho\,\sigma_y^*\,\mu_y^*}{\mu_y^{*2}+(1+\rho)\,\sigma_y^{*2}} < \frac{\mu_r}{\sigma_r},$$

where the inequality $\mu_r/\sigma_r < \mu_y^*/\sigma_y^*$ holds as a consequence of Lemma 3, and the expression on the left hand side of the last inequality is the slope of the indifference curve at the optimum (cf. Equation A.27). Hence, the optimum cannot be in the convex domain of the income possibility frontier.

Ad (ii). For $\rho = 0$, the indifference curves are horizontal lines. Hence, the maximum of the income possibility frontier, which is at the corner $(\mu_y, \sigma_y) = (\mu_r, \sigma_r)$, is the optimum.

For $\rho > 0$ corner solutions are excluded. At the corner $(\mu_y, \sigma_y) = (\mu_r, \sigma_r)$ the slope of the income possibility frontier is zero (Lemma 3), whereas the indifference curves have a positive slope, provided $\rho > 0$. At the corner $(\mu_y, \sigma_y) = (0, 0)$, the income possibility frontier is increasing with a slope μ_r/σ_r (Lemma 3), but the slope of the indifference curves is zero for $\sigma_r = 0$ (cf. Appendix A.3).

A.9 Proof of Result 1

We have shown that the unique optimum is in the concave domain of the income possibility frontier (Appendix A.8), and that the slope of the farmer's indifference curves increases with ρ (Appendix A.3). Thus, the optimal mean income μ_y^* decreases if ρ increases. Since for efficient strategies the mean μ_y^* is increasing in $\underline{r}$, the rain threshold $\underline{r}^*$ of the optimal strategy decreases if ρ increases.

A.10 Sensitivity analysis of Result 2

The aim of this Appendix is to show in a sensitivity analysis how the qualitative results shown in Figure 6 and stated in Result 2 depend on the parameters of the model. The sensitivity analysis was performed using a Monte Carlo approach, repeating the computations with multiple randomly selected parameter sets. We focused on three parameters, namely the growth parameter of green biomass w_G, the influence c of grazing on the growth of reserve biomass, and the standard deviation σ_r of rainfall. The other parameters either affect the outcomes in the same direction as the selected parameters (this is the case for the growth parameter of the reserve biomass w_R and the expected value of rainfall μ_r), or in the inverse direction (this is the case for the death rate of the reserve biomass d).[15] Hence their variation enables no further insights.

A sample size of $N = 20$ parameter sets was created according to the Latin Hypercube sampling method (Saltelli et al. 2000).[16] The three parameters were assumed to be

[15]For the two parameters K and R, no substantial influence is to be expected: they just rescale the problem.

[16]This method, by stratifying the parameter space into N strata, ensures that each parameter has all proportions of its distribution represented in the sample parameter sets.

independent uniformly distributed, with $0 \leq w_G \leq 5$, $0 \leq \sigma_r \leq 2.4$ and $0 \leq c \leq 1$, the upper bounds for w_G and σ_r are guesses which proved to be suitable. The respective simulation results were compared to the results shown in Figure 6. The following types of long-term dynamics of mean reserve biomass and mean income (distinct from those stated in Result 2) were found:[17]

(i) If the growth parameter of the green biomass w_G is very low, i.e. if $w_G \cdot w_R < d$, the reserve biomass is not able to persist at all. Keeping livestock is not possible, independent of the chosen grazing management strategy.

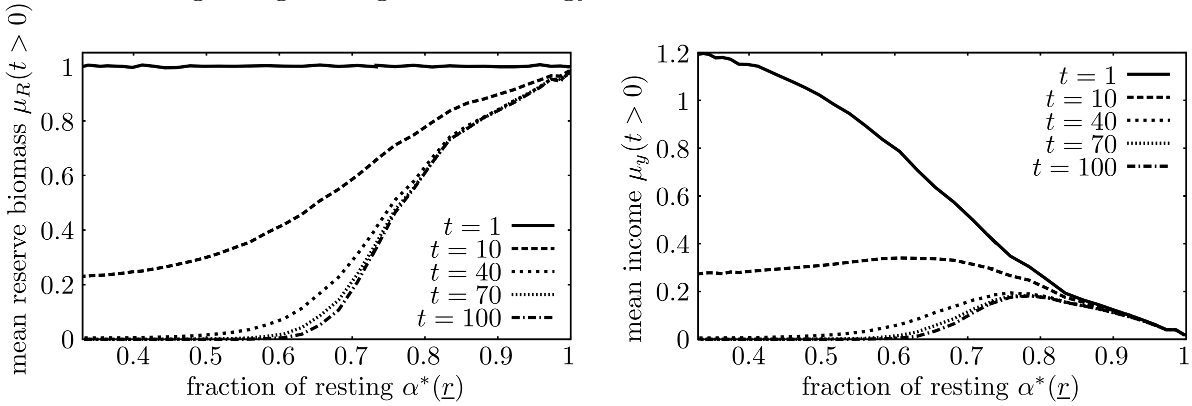

Figure 7: Parameter values are as in Figure 6, except for $c = 0.9$.

(ii) If the impact c of grazing on the growth of the reserve biomass is very high, the mean reserve biomass declines to zero in finite time, unless the grazing management strategy is very conservative. This is illustrated in Figure 7, where we have chosen $c = 0.9$.

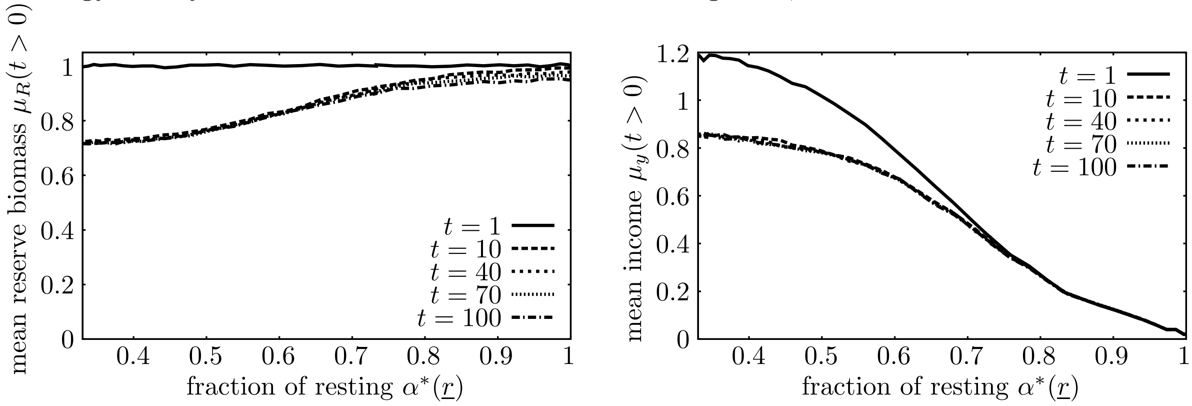

Figure 8: Parameter values are as in Figure 6, except for $w_G = 4$.

(iii) If the growth parameter of the green biomass is very high or the impact of grazing on the growth of the reserve biomass is very low, the future mean income is the higher the less conservative the strategy is, i.e. resting is not required to preserve the ecosystem. This is illustrated in Figure 8 for a very high growth rate of the biomass, $w_G = 4$. Qualitatively the same outcome arises for very low c (see also Müller et al. 2004).

(iv) If the standard deviation of rainfall σ_r is very small, resting is almost deterministic: for $\underline{r} > \mu_r$, resting will take place in hardly any year, such that mean reserve biomass μ_R

[17] To illustrate them, additional calculations were done, where one parameter was chosen differently from the original parameter set of Figure 6 in each case.

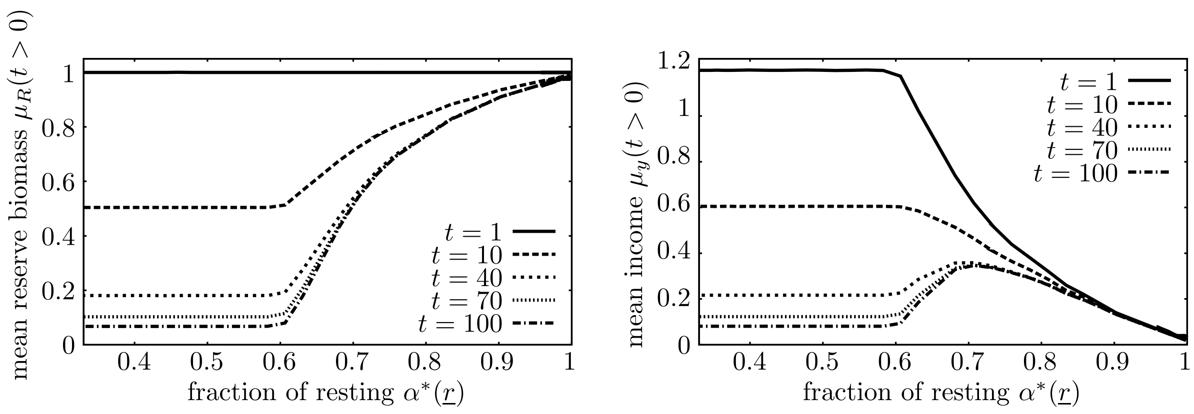

Figure 9: Parameter values are as in Figure 6, except for $\sigma_r = 0.05$.

and mean income μ_y are independent of the strategy. For $\underline{r} < \mu_r$, resting will take place in almost every year, i.e. the fraction $\alpha^*(\underline{r})$ of rested paddocks determines the outcome, as illustrated in Figure 9 for $\sigma_r = 0.05$.

A.11 Proof of Lemma 5

In order to determine the steady-state mean value R^{stst} of the reserve biomass, we plug Equation (1) into Equation (2) an take the expected value on both sides of the resulting equation. In the long-term, the expectation value of R^i_t and R^i_{t+1} are the same and equal to R^{stst}. Given that in the long-term each camp will be rested with equal probability, we derive

$$d\,R^{\text{stst}}\left(1+\frac{R^{\text{stst}}}{K}\right) = w_R\,w_G\,R^{\text{stst}}\left(1-\frac{R^{\text{stst}}}{K}\right)\left(\mu_r - c\,\mu_{y(1)}(\alpha^\star(\underline{r},\underline{r})\right).$$

This equation is solved by $R^{\text{stst}} = 0$ and by

$$R^{\text{stst}} = K\,\frac{w_G\,w_R\,(\mu_R - c\,\mu_{y(1)}(\alpha^\star(\underline{r}),\underline{r})) - d}{w_G\,w_R\,(\mu_R - c\,\mu_{y(1)}(\alpha^\star(\underline{r}),\underline{r})) + d}. \tag{A.42}$$

If it is positive, the last expression is the solution; otherwise $R^{\text{stst}} = 0$ is the solution, since the reserve biomass cannot become negative. It is easily confirmed that R^{stst} is monotonically decreasing in $\mu_{y(1)}$. With a very similar argument as in Lemma 2, it is shown that $\mu_{y(1)}$ is monotonically increasing in $\underline{r}$. Hence, R^{stst} is monotonically decreasing in $\underline{r}$.

Income in each year is given by $y(t) = R(t)/(I\,R)\sum_{i=1}^{I} x^i\,r$. Given that each camp is equally likely to be rested in the long-term, the long-term expected value of income is

$$\mu_y^{\text{stst}} = \frac{\mu_R^{\text{stst}}}{R}\,\mu_{y(1)}(\alpha^\star(\underline{r}),\underline{r}). \tag{A.43}$$

The unique interior extremum for which $R^{\text{stst}} > 0$ is given by

$$\hat{\mu}_{y(1)} = \frac{w_G\,w_R\,\mu_r + d - \sqrt{2\,d\,(w_G\,w_R\,\mu_r + d)}}{c\,w_G\,w_R}\,. \tag{A.44}$$

Is is a maximum, since for both corners $\mu_{y(1)} = 0$ and $\mu_{y(1)} = \mu_r$ we have $\mu_y^{\text{stst}} = 0$. Since $\mu_{y(1)}$ is monotonically increasing in $\underline{r}$, a unique $\hat{\underline{r}}$ exists, for which $\mu_{y(1)} = \hat{\mu}_{y(1)}$.

A.12 Proof of Lemma 6

If $c > 1 - d/(w_G\, w_R\, \mu_r)$, $\mu_R^{\text{stst}} = 0$ for $\underline{r} \to \infty$, by Lemma 5. That is, a strategy without resting is unsustainable. If, however, $\underline{r} \to 0$, $\mu_{y(1)} = 0$ (by Lemma 2). Hence, as $d < w_G\, w_R\, \mu_r$, the strategy with complete resting is sustainable. By Lemma 2 $\mu_{y(1)}$ is monotonically increasing with $\underline{r}$, which concludes the proof.

A.13 Proof of Result 3

By Lemma 6, all strategies are sustainable if $c \leq \bar{c} = \frac{w_G\, w_R\, \mu_r - d}{w_G\, w_R\, \mu_r}$. Hence, even the strategy chosen by risk-neutral farmers is sustainable. The interesting case is $c > \bar{c}$. In that case, the strategy chosen by a risk-neutral farmer is unsustainable. What remains to be shown is that for sufficiently large σ_r and sufficiently small c, a ρ' exists, such that all farmers with risk aversion $\rho > \rho'$ will choose a sustainable strategy. A necessary and sufficient condition for this statement is that

$$\lim_{\rho \to \infty} \mu_y(\alpha^\star(\underline{r}^\star(\rho)), \underline{r}^\star(\rho)) < \frac{w_G\, w_R\, \mu_r - d}{c\, w_G\, w_R}\,, \tag{A.45}$$

where $((\alpha^\star(\underline{r}^\star(\rho)), \underline{r}^\star(\rho))$ is the optimal strategy for a myopic farmer with risk aversion ρ. For, if Condition (A.45) holds, the strategy chosen by an infinitely risk averse farmer is sustainable (cf. Lemma 6). Condition (A.45) is fulfilled, if (i) the right hand side is large enough and (ii) the left hand side is small enough. The right hand side is large, if c and d are small. The right hand side is small, if σ_r is large compared to μ_r. This has been shown in Appendix A.7: if σ_r is large, the income-possibility frontier is very flat in its concave domain. Hence, the optimal μ_y is only slightly smaller than μ_r, and Condition (A.45) is violated, unless c is very small. In Figure 10, the threshold degree of risk aversion is plotted against σ_r (left hand side) and c (right hand side). For both, low σ_r and high c, this threshold value exceeds plausible values of ρ. But for high σ_r and comparatively low c, the threshold value ρ' lies well within the range of degrees of risk aversion which are commonly considered as reasonable ($\rho \leq 4$; see, e.g., Gollier 2001).

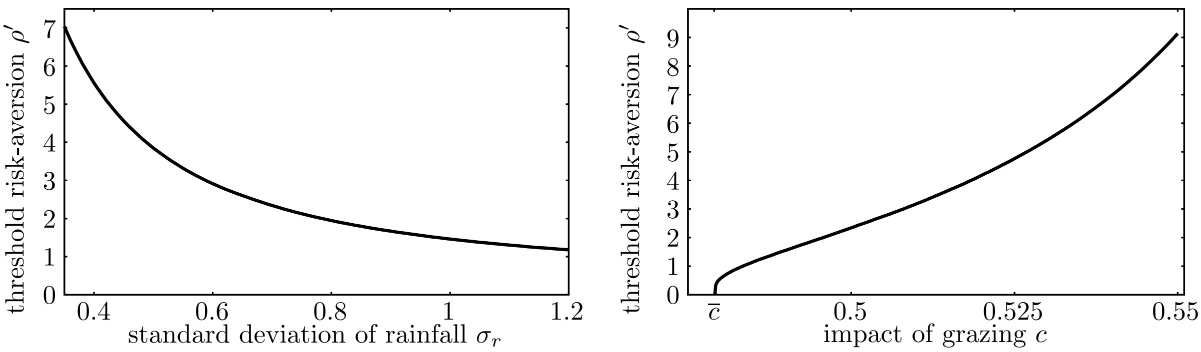

Figure 10: The threshold value of risk aversion, above which a myopic farmer chooses a sustainable strategy. On the left hand side plotted against the standard deviation σ_r of rainfall, on the right hand side plotted against the impact of grazing on vegetation. For $c \leq \bar{c} = \frac{w_G\, w_R\, \mu_r - d}{w_G\, w_R\, \mu_r}$, all strategies are sustainable (Lemma 6). The remaining parameter values are as in Figure 6.