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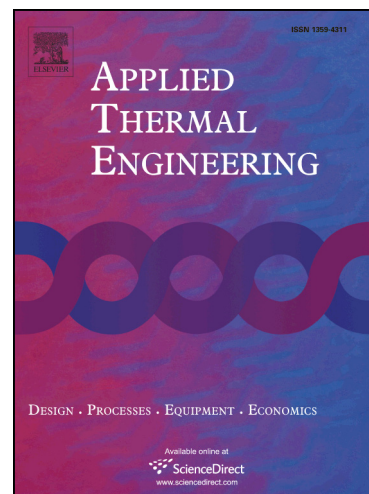
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# Thermo-mechanical analysis of heat exchanger design for thermal energy storage systems

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## Abstract

Significant tensile stresses inside solid thermal energy storage media are induced due to incompatible thermal expansion characteristics. These stresses can cause damage to the often brittle storage material which is associated with a performance loss of thermal properties or the partial loss of long-term mechanical stability. In the present paper, a previously introduced analytical approach is extended to estimate the effects of the dominant physical and geometrical quantities on critical tensile stresses around tubular heat exchangers. Results are presented in terms of three composite dimensionless parameters representing the geometrical and material parameters of the system. Analytical sensitivities furthermore provide a direct quantification of how these sensitivities depend on selected system parameters, thus giving clues regarding the most promising optimisation handles. A representative case study was performed and can serve as a guide-line for making design decisions from a mechanical perspective as a complement to the typically performed thermodynamic design.

*Keywords:*

Sensible heat storage, Solid storage media, Thermo-mechanics, Heat exchanger design, Water-saturated cement

## 1. Introduction

Driven by the purpose of saving fossil resources and reducing air pollution, alternative technologies are being developed to capture and use renewable sources of energy, such as solar, wind and hydro power, or geothermal heat. In this context, thermal energy storage (TES) is a technology that is primarily used for alleviating the mismatch between energy demand and supply at varying temperature conditions (Cabeza (2014)) and allows a better management of the intermittent renewable energy; cf., for example, Braun et al. (1981), Herrmann and Kearney (2002), Hesarakı et al. (2015).

Among the numerous technologies available, this article is concerned with sensible heat storage in solid media, which is extensively used in various heat storage applications (Herrmann et al., 2004; Laing et al., 2008; Bauer et al., 2010; Gil et al., 2010; Laing et al., 2012; Duffy et al., 2015; Jian et al., 2015b). This

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**Nomenclature**

$\alpha_{1/2}$	Linear thermal expansion coefficient of the heat exchanger/the storage material.
$\mathcal{C}$	Fourth-order stiffness tensor.
$\epsilon$	Small strain tensor.
$\sigma$	Cauchy stress tensor.
$\nu_{1/2}$	Poisson's ratio of the heat exchanger/the storage material.
$\sigma_{\phi\phi}^{1/2}$	Circumferential stress in the heat exchanger/the storage material.
$\sigma_{rr}^{1/2}$	Radial stress in the heat exchanger/the storage material.
$\varphi$	Airy stress function.
$\vartheta$	Temperature difference with respect to $T_0$ .
$A_i, C_i$	Integration constants.
$E_{1/2}$	Young's modulus of the heat exchanger/the storage material.
$p_{0/2}$	Pressure applied on the inner/outer boundary.
$r_{0/1}$	Inner/outer radius of the tube.
$S$	Tube pitch ( $S = 2r_2$ ).
$T_0$	Homogeneous reference temperature.
$T_u$	Ultimate storage temperature.

1 mature technology is cost-effective and can be qualified for most domestic and commercial applications  
2 while being environmentally benign.

3 The continuing development of TES systems is accompanied by specific requirements for heat exchang-  
4 ers. Not only does the heat exchanger need to be designed to achieve a high heat transfer efficiency (Zheng  
5 et al., 2015), but also the thermo-mechanical and thermo-hydraulic performance of the heat exchanger and  
6 the surrounding storage media must be improved to maintain functional reliability during the operational  
7 life.

8 As there are numerous design aspects implied in the multi-disciplinary design of a heat exchanger for  
9 solid sensible TES, presenting a comprehensive optimum may not be the most efficient and economical  
10 procedure (Bao et al., 2013), and is not a uniquely posed problem. Accordingly, it is more reasonable  
11 to concentrate on those factors that have been identified as the most dominant in a given specific kind of  
12 TES system. Most existing studies focus on improving heat transfer performance along with overall cost  
13 reduction. Although, the analyses of mechanical fields for TES materials were presented in literature (cf.,  
14 for example Su et al. (2005); Zhang and Zhao (2011); Yuan et al. (2012)), to the authors' knowledge, there  
15 are few studies taking into account thermo-mechanical effects in the solid TES with heat exchangers, such  
16 as critical stresses inside solid sensible TES during thermal cycling as well as quantifying the dependence of  
17 these stresses on key parameters of the heat exchanger and the storage material and using that information

1 for design optimisation.

2 In this paper, we focus on estimating peak tensile stresses around tubular heat exchangers embedded in a  
 3 solid heat storage medium as an indicator for the thermo-mechanical integrity of the heat store. The specific  
 4 motivation for this problem comes from the observation of significant tensile stress within the storage ma-  
 5 terial which is typically an affordable building material with a relatively low tensile strength (Skinner et al.,  
 6 2014; Miao et al., 2016). Thus, such thermally induced stresses might quite possibly exceed the material's  
 7 strength limits for an inappropriate combination of geometry and material. The purpose of the present study  
 8 is twofold: i) to investigate the effects of geometrical/physical parameters of the heat exchanger on the  
 9 thermo-mechanical performance of the heat store unit in detail by using a previously developed analytical  
 10 approach; ii) to show how this analytical approach can be modified and used to select a heat exchanger  
 11 which is optimal from a mechanical perspective and does not stand in contrast to thermodynamic require-  
 12 ments that are critical for the system's operation as a heat store.

13 Generally, a designated TES system is characterized by the geometric layout of its heat exchanger as  
 14 well as the selected storage material which must satisfy the thermal performance requirements as well as  
 15 provide long-term reliability. Mechanically, the incompatibility of the heat exchanger with the storage  
 16 material in terms of material properties dominates the stresses induced by thermo-mechanical loads.

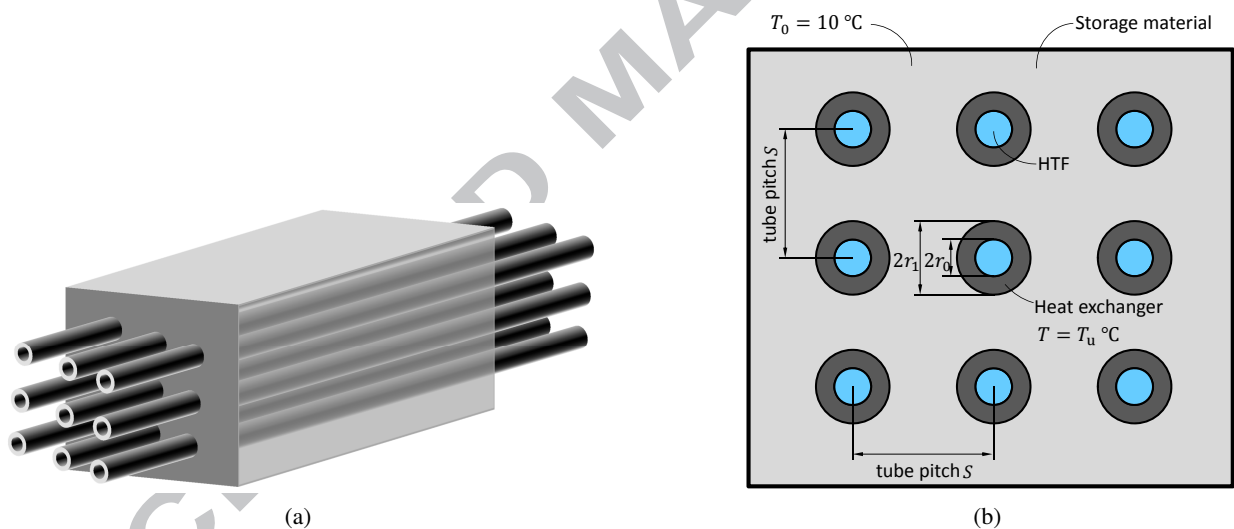


Figure 1: Schematic and details of standard tubular heat exchanger's geometrical layout.

17 The heat exchanger layout investigated here is based on Laing et al. (2006) and illustrated in Fig. 1a.  
 18 According to the analytical solution presented in Miao et al. (2016), three quantities were considered as  
 19 the main geometric design parameters in the analysis: the outer and inner diameters of the heat exchanger,  
 20 and the tube pitch which is defined as the centre-to-centre distance between two adjacent cylindrical heat  
 21 exchangers, see Fig. 1b. The above configuration aside, our approach applies to other configurations com-  
 22 posed of piping bundles embedded within a solid storage material as well (for example Agyenim et al.  
 23 (2010); Raju and Kumar (2012); Wu et al. (2014); Jian et al. (2015a)).

24 This paper is organised as follows. In Section 2, the analytical solution used in the analysis is briefly re-  
 25 viewed. Dimensionless geometrical/physical quantities are derived to reduce the number of free parameters  
 26 and to simplify the illustration of the analysis. In Section 3, result are presented for the optimal layout of a  
 1 specific system based on the identification of dominant thermo-mechanical aspects in extensive sensitivity

2 analyses ([Appendix B](#)). The main conclusions of this paper are stated in Section 4.

## 3 2. Theoretical modelling

4 Heat transport models of the heat store were validated by distributed thermo-couple measurements [data  
5 not shown]. The stress fields within the cast cement block around the heat exchangers, however, cannot be  
6 readily measured and only indirect experimental evidence on their effects is available ([Skinner et al., 2014](#)).  
7 This motivated us to use theoretical modelling as the apparently most feasible way for their quantification.  
8 An extensively validated and simple linear thermo-elastic theory was employed and material parameters  
9 measured in the laboratory ([Miao et al., 2016](#)) were used.

10 To keep the approach simple and broadly applicable, it is based on a well-established engineering the-  
11 ory. The model describes a cylindrical region comprising the heat exchanger and the surrounding storage  
12 material and allows for an internal pressure as well as an external pressure acting on the cylinder walls.

13 The following assumptions were made to arrive at an analytical solution:

- 14 (i) All materials are assumed to behave linearly elastic and to be isotropic.
- 15 (ii) Body forces are not considered.
- 16 (iii) Axisymmetry of geometry and loading is assumed.
- 17 (iv) Radial displacement and radial stress are continuous at the interface.

1 Assumption (iv) carries the implicit assumption that  $\alpha_1 \geq \alpha_2$ , which is fulfilled in most practical cases.

The basic constitutive equation with respect to stresses, strains and temperature changes is written based  
on the thermo-elastic relation

$$\boldsymbol{\sigma} = \boldsymbol{\mathcal{C}} : (\boldsymbol{\epsilon} - \alpha \vartheta \mathbf{I}) \quad (1)$$

Introducing a scalar valued stress function  $\varphi$  ([Kreißig and Benedix \(2013\)](#)), the biharmonic equation

$$\Delta \Delta \varphi = 0 \quad (2)$$

can be shown to hold. Due to axisymmetry, Eq. (2) can be expressed as an ordinary differential equation

$$\frac{d}{dr} \left\{ r \frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d\varphi}{dr} \right) \right] \right\} = 0 \quad (3)$$

As outlined in [Miao et al. \(2016\)](#), a solution of Eq. (3) yields the following expressions for the stresses

$$\sigma_{rr}^i = \frac{A_i}{r^2} + 2C_i \quad (4)$$

$$\sigma_{\phi\phi}^i = -\frac{A_i}{r^2} + 2C_i \quad (5)$$

$$\sigma_{zz}^i = 4\nu_i C_i - E_i \alpha_i \vartheta_i \quad (6)$$

2 The integration constants  $A_i, C_i$  can be determined by the continuity and boundary conditions and are  
3 listed in [Appendix A](#). For further details on the derivation as well as a comparison to numerical modelling  
4 results, see [Miao et al. \(2016\)](#).

5 *2.1. Dimensionless peak tensile stress*

As outlined in Miao et al. (2016), the peak tensile stress typically occurs within the storage material in the circumferential direction at the interface:

$$\sigma_{\phi\phi}^2|_{r_1} = -\frac{A_2}{r_1^2} + 2C_2 \quad (7)$$

6 As the peak tensile stress is the most critical stress for commonly used brittle storage media such as cemen-  
7 titious or ceramic materials, it will also be referred to as critical stress. For ease of notation, in the sequel  
8 we will simply write  $\sigma_{\phi\phi}$  when referring to this quantity. To simplify the analysis, all variables influencing  
9 the critical tensile stress  $\sigma_{\phi\phi}$  will be condensed into dimensionless quantities.

Suppose that negligible pressures are imposed on the boundaries, i.e.,  $p_0 = 0$  and  $p_2 = 0$ , and  $T_u = \text{const.}$ , i.e., the heat store is fully charged and at maximum homogeneous temperature which was confirmed to be the mechanically most critical practically occurring state by full transient heat transport simulations (Miao et al., 2016). Then,  $A_2$  can be rewritten as

$$A_2 = \frac{R_0 E_1 E_2 r_0^2 r_1^2 r_2^2 (\alpha_2 - \alpha_1) \vartheta}{E_1 r_0^2 R_0 [r_2^2 + r_1^2 + \nu_2 (r_2^2 - r_1^2)] - E_2 r_2^2 [r_0^2 + r_1^2 + \nu_1 (r_0^2 - r_1^2)]} \quad (8)$$

By introducing the dimensionless quantities

$$(r_0^*, r_1^*) = \frac{1}{r_2} (r_0, r_1), \quad R_0^* = \frac{1 - \frac{r_1^2}{r_0^2}}{r_1^{*2} - 1}, \quad (9)$$

$$E^* = \frac{E_1}{E_2}, \quad \vartheta^* = (\alpha_2 - \alpha_1) \vartheta \quad (10)$$

$$\nu_1^* = \frac{1 + \nu_1}{1 + \nu_2}, \quad \nu_2^* = \frac{1 - \nu_1}{1 - \nu_2}, \quad M^* = \frac{\nu_2^*}{E^*}, \quad (11)$$

the critical tensile stress normalised by the Young's modulus of the storage material can be written as:

$$\frac{\sigma_{\phi\phi}}{E_2} = \frac{E^* \vartheta^* R_0^* (r_0^{*2} + r_0^{*2} r_1^{*2})}{r_0^{*2} (1 + \nu_2) (E^* R_0^* - \nu_1^*) + r_1^{*2} (1 - \nu_2) (E^* R_0^* r_0^{*2} - \nu_2^*)} \quad (12)$$

10 The dimensionless parameter  $R_0^*$  includes geometric sizes characteristic of the analytical solution, such as  
11 the inner and outer pipe radii as well as the tube pitch  $S = 2r_2$ .  $M^*$  indicates the stiffness of the storage  
12 material in comparison to that of the heat exchanger. The higher  $M^*$  is, the stiffer the storage material gets  
13 relatively.  $\theta^*$  represents the difference in thermal strain between both materials at maximum  $\Delta T$ .

14 The three quantities  $R_0^*$ ,  $M^*$  and  $\theta^*$  contain all ten parameters influencing the solution. In the following,  
15 the solution is used to obtain an optimum tube pitch, perform sensitivity analyses as well as a case study on  
16 how to select a suitable heat exchanger from a mechanical perspective.

### 17 3. Results

#### 18 3.1. Sensitivity-based optimisation criterion

19 The dependence of the peak stress on multiple parameters provides flexibility regarding possible technical solutions satisfying specific design requirements of the heat store. Sensitivity analyses (see Appendix B) can serve as a guide-line for the design of the tube-type thermal storage in this respect.

20 As seen in Appendix B, increasing the tube pitch can lower the peak stress at the material interface up to a threshold value of  $S$ , beyond which a further increase of  $S$  has no effect (Fig. B.5c). Following that, Fig. 2 illustrates the peak tensile stress as it varies with the tube pitch.

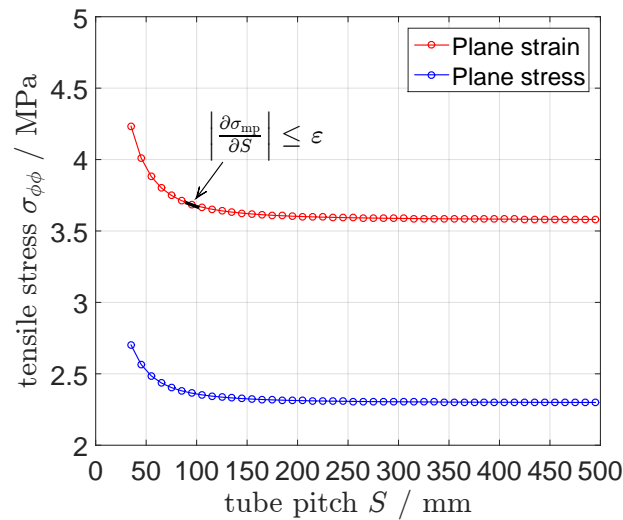


Figure 2: Critical tensile stress  $\sigma_{\phi\phi}$  varying with the tube pitch  $S$ .

6 After a sharp decline, the stress achieves asymptotically a constant value consistent with the observations in Appendix B. From an economic point of view, it is not necessary to extend the tube pitch any further as it does not lead to lower peak stresses. In combination with the observation that a fast heat input into the heat store and an equally fast heat extraction from it, i.e., a high thermal power, require a sufficiently tight packing of heat exchanger tubes or bundles, a certain optimum balance can be postulated: There is a maximum spacing allowed from a thermal perspective and a minimum spacing required from a mechanical perspective. Both measures can be combined into an objective function for a comprehensive optimisation. Here, the focus is exclusively on the mechanical contribution.

7 To define an optimal value, the lowest achievable peak stress  $\sigma_{mp}$  is here defined based on a threshold criterion as that stress where  $|\mathcal{D}_S| \leq \varepsilon$  (see Fig. 2). Simultaneously, this defines the optimal tube pitch  $S_{mp}$  as that value of  $S$  that minimises the peak stress.  $S_{mp}$  is considered as a design parameter and an optimisation criterion. In this article,  $\varepsilon = 1\%$  has been used.

#### 18 3.2. Optimal tube-pitch in a novel heat store

19 All sensitivities investigated above change monotonically, except for those with respect to the inner and outer radii of the heat exchanger, which show a more complex variation, see Fig. B.5a and Fig. B.5b. To further study the effects of the radii on the peak stress and the tube pitch, a case study with prescribed geometrical sizes of the heat exchanger used in the present project (Miao et al., 2016) is conducted. Since



the heat exchanger material has been selected to be an aluminium-polyethylene composite,  $M^*$  can be considered fixed and so can  $\theta^*$  for a given maximum storage temperature of  $90^\circ\text{C}$ .

Fig. 3a illustrates the minimum peak tensile stress values achievable ( $\sigma_{\text{mp}}$ ) with the given heat exchanger for the available pipe geometries according to the  $|\mathcal{D}_S| \leq 1\%$  criterion. Corresponding to these stress values, Fig. 3b plots the optimised tube pitch values at which the lowest critical stress values are achieved. A comparison of both figures indicates the complex interactions between the peak stress, the tube pitch and the dimensions of the heat exchanger. The appropriate dimension which leads to the minimal peak stress does not coincide with the minimal tube pitch, see Fig. 3a and Fig. 3b.

Considering the results, a good compromise may be to choose the  $25 \times 2.5$  pipe, as it yields the lowest stress value of all pipes at a tube spacing of  $75\text{ mm}$ , which—based on numerical studies not shown here—also provides the required thermal performance characteristics.

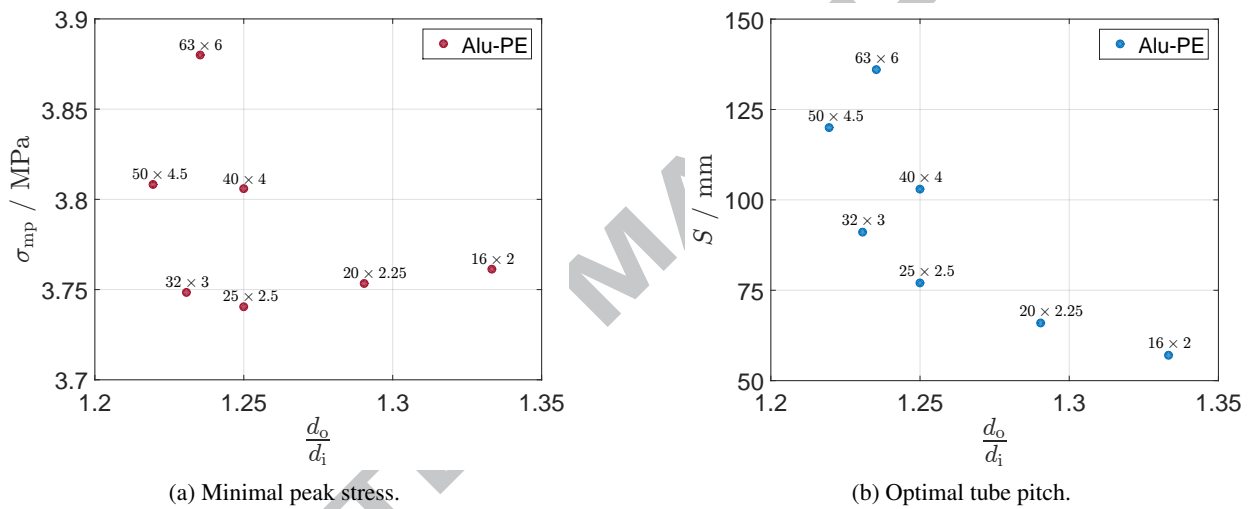


Figure 3: The peak stress and the tube pitch based on different dimensions of the heat exchanger made from an aluminium-polyethylene composite.

#### 4. Conclusions

In this paper, a new analytical approach for aiding design choices in solid thermal energy storage with tubular heat exchangers was proposed based on thermo-mechanical arguments. It captured the dominant factors that affect the mechanical integrity of the heat store, and allowed the identification of optimal properties in the sense of minimal induced stresses while maintaining thermal performance requirements. Sensitivity analyses indicated the flexibility of selecting the appropriate heat exchanger for satisfying the design requirements mechanically or thermally. Additionally, analytical sensitivities directly yield their dependence on all relevant parameters, which is a significant benefit of the analytical over a numerical approach. By studying a currently developed system it was shown how a specific design choice regarding the optimal tube pitch can be made.

#### Acknowledgements

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5 heat storage system", project acronym IGLU) and is gratefully acknowledged.

## 6 Appendix A. Integration constants

The integration constants required for Eqs. (4)-(6) are given by

$$A_1 = \frac{E_2(1-\nu_1)r_0^2r_1^2r_2^2p_0 + E_1r_0^2[-r_1^2r_2^2p_2 - R_1(r_1^2 + r_2^2)] + E_1\nu_2r_0^2[r_1^2r_2^2p_2 + R_1(r_1^2 - r_2^2)]}{E_1r_0^2R_0[r_2^2 + r_1^2 + \nu_2(r_2^2 - r_1^2)] - E_2r_2^2[r_0^2 + r_1^2 + \nu_1(r_0^2 - r_1^2)]} + \frac{E_1E_2r_0^2r_1r_2^2[\alpha_2(\psi_2(r)|_{r_1} - r_1T_0) - \alpha_1(\psi_1(r)|_{r_1} - r_1T_0)]}{E_1r_0^2R_0[r_2^2 + r_1^2 + \nu_2(r_2^2 - r_1^2)] - E_2r_2^2[r_0^2 + r_1^2 + \nu_1(r_0^2 - r_1^2)]} \quad (\text{A.1})$$

$$A_2 = R_0A_1 + R_1 \quad (\text{A.2})$$

$$C_1 = -\frac{1}{2}\left(p_0 + \frac{A_1}{r_0^2}\right) \quad (\text{A.3})$$

$$C_2 = -\frac{1}{2}\left(p_2 + \frac{A_2}{r_2^2}\right) \quad (\text{A.4})$$

7 where  $R_0 = \frac{r_2^2(r_0^2 - r_1^2)}{r_0^2(r_1^2 - r_2^2)}$  and  $R_1 = -\frac{r_1^2r_2^2(p_0 + p_2)}{r_1^2 - r_2^2}$ .

## 8 Appendix B. Sensitivity analyses

9 To determine the dominant physical parameters affecting the peak stress, sensitivity analyses were per-  
10 formed. To maintain physical intuition and facilitate interpretation, the dimensions of the problem are  
11 maintained. The values and trends of the sensitivities will strongly depend on the reference state, i.e., the  
1 point in the parameter space at which the derivatives will be taken. In order to maintain a specific link to  
2 an ongoing project and to practically relevant parameter values, the baseline system parameters were set to  
3 values employed in the IGLU project, compare Tab. B.1 and Fig. 3, and the maximum storage temperature  
4 of 90 °C was used aligned with domestic requirements. For a more detailed description of the prototype,  
5 the interested reader is referred to Miao et al. (2016).

To simplify the following expressions, we define

$$G(E_1, E_2, \alpha_1, \alpha_2, r_0, r_1, r_2) = E_1E_2R_0r_0^2(r_1^2 + r_2^2)(\alpha_2 - \alpha_1)\vartheta \quad (\text{B.1})$$

$$F(E_1, E_2, \nu_1, \nu_2, r_0, r_1, r_2) = E_1R_0r_0^2[r_2^2 + r_1^2 + \nu_2(r_2^2 - r_1^2)] - E_2r_2^2[r_0^2 + r_1^2 + \nu_1(r_0^2 - r_1^2)] \quad (\text{B.2})$$

The first-order partial derivatives of the circumferential tensile stress in the storage material at the in-  
terface  $\sigma_{\phi\phi}$  with respect to the independent material parameters ( $E_1, E_2, \nu_1, \nu_2, \alpha_1, \alpha_2$ ) and the tube pitch  
( $S = 2r_2$ ) as well as the tube dimensions ( $r_0, r_1$ ) can be found as

$$\mathcal{D}_{E_1} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial E_1} \right|_{r_1} = -\frac{E_2R_0r_0^2(r_1^2 + r_2^2)(\alpha_2 - \alpha_1)\vartheta F - GR_0r_0^2[r_2^2 + r_1^2 + \nu_2(r_2^2 - r_1^2)]}{F^2} \quad (\text{B.3})$$

$$\mathcal{D}_{E_2} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial E_2} \right|_{r_1} = -\frac{E_1R_0r_0^2(r_1^2 + r_2^2)(\alpha_2 - \alpha_1)\vartheta F + Gr_2^2[r_0^2 + r_1^2 + \nu_1(r_0^2 - r_1^2)]}{F^2} \quad (\text{B.4})$$

$$\mathcal{D}_{\nu_1} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial \nu_1} \right|_{r_1} = \frac{GE_2r_2^2(r_1^2 - r_0^2)}{F^2} \quad (\text{B.5})$$

$$\mathcal{D}_{\nu_2} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial \nu_2} \right|_{r_1} = \frac{GE_1 R_0 r_0^2 (r_2^2 - r_1^2)}{F^2} \quad (\text{B.6})$$

$$\mathcal{D}_{\alpha_1} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial \alpha_1} \right|_{r_1} = \frac{E_1 E_2 R_0 r_0^2 (r_1^2 + r_2^2) \vartheta}{F} \quad (\text{B.7})$$

$$\mathcal{D}_{\alpha_2} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial \alpha_2} \right|_{r_1} = -\frac{E_1 E_2 R_0 r_0^2 (r_1^2 + r_2^2) \vartheta}{F} \quad (\text{B.8})$$

$$\mathcal{D}_{r_0} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial r_0} \right|_{r_1} = -\frac{2GF - G \{E_1 R_0 2r_0^2 [r_2^2 + r_1^2 + \nu_2 (r_2^2 - r_1^2)] - E_2 r_2^2 2r_0^2 (1 + \nu_1)\}}{r_0 F^2} \quad (\text{B.9})$$

$$\mathcal{D}_{r_1} = \left. \frac{\partial \sigma_{\phi\phi}}{\partial r_1} \right|_{r_1} = -\frac{E_1 E_2 R_0 r_0^2 2r_1 (\alpha_2 - \alpha_1) \vartheta F - G [E_1 R_0 r_0^2 2r_1 (1 - \nu_2) - E_2 r_2^2 2r_1 (1 - \nu_1)]}{F^2} \quad (\text{B.10})$$

$$\mathcal{D}_S = \left. \frac{\partial \sigma_{\phi\phi}}{\partial S} \right|_{r_1} = -\frac{E_1 E_2 R_0 r_0^2 S (\alpha_2 - \alpha_1) \vartheta F - G \{E_1 R_0 r_0^2 S (1 + \nu_2) - E_2 S [r_0^2 + r_1^2 + \nu_1 (r_0^2 - r_1^2)]\}}{F^2} \quad (\text{B.11})$$

6 These sensitivities represent the gradient of  $\sigma_{\phi\phi}$  in the cement at the interface with respect to the speci-  
7 fied parameters. Note, that only one parameter is changed at a time, all other parameters remaining fixed.

8 Eq. (B.7) and Eq. (B.8) imply that the relation between the peak stress and the thermal expansion  
9 coefficient is linear, i.e. the sensitivity  $\mathcal{D}$  is independent of the parameter itself.  $\mathcal{D}_{\alpha_1}$  is positive while  $\mathcal{D}_{\alpha_2}$   
10 is negative. In the reference state,  $\mathcal{D}_{\alpha_1} = 125 \text{ GPa K}$  and  $\mathcal{D}_{\alpha_2} = -125 \text{ GPa K}$ . This is reminiscent of the fact  
11 that due to the inherent assumption of  $\alpha_1 \geq \alpha_2$  a decrease in stress can be brought about by a decrease of  
12  $\alpha_1 - \alpha_2$ , which can be achieved either by increasing  $\alpha_2$  or by decreasing  $\alpha_1$ .

13 As shown in Fig. B.4a and Fig. B.4b, the sensitivity to changes in the Young's modulus of the storage  
14 material is about three orders of magnitude higher than to that of the heat exchanger. Generally, the sensi-  
15 tivities decrease with an increase in the Young's moduli. This trend levels off for higher Young's moduli.  
16 As the stiffness of the pipe increases to very high values in comparison to the filler it completely dominates  
17 the deformation in the cement to the point where small changes in  $E_1$  have little effect. At the reference  
18 state, the sensitivities are approximately  $2.56 \text{ kPa GPa}^{-1}$  ( $E_1$ ) and  $1.2 \text{ MPa GPa}^{-1}$  ( $E_2$ ). Both sensitivities  
19 are positive, i.e. an increase in any Young's modulus causes an increase in stresses.

Table B.1: Material properties of the specific heat exchangers and the storage material used in IGLU project

Materials	Density $\rho_{\text{SR}} / \text{kg m}^{-3}$	Thermal Conductivity $\lambda_{\text{TS}} / \text{W m}^{-1} \text{K}^{-1}$	Thermal Expansion $\alpha_{\text{TS}} / \text{K}^{-1} \cdot 10^{-6}$	Heat Capacity $c_{\text{TS}} / \text{J kg}^{-1} \text{K}^{-1}$	Elastic Modulus $E / \text{GPa}$
<b>Heat exchanger</b>					
Alu-PE composite	1825	0.4	30	2200	68.9
<b>Storage material</b>					
Füllbinder L	1583	0.96	10.7	2083	1.9

20 The effect of the Poisson's ratio on the peak stress is sublinearly increasing, and the one of the storage  
21 material again has a stronger influence than that of the heat exchanger, see Fig. B.4c and Fig. B.4d. The  
22 sensitivity with respect to the Poisson's ratio is higher than the one with respect to changes in the Young's  
1 moduli. At the reference state, the sensitivities are approximately  $0.04 \text{ MPa}$  ( $\nu_1$ ) and  $-1.53 \text{ MPa}$  ( $\nu_2$ ). In  
2 particular  $\mathcal{D}_{\nu_1} > 0$  and  $\mathcal{D}_{\nu_2} < 0$  highlight the qualitatively opposite impact of both parameters. Increasing

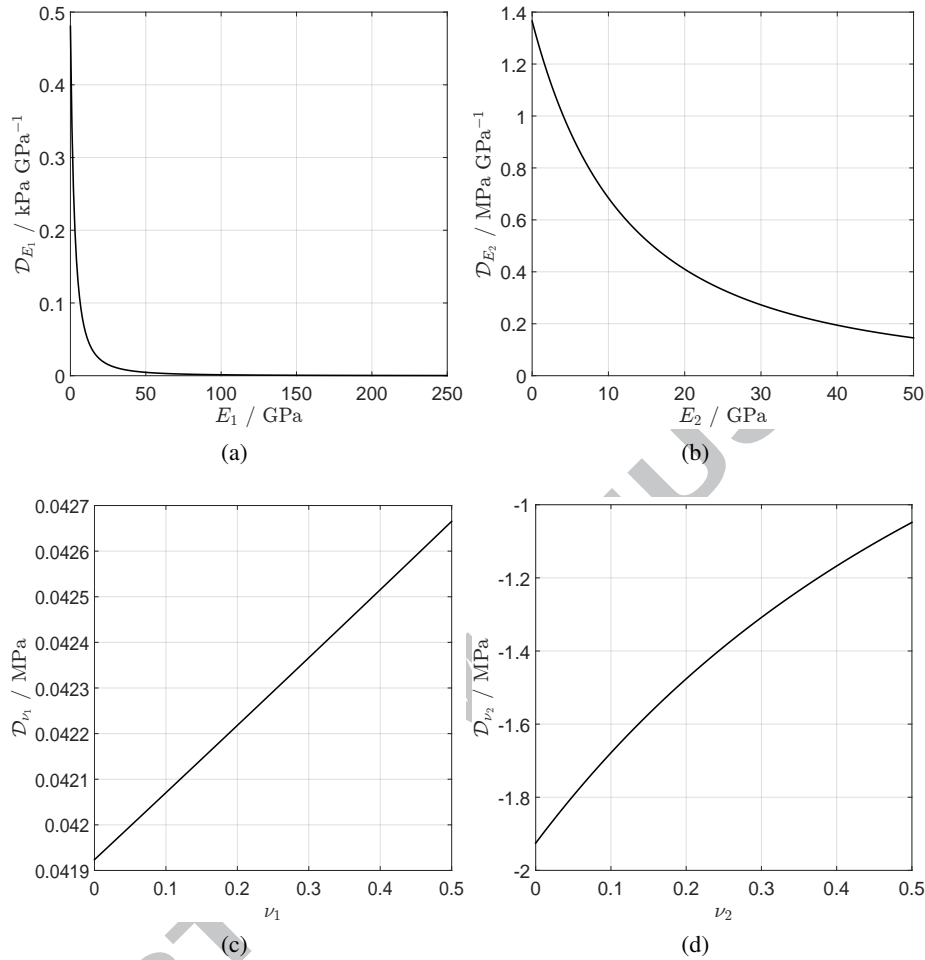


Figure B.4: Sensitivity of the objective tensile stress with respect to the material parameters. (a)/(b): Young's modulus of the heat exchanger/the storage material; (c)/(d): Poisson's ratio of the heat exchanger/the storage material.

3 the Poisson's ratio of the storage material can have a significant reducing effect on the peak stress.

4 The influence of the geometrical configuration of the heat exchanger is more polymorph. The sensitivity  
 5 of the inner diameter is positive and exhibits a comparatively dominant influence in the region where the  
 6 thickness of the heat exchanger is high, while the outer diameter has negative sensitivity when the thickness  
 7 is extremely low. As the thickness increases, the sensitivity is positive and exhibits a peak value. In the  
 8 reference state, the sensitivities are approximately  $0.014 \text{ MPa mm}^{-1}$  ( $r_0$ ) and  $0.003 \text{ MPa mm}^{-1}$  ( $r_1$ ). Under  
 9 standard conditions, the sensitivities of  $r_0$  and  $r_1$  have approximately the same order of magnitude as one  
 10 of  $S$  with respect to the peak stress.

11 The first-order partial derivative of the peak stress with respect to the tube pitch in Fig. B.5c shows the  
 12 existence of a threshold value above which the sensitivity is practically zero, i.e., a further increase in tube  
 1 pitch has no impact. At the reference state, the sensitivity is  $-0.005 \text{ MPa mm}^{-1}$ , i.e. increasing the tube  
 2 pitch will lower the peak tensile stress.

3 So far, the effect of each parameter was studied in isolation. In practical applications, an independent  
 4 choice may not always be possible. In the subsequent sections, the dimensionless solution is used to study

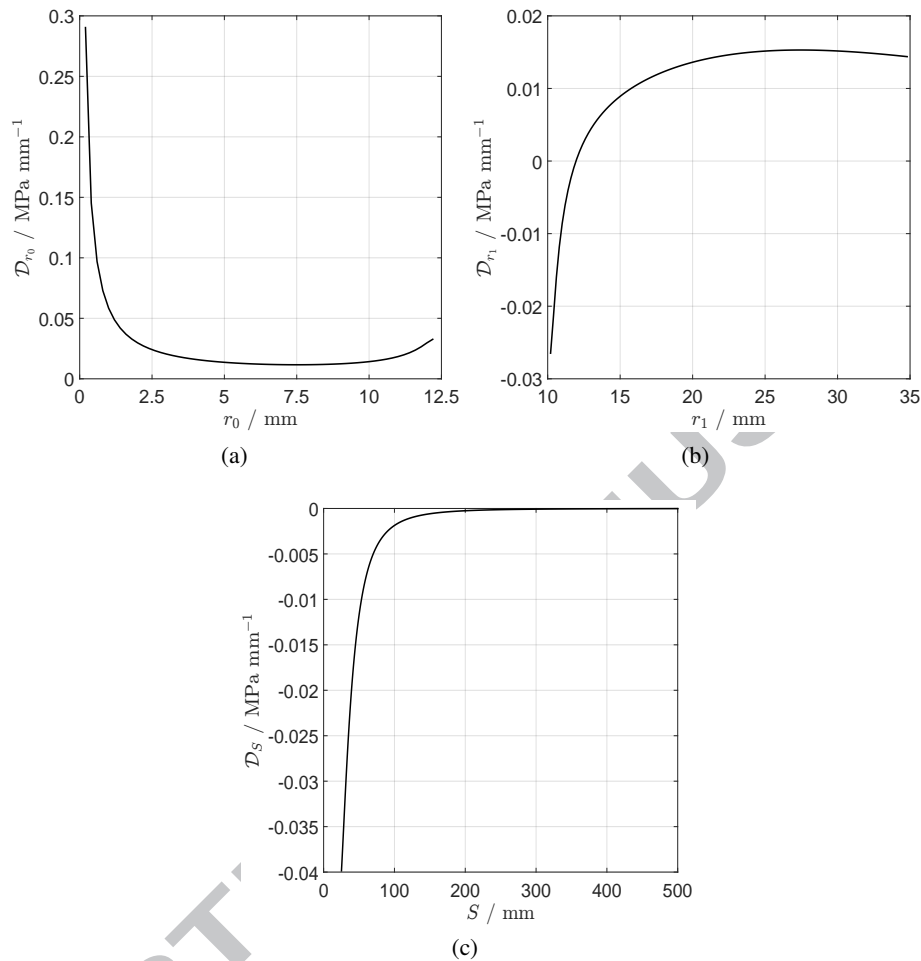


Figure B.5: Sensitivity of the objective tensile stress with respect to the geometrical parameters, (a)/(b) the inner/outer radius of the heat exchanger, (c) the tube pitch.

5 the effect of multiple parameters at once.

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- Analytical solution in the near field of the heat exchanger for extensive sensitivity analyses.
- Optimisation of thermal energy stores from a thermo-mechanical perspective.
- Case studies for typical heat exchanger materials and geometries.
- Analysis of water-saturated cement heat store.

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