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# Energy considerations for the persistence of a monimolimnion – molecular diffusion

By Bertram Boehrer
UFZ - Gewässerforschung
Brückstraße 3a, D - 39114 Magdeburg, Germany
boehrer@monimo.gm.ufz.de

## SCOPE

In an infinitly large water body with fresh water overlying salty water, the smoothing of an initially absolutely sharp salinity jump is considered (Fig. 1). No advective processes are present. Due to diffusion, salt is transfered vertically and the water column gains potential energy which can be evaluated analytically and numerically and a mathematical relation between potential energy gain and diffusion coefficient is achieved.

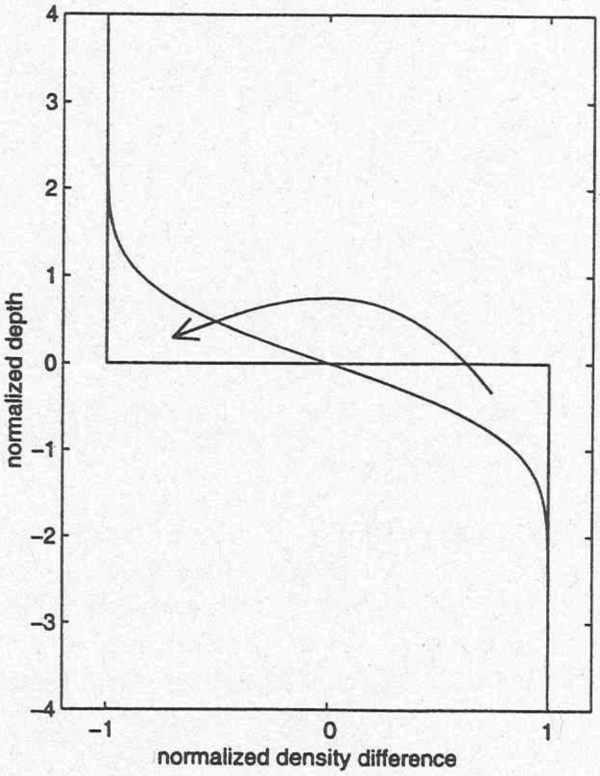


Figure 1: An initially sharp salinity gradient is smoothed by molecular diffusion. A salinity parcel gained potential energy by vertical transport

## INTRODUCTION

Meromixis can establish, if the deep water body of a lake has sufficiently high density to resist the deep recirculation in the cold or windy season. Either this water of higher density has to be introduced from the outside by a salty inflow (like Fig. 2), or vica versa by a fresh inflow into a salt-lake from surface ('exogenic', Wetzel 1983) or groundwater sources ('creogenic', Wetzel 1983). On the other hand, these denser waters can be produced internally by evaporation in a salt lake or by biological activity ('biogenic', Wetzel 1983), as the plants and animals contain a higher concentration of dissolved matter withing their shells. In addition, chemical transformation of solutes can cause a change in density, especially if supported by microbiological activity.

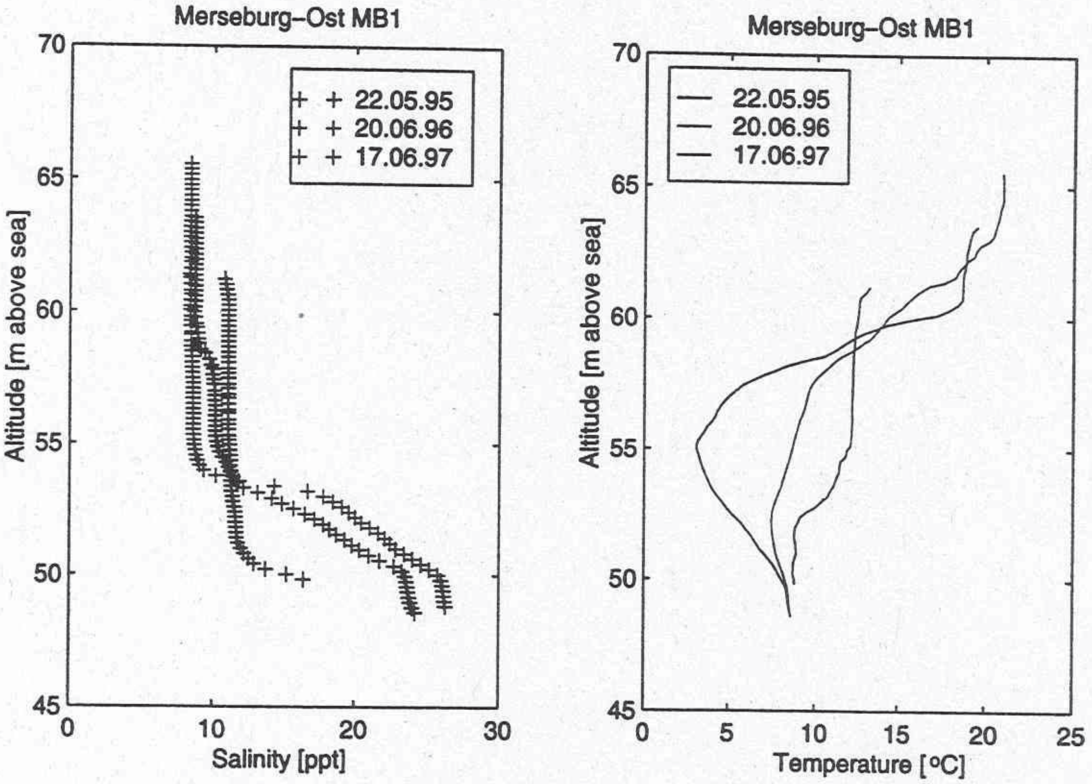


Figure 2: Salinity and temperature profiles in Merseburg-Ost Lake 1b at sampling point MB1 in the years 1995, 96, 97 while the waterlevel was recovering from its artificially low level by unforced groundwater inflows (also Boehrer and Zippel, 1997)

When a deep layer of dense water exists, a number of processes is involved in its dissolution into the overlying mixolimnion. As the stable stratification is at a lower potential energy than the perfectly mixed water column would be, energy has to be supplied from various sources to shift the lake towards a perfectly mixed water body.

Potential energy considerations allow a comparison of the importance of the various processes. In some meromictic lakes several of the contributions might be at a similar magnitude, while in others one factor might be dominant. Especially for artificial lakes, e.g. the mining lakes from opencast mines, predictions of the recirculation pattern would be desirable, even before the lakes exist in final form (e.g Boehrer et al. 1998).

The discussion of all contributions to meromixis, and all diluting processes would be beyond the scope of this contribution. We concentrate on a model of an infinite water column, where a fresh water overlies salty water initially separated by a sharp interface. Temperature and advective processes are not considered. We evaluate the contribution of the diffusion to the potential energy of the water column, yielding a power per area.

## DIFFUSION

Diffusion is the consequence of a stochastic movement of the individual particles of an observed substance. The process results in a down-gradient net transport. The salinity in the lower water body (corresponds to the monimolimnion of a meromictic lake) is higher than in the upper water body (corresponds to the mixolimnion of a lake). A net salt transport opposed to gravity is the result. Thus the potential energy of the stratification is increased.

We quantify this process, for an initially sharp density step. Fig. 1 shows, how the sharp density gradient has smoothed after a time span $t$. $\kappa$ denotes the diffusion coefficient. For simplicity the vertical coordinate is chosen $z = 0$ in the density step, the density $\rho$ is normalized by $M = 2(\rho - \bar{\rho})/\Delta\rho$, where $\bar{\rho}$ is the density of the fluid half across the density difference $\Delta\rho$. The problem is considered not to have any bounds. The diffusion obeys (e.g. Batchelor, 1967 pp 187):

$$\frac{\partial M}{\partial t} = \kappa \frac{\partial^2 M}{\partial z^2}, \qquad \text{with } M(z, t = 0) = \pm 1 \text{ for } z <> 0 \tag{1}$$

The solution is known

$$M(z,t) = \frac{-1}{\sqrt{\pi \kappa t}} \int_0^z \exp(-\frac{z'^2}{4\kappa t}) dz' = -\text{erf}(\eta) \tag{2}$$

including $\eta = z/\sqrt{4\kappa t}$. The question is now, how much potential energy was gained through the vertical transport of salinity of the lower shaded area to the upper shaded area. Because of symmetry, the energy for the tranport is double the potential energy gain due to the shaded area on one side of $z = 0$:

$$E_d = 2g \int_0^\infty z(\rho(z) - \bar{\rho} - \frac{1}{2}\Delta\rho) dz \tag{3}$$

substitute $\rho$ by $M$:

$$E_d = g\Delta\rho \int_0^\infty z(M(z,t) + 1) dz \tag{4}$$

and $\eta = z/\sqrt{4\kappa t}$

$$E_d = 4g\kappa t \Delta\rho \int_0^\infty \eta(-\text{erf}(\eta) + 1) d\eta \tag{5}$$

The evaluation of the integral yields 0.25, see appendix, and thus we can calculate the potential energy input by the molcular diffusion:

$$E_d = g\,\kappa\,\Delta\rho t \tag{6}$$

## CONCLUSIONS

A number of aspects of the result (eq. 6) disserve a closer look. Firstly a diffusing stable stratification is gaining potential energy. The amount of gained energy over time (power) is the derivative of eq. 6 after time

$$P_d = g\,\kappa\,\Delta\rho \tag{7}$$

and is constant with time. A density difference, as found in Merseburg-Ost Lake 1b, (Boehrer and Zippel, 1997) would indicate:

$$P_d = 9.81\mathrm{m/s^2} \cdot 1 \cdot 10^{-9}\mathrm{m^2/s} \cdot 15\,\mathrm{kg/m^3} \approx 1.5 \cdot 10^{-7}\mathrm{W/m^2} \tag{8}$$

Secondly the energy input over time (power) is linear in $\Delta\rho$. This means any shape of halocline can be approximated by a number of steplike salinity profiles. Therefore it can be concluded that the gain of potential energy over time depends only on the density difference between the waters at infinity, but not the shape of the gradient.

## REFERENCES


BATCHELOR, G.K., 1983. *An introduction to fluid dynamics*, Cambridge University Press, Cambrigde, ISBN 0 521 04118x

BOEHRER, B., HEIDENREICH, H., SCHIMMELE, M. AND SCHULTZE, M., 1998. *Numerical prognosis for salinity profiles of future lakes in the opencast mine Merseburg-Ost*, Int. J. Salt Lake Research (in press)

BOEHRER, B. AND ZIPPEL, B., 1997. *Jahresbericht 1997 für den Tagebau Merseburg-Ost.* UFZ-Gewässerforschung, Brückstr. 3a, D-39114 Magdeburg.

BRONSTEIN, I.N. AND SEMENDJAJEW, K.A., 1981. *Taschenbuch der Mathematik.* 21st ed., Verlag Harry Deutsch, Thun und Frankfurt(Main), ISBN 3 87144-492-8.

WETZEL, R.G., 1983. *Limnology*, 2nd ed., Saunders College Publishing, ISBN 0-03-057913-9.


## APPENDIX

With the help of Rainer Feistel (many thanks again!), the integral was evaluated analytically. We start from the eq. 4 including $M(z,t)$ from eq. 2.

$$E_d = g\Delta\rho \int_0^\infty z\Big(\frac{-1}{\sqrt{\pi\kappa t}} \int_0^z \exp(-\frac{z'^2}{4\kappa t})dz' + 1\Big)dz' \tag{9}$$

Partial integration yields

$$E_d = g\Delta\rho\Big[\frac{z^2}{2}\Big(\frac{-1}{\sqrt{\pi\kappa t}}\int_0^z \exp(-\frac{z'^2}{4\kappa t})dz' + 1\Big)\Big]_0^\infty + g\Delta\rho\int_0^\infty \frac{z^2}{2}\frac{1}{\sqrt{\pi\kappa t}}\exp(-\frac{z^2}{4\kappa t})dz \tag{10}$$

From Bronstein (1981, p.66), the first term is zero, and the second term is:

$$E_d = g\Delta\rho\kappa t \tag{11}$$

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**Bertram Boehrer and Michael Schimmele (eds.)**

UFZ-Umweltforschungszentrum Leipzig-Halle GmbH
Permoserstraße 15, D-04318 Leipzig

Sektion Gewässerforschung Magdeburg
Brückstraße 3a, D-39114 Magdeburg

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