UFZ-Discussion Papers

Department of Ecological Modelling

27/2005

The Private and Public Insurance Value of Conservative Biodiversity Management

Stefan Baumgärtner, Martin Quaas

December 2005

UFZ Centre for Environmental Research Leipzig-Halle GmbH,
Permoserstr. 15, 04318 Leipzig, Germany

Email of corresponding author: Martin.Quaas@ufz.de
The Private and Public Insurance Value of Conservative Biodiversity Management

Stefan Baumgärtner\textsuperscript{a,§} and Martin F. Quaas\textsuperscript{b,*}

\textsuperscript{a} Department of Economics, University of Heidelberg, Germany
\textsuperscript{b} Department of Ecological Modelling, UFZ-Centre for Environmental Research Leipzig-Halle, Leipzig, Germany

22 December 2005

Abstract. The ecological literature suggests that biodiversity reduces the variance of ecosystem services. Thus, conservative biodiversity management has an insurance value to risk-averse users of ecosystem services. We analyze a conceptual ecological-economic model in which such management measures generate a private benefit and, via ecosystem processes at higher hierarchical levels, a positive externality on other ecosystem users. We find that ecosystem management and environmental policy depend on the extent of uncertainty and risk-aversion as follows: (i) Individual effort to improve ecosystem quality unambiguously increases. The free-rider problem may decrease or increase, depending on the characteristics of the ecosystem and its management; in particular, (ii) the extent of optimal regulation may decrease or increase, depending on the relative size of private and external effects of management effort on biodiversity; and (iii) the welfare loss due to free-riding may decrease or increase, depending on how biodiversity influences ecosystem service provision; it increases, unless higher biodiversity greatly decreases the variance of ecosystem services.

JEL-Classification: Q57, H23, D81, D62

Keywords: biodiversity, ecosystem services, ecosystem management, free-riding, insurance, public good, risk-aversion, uncertainty

\textsuperscript{§}Corresponding author: Alfred-Weber-Institute of Economics, University of Heidelberg, Bergheimer Str. 20, D-69115 Heidelberg, Germany. Phone: +49.6221.54-8012, fax: +49.6221.54-8020, email: baumgaertner@uni-hd.de.

\textsuperscript{*}We are grateful to Salvatore di Falco and Ralph Winkler for helpful discussion and comments. Financial support by the Volkswagen Foundation under grant II/79628 is gratefully acknowledged.
1 Introduction

Human well-being depends in manifold ways on ecosystem services, which are understood as the various benefits provided by natural or managed ecosystems (Daily 1997, Millennium Ecosystem Assessment 2005). Examples include goods such as food, fuel or fibre; and services such as pollination or the regulation of local climate, pests, diseases or water runoff from a watershed. In a world of uncertainty, human well-being depends not only on the mean level at which such services are being provided, but also on their statistical distribution. Biodiversity can reduce the variance at which desired ecosystem services are provided. This means, biodiversity can provide insurance to risk averse users of these systems, e.g. crop, orchard or livestock farmers, or water utility managers. In this paper, we analyze how risk-averse ecosystem managers make use of this insurance function of biodiversity when management measures generate both a private benefit and, via ecosystem processes at higher hierarchical levels, positive externalities on other ecosystem users. We study the implications of uncertainty and risk-aversion for ecosystem management and environmental policy.

The analysis is based on a conceptual ecological-economic model. Ecosystem services (e.g. pollination of orchards by insects) are random because of exogenous sources of risk (e.g. winter temperature); their distribution (mean and variance) is determined by ecosystem quality (biodiversity). Ecosystem quality, in turn, can be influenced by management action (e.g. setting aside land for wetlands and hedges as habitat for insects) that affects ecosystem processes at different scales. Ecosystem users are risk-averse and choose a management action such as to maximize utility from ecosystem services (e.g. income from orchard farming). Our modelling of biodiversity and the provision of ecosystem services captures three stylized facts about ecosystem functioning that emerged from recent theoretical, experimental and observational research in ecology (which is surveyed in Section 2):

- The mean level of ecosystem services increases with biodiversity.
- The variance of ecosystem services decreases with biodiversity.
Local biodiversity is affected by ecosystem processes at different hierarchical scales. These stylized ecological facts are of economic relevance. Biodiversity increasing management creates benefits in terms of a higher mean level and a reduced variance of ecosystem services. In particular, an individual manager’s action affects biodiversity via ecosystem processes at different scales. At a lower scale, benefits accrue exclusively to him. At a higher scale, his action contributes to increasing local biodiversity for other users, thereby generating a positive externality. For example, by setting aside land on his farm as habitat for insects, an individual farmer increases the local level of biodiversity on his farm and also contributes – via metapopulation dynamics – to biodiversity on other farms.

Our analysis of endogenous environmental risk and ecosystem management is inspired by Crocker and Shogren (1999, 2001, 2003) and Shogren and Crocker (1999), who have developed the idea that environmental risk is endogenous, that is, economic decision makers bearing environmental risk influence their risk through their actions. They have formalized decision making under uncertainty in this context by conceptualizing ecosystems as lotteries. The role of biodiversity as a natural insurance has already been studied for the case of a single decision maker managing a private resource (Baumgärtner, forthcoming, Quaas et al., forthcoming). In the field of agricultural economics a number of studies have analyzed the influence of crop diversity on the mean and variance of agricultural yields (Smale et al. 1998, Schläpfer et al. 2002, Widawsky and Rozelle 1998, Zhu et al. 2000) and on the mean and variance of farm income (Di Falco and Perrings 2003, 2005, Di Falco et al. 2005). It has been conjectured that risk averse farmers use crop diversity in order to hedge their income risk (Birol et al. 2005a, 2005b, Di Falco and Perrings 2003). However, biodiversity has not only a private insurance function, but provides public insurance benefits as well. This public-good aspect and

\footnote{In this respect, biodiversity plays a similar role for farmers as other risk changing production factors, such as e.g. nitrogen fertilizer or pesticides (Horowitz and Lichtenberg 1993, 1994a, 1994b).}
the associated environmental policy issues have not been studied so far.

The conventional wisdom on the use (or provision) of a public good under uncertainty seems to be that the more uncertainty and the higher the risk aversion of individual decision makers, the less severe is the problem of overuse (or under-provision) of the public good (Bramoullé and Treich 2005, Sandler and Sterbenz 1990, Sandler et al. 1987). In a sense, this literature suggests that private uncertainty and risk-aversion increase the efficiency of the private provision of public goods. The focus is on the properties of the utility function which are necessary and sufficient for this result. Bramoullé and Treich (2005) derive conditions on the curvature properties of the marginal utility function. Sandler et al. (1987) discuss in addition the role of separability between utility from the private and the public good. Both contributions are not interested in the ‘technology’ of public good provision. They model the production of the public good (or public bad) in a trivial way, i.e. one unit of money spent on providing the public good equals one unit of the public good provided. Sandler and Sterbenz (1990) consider the open access harvesting of a renewable resource, thus taking a more detailed look on how individual harvesting efforts cause externalities for other users of the ecosystem. Also, all these contributions study how uncertainty affects individual behaviour in equilibrium, but do not explicitly address the questions of how severe is the problem of market failure in welfare terms, or how to solve this problem by suitable policy measures such as e.g. Pigouvian taxes or subsidies. In this regard, Aronsson and Blomquist (2003) study the optimal and second-best taxation of a dirty consumption good which causes a (public bad) pollution problem. They show that the optimal tax increases with uncertainty.

Against this background, our analysis makes three contributions. First, we employ a detailed and differentiated model of ecosystem functioning, which captures how individual actions translate into private and public benefits. Second, we explicitly study the extent of market distortion and optimal regulation, and how those depend on the degree of uncertainty and risk-aversion, by employing a measure of social welfare. Third, we analyze how the relationship between uncer-
tainty and the free-rider problem depends on ecosystem properties and processes. Our analysis thereby yields insights into how the optimal regulation of biodiversity management under uncertainty hinges upon ecosystem characteristics. Of particular importance in this respect is the insurance function of biodiversity.

We show that with increasing uncertainty and risk-aversion the private efforts to improve ecosystem quality increase, because ecosystem managers, when choosing a management action under uncertainty, take into account biodiversity’s insurance value and manage the ecosystem such as to obtain the optimal balance between high expected yield and insurance. As a consequence, the higher the uncertainty and the more risk-averse the ecosystem managers are, the higher is the resulting ecosystem quality. Thus, under uncertainty the ecosystem management is more conservative, and the resulting ecosystem quality is higher, than it would be in a world of certainty. Yet, the effect of uncertainty on the free-rider problem is ambiguous. The extent of the optimal regulatory intervention may decrease or increase with uncertainty depending on the relative effects of management measures on biodiversity via the lower, i.e. individual, and the higher, i.e. public, scale. Additional information about ecosystem functioning is required in order to assess how uncertainty influences the welfare loss due to free-riding, which also may decrease or increase with uncertainty. If biodiversity reduces the variance of ecosystem services very strongly, the welfare loss decreases with uncertainty. If, on the other hand, biodiversity hardly reduces the variance of ecosystem services, the welfare loss increases with uncertainty. In other words, for poorly manageable ecosystems, the free-rider problem increases with uncertainty.

The paper is organized as follows. Section 2 briefly reviews the ecological background on ecosystem functioning and how biodiversity affects the provision of ecosystem services. In Section 3, we specify an ecological-economic model of an ecosystem which is managed for the ecosystem services that it provides. The analysis and results are presented in Section 4, with all proofs and formal derivations contained in the Appendix. Section 5 concludes.
2 Ecological background: biodiversity and the provision of ecosystem services

Over the past fifteen years, there has been intensive research in ecology on the role of biodiversity for ecosystem functioning and the provision of ecosystem services. *Biodiversity* has been defined as ‘the variability among living organisms from all sources ... and the ecological complexes of which they are part’ (CBD 1992), which encompasses a wide spectrum of biotic scales, from genetic variation within species to biome distribution on the planet (Gaston 1996, Purvis and Hector 2000, Wilson 1992). Biodiversity can be described in terms of numbers of entities (e.g. genotypes, species, or ecosystems), the evenness of their distribution, the differences in their functional traits, and their interactions. The simplest measure of biodiversity at, say, the species level is therefore simply the number of different species (‘species richness’). Much of ecological research has relied on this measure when quantifying biodiversity, although more encompassing information has also been employed (Baumgärtner 2004).

Research on the role of biodiversity for ecosystem functioning and the provision of ecosystem services builds on (i) observations of existing ecosystems, (ii) controlled experiments both in the laboratory and in the field (‘pots and plots’) and (iii) theory and model analysis. While the discussion of results has been, at times, heated and controversial, there now seems to be a consensus over some of the basic results from this research (Hooper et al. 2005, Kinzig et al. 2002, Loreau et al. 2001, 2002). Among other insights three ‘stylized facts’ about biodiversity and ecosystem functioning emerged which are of crucial importance for the issue studied here: 2

---

2The article by Hooper et al. (2005) is a committee report commissioned by the Governing Board of the Ecological Society of America. Some of its authors have previously been on opposite sides of the debate. This report surveys the relevant literature, identifies a consensus of current knowledge as well as open questions, and can be taken to represent the best currently available ecological knowledge about biodiversity and ecosystem functioning.
1. **Biodiversity may enhance the mean level of ecosystem services.** In many instances, an increase in the level of biodiversity monotonically increases the mean absolute level at which certain ecosystem services are provided. This effect decreases in magnitude with the level of biodiversity.

2. **Biodiversity may reduce the variance of ecosystem services.** In many instances, an increase in the level of biodiversity monotonically decreases the temporal and spatial variability of the level at which these ecosystem services are provided under changing environmental conditions. This effect decreases in magnitude with the level of biodiversity.

3. **Local biodiversity is affected by ecosystem processes at different hierarchical scales.** Ecosystems are hierarchically structured, with processes operating at different spatial and temporal scales and interacting across scales. Species diversity is typically influenced differently by processes at different scales. Accordingly, biodiversity management measures at different scales have different impact on local biodiversity.

These three stylized facts are now briefly discussed in turn.\(^3\)

### 2.1 Biodiversity may enhance the mean level of ecosystem services

There are two primary mechanisms through which species or functional diversity may increase the mean absolute level at which certain ecosystem services are provided (Figure 1):

(i) Only one or a few species might have a large effect on any given ecosystem service. Increasing species richness, i.e. the number of different species, increases the likelihood that those key species would be present in the system.

\(^3\)For a more detailed and encompassing discussion of these findings, and references to the literature, see Hooper et al. (2005).
This is known as the ‘sampling effect’ or the ‘selection probability effect’ (Figure 1A).

(ii) Species or functional richness could increase the level of ecosystem services through complementarity – i.e. species use different resources, or the same resources but at different times or different points in space – and facilitation – i.e. positive interactions among species so that e.g. certain species alleviate harsh environmental conditions or provide a critical resource for other species (Figure 1B).

Figure 1: Ecological theory has suggested two basic mechanisms of how biodiversity could increase the mean absolute level of ecosystem services: sampling or selection probability effect (A), and complementarity or facilitation (B). Points show individual treatments, and lines show the average response. (Figures are taken from Tilman 1997, as compiled by Hooper et al. 2005.)

Complementarity, facilitation and sampling effects will all lead to a saturating average impact of species richness on the level of some ecosystem service (Figure 1A, B).

Experiments have confirmed the important role of these two primary mechanisms through which biodiversity may increase the mean absolute level of certain ecosystem services. In these experiments, the responses to changing diversity are
strongest at low levels of species richness and generally saturate at 5-10 species. It has also become evident that complementarity, facilitation and sampling/selection effects are all relevant and can be observed. They are not necessarily mutually exclusive, but they may be simultaneously or sequentially at work in one system.

These theoretical and experimental findings need to be qualified in a number of respects:

- The exact response of ecosystem services on changes in biodiversity is determined at least as much by differences in species composition, i.e. which species and functional traits are lost and remain behind, as by species richness, i.e. how many species are lost.

- Patterns of response to experimental manipulation of species richness vary for different ecosystem processes and services, different ecosystems, and even different compartments within ecosystems.

- Varying the diversity and composition of an ecological community at more than one trophic level can lead to more idiosyncratic behavior than varying diversity of primary producers alone.

- The different patterns identified may or may not reflect actual patterns seen for a particular ecosystem under a particular scenario of species loss or invasion, which will depend not only on the functional traits of the species involved, but also on the exact pattern of environmental change and the species traits that determine how species respond to these changes.

2.2 Biodiversity may reduce the variance of ecosystem services

Ecological theory, both via simple reasoning and via mathematical models, has lead to the understanding that a diversity of species with different sensitivities to a suite of environmental conditions should lead to greater stability of ecosystem properties. The basic idea is that with increasing number of functionally different
species, the probability increases that some of these species can react in a functionally differentiated manner to external disturbance of the system and changing environmental conditions. In addition, the probability increases that some species are functionally redundant, such that one species can take over the role of another species when the latter goes extinct. This is what ecologists have been calling an ‘insurance effect’ of biodiversity in carrying out ecological processes (e.g. Yachi and Loreau 1999). With this logic, the number of species or functional traits necessary to maintain ecosystem processes under changing environmental conditions increases with spatial and temporal scales.

Several mathematical models generally support these hypotheses and highlight the role of statistical averaging – the so-called ‘portfolio effect’ – for the result: if species abundances are negatively correlated or vary randomly and independently from one another, then overall ecosystem properties are likely to vary less in more diverse communities than in species-poor communities.\(^4\) The strength of the modeled effects of diversity depends on many parameters, including the degree of correlation among different species’ responses, the evenness of distribution among species’ abundances, and the extent to which the variability in abundances scales with the mean.

While theory is well developed, controlled experiments are very difficult to carry out, because one needs to make sure that the effect of species diversity is not confounded by other variables, such as e.g. soil fertility or disturbance regime. Nevertheless, considerable evidence exists from experimental studies in a variety of ecosystems that increasing species diversity can increase the stability of ecosystem processes and services in response to changing environmental conditions and species loss. As an example, Figure 2 shows experimental results for aboveground plant biomass production in response to climatic variability in a Minnesota grassland (Figure 2A), and net ecosystem CO\(_2\) flux in a microbial microcosm (Figure 2B). However, results of these experiments may be confounded by a variety of variables other than species richness or diversity, which has raised considerable

\(^4\)This is similar to the effect of diversifying a portfolio of financial assets, e.g. stocks.
controversy over the interpretation of these results. And while species richness or the Shannon-Wiener-index of species diversity was statistically significant in all these experiments, species composition (where investigated) had an at least equally strong effect on stability. Also, while the overall stability patterns found are as predicted from theory, the experiments so far give little insights about the underlying basic mechanisms.

2.3 Local biodiversity is influenced by ecosystem processes at different hierarchical scales

Ecosystems are hierarchically structured (Holling 2001, O’Neill 1986), with ecological processes operating at different scales and interacting across scales. A hierarchy of different scales is often apparent in the temporal and spatial structure of ecosystems, but it can be important in other dimensions as well, for instance with respect to different taxonomic levels (Godfray and Lawton 2001). As for biodiversity, a hierarchical temporal structure is captured in most basic models of
population dynamics: populations introduced into new areas typically grow exponentially on a fast time scale, before density dependent restrictions limit growth and determine the long-run carrying capacity (Levin 2000). Very often, there is a close relationship between hierarchically structured spatial and temporal scales (Gillson 2004, Leibold et al. 2004). Higher hierarchical levels are characterized by slower and longer-range processes. Processes on different hierarchical levels typically interact: higher level processes impose constraints on lower levels; and lower level processes provide the underlying mechanisms from which higher level properties emerge (Levin 2000).

The (spatial) hierarchical structure of an ecosystem has a particular influence on its biodiversity (Tilman 1994), since species diversity is influenced differently by processes at different spatial scales. This has been shown both theoretically (Bond and Chase 2002, Weitz and Rothman 2003) and empirically in experiments (Cadotte and Tadashi 2005) and field work (Chase and Leibold 2002, Cushman and McGarigal 2002). Local processes at the patch level and regional processes, in particular the dispersal processes between patches, jointly regulate species diversity and composition in many systems (e.g., Shurin and Allen 2001).

The hierarchical structure of ecosystems constitutes a particular challenge for ecosystem management, since it is necessary to adapt the scales at which management operates to the relevant scales of the ecosystem (Levin 2000, Peterson et al. 1998). In our model, we capture this by identifying the management actions affecting processes at the higher hierarchical level with the aggregate action of ecosystem managers, while the individual management actions influence the ecosystem processes at the lower hierarchical level.

3 Ecological-economic model

We consider an ecosystem which is managed for some ecosystem service that it provides. Due to stochastic fluctuations in environmental conditions the provision of the ecosystem service is uncertain. Its statistical distribution depends on the
state of the ecosystem in terms of biodiversity (‘ecosystem quality’), which is influ-
enced by how the system is being managed. As a result, the statistical distribution
of ecosystem service and, hence, of income from ecosystem use depend on ecosys-
tem management. We capture these relationships in a stylized ecological-economic
model as follows.

3.1 Ecosystem management

There are \( n \) ecosystem managers, numbered by \( i = 1, \ldots, n \). Each ecosystem
manager can choose a level \( x_i \) of individual effort to improve ecosystem quality.
The level of ecosystem quality \( q_i \) is specific to user \( i \). It increases with user \( i \)'s
individual effort \( x_i \) and the aggregate effort \( X \):

\[
q_i = q(x_i, X) \quad \text{with} \quad q_x \geq 0, \quad q_{xx} \leq 0, \\
q_X \geq 0, \quad q_{XX} \leq 0, \quad q_{xx} = q_{Xx} = 0, 
\]

(1)

where

\[
X = \sum_{i=1}^{n} x_i 
\]

(2)

and subscripts \( x \) and \( X \) denote partial derivatives with respect to \( x_i \) and \( X \) re-
spectively. We assume that \( q_x > 0 \) if \( q_X = 0 \), and that \( q_X > 0 \) if \( q_x = 0 \) (otherwise
results are trivial) and that all individuals face the same type of ecosystem, so that
the function \( q(\cdot, \cdot) \) has no index \( i \).

Assumption (1) expresses the idea that the level of ecosystem quality relevant
to user \( i \) is determined by both the individual management action \( x_i \) taken by
user \( i \) and positive externalities from the joint effort \( X \) of all ecosystem managers.
How the function \( q_i \) depends on \( x_i \) and \( X \) reflects the hierarchical structure of the
ecosystem (cf. Section 2.3): it captures how the individual effort \( x_i \) affects local
ecological processes, how the aggregate effort \( X \) affects ecological processes at a
higher scale, and how these processes interact to determine local ecosystem quality.

In the extreme, \( q_x > 0 \) and \( q_X \equiv 0 \) corresponds to a situation where only
local ecological processes are relevant and therefore management effort is purely
private with no spill-overs to others. The other extreme, \( q_x \equiv 0 \) and \( q_X > 0 \),
corresponds to a situation where local ecosystem quality is completely determined by higher-scale ecological processes, such that management effort is a pure public good.

Given ecosystem quality $q_i$, the ecosystem provides user $i$ with the ecosystem service at level $s_i$ which is a random variable that follows a normal distribution. Its mean, $\mathcal{E}s_i$, and variance, $\text{var} s_i$, depend on ecosystem quality $q_i$:

$$\mathcal{E}s_i = \mu(q_i) \quad \text{and} \quad \text{var} s_i = \theta \sigma^2(q_i) ,$$ \hspace{1cm} (3)

where $\mathcal{E}$ is the expectancy operator. An increase in the parameter $\theta > 0$ models a mean-preserving spread of risk (Rothschild and Stiglitz 1970). This allows us to discuss the effects of increased uncertainty in a convenient way. Again, since all individuals face the same type of ecosystem, the probability distribution of the ecosystem service is the same for all users who have the same ecosystem quality $q$. In accordance with ecological evidence (cf. Sections 2.1 and 2.2), the functions $\mu$ and $\sigma^2$ are assumed to have the following properties:

$$\mu' > 0, \quad \mu'' \leq 0 \quad \text{and} \quad \sigma'^2 < 0, \quad \sigma''^2 \geq 0 ,$$ \hspace{1cm} (4)

where the prime denotes a derivative. For each user, the mean level of ecosystem service provision increases, and its variance decreases, with ecosystem quality $q$. Both effects decrease in magnitude with the level of ecosystem quality.

### 3.2 Income

Improving ecosystem quality carries costs, which are purely private and are described by the cost function

$$c(x_i) \quad \text{with} \quad c' > 0, \quad c'' \geq 0 .$$ \hspace{1cm} (5)

Balancing the benefits from ecosystem services and the costs of ecosystem management, manager $i$’s net income from ecosystem use is

$$y_i = s_i - c(x_i) ,$$ \hspace{1cm} (6)
where we have assumed that the ecosystem service directly translates into monetary income. Since the ecosystem service is a random variable, net income $y_i$ is a random variable, too. With the normal distribution of ecosystem service $s_i$, where the mean and variance are given by $\mathcal{E}s_i = \mu(q(x_i, X))$ and $\text{var } s_i = \sigma^2(q(x_i, X))$ (Equations 3 and 1), the manager’s income $y_i$ is normally distributed as well, with mean $\mathcal{E}y_i$ and variance $\text{var } y_i$:

$$
\begin{align*}
\mathcal{E}y_i &= \mathcal{E}s_i - c(x_i) = \mu(q(x_i, X)) - c(x_i) \quad \text{and} \\
\text{var } y_i &= \text{var } s_i = \sigma^2(q(x_i, X)).
\end{align*}
$$

(7) \hspace{2cm} (8)

That is, the mean income is given by the mean ecosystem service minus the costs of managing ecosystem quality; the variance of income equals the variance of ecosystem service.

### 3.3 Preferences

All ecosystem managers are assumed to have identical preferences over their uncertain income $y_i$. These are represented by a von Neumann-Morgenstern expected utility function

$$
U_i = \mathcal{E}u(y_i).
$$

(9)

The Bernoulli utility function $u(y_i)$ is increasing ($u' > 0$) and strictly concave ($u'' < 0$), i.e. the decision maker is non-satiated and risk-averse.\footnote{While risk-aversion is a natural and standard assumption for farm households (Besley 1995, Dasgupta 1993: Chapter 8), it appears as an induced property in the behavior of (farm) companies which are fundamentally risk neutral but act as if they were risk averse when facing e.g. external financing constraints or bankruptcy costs (Caillaud et al. 2000, Mayers and Smith 1990).} In order to obtain simple closed-form solutions, we assume that manager $i$’s preferences are given by the constant absolute risk aversion Bernoulli utility function

$$
u(y_i) = -e^{-\rho y_i},$$

(10)

where $\rho > 0$ is a parameter describing the manager’s Arrow-Pratt measure of absolute risk aversion (Arrow 1965, Pratt 1964). Since income follows a normal
distribution with mean $\mathbb{E}y_i$ and variance $\text{var} y_i$, the von Neumann-Morgenstern expected utility function (9) is (see Appendix A.1):

$$U_i = \mathbb{E}y_i - \frac{\rho}{2} \text{var} y_i.$$  \hspace{1cm} (11)

4 Analysis and results

After introducing the notion of insurance value in Section 4.1, the analysis proceeds in three steps: First, we discuss the laissez-faire equilibrium, which arises if the $n$ different ecosystem managers optimize their management effort taking the actions of the other managers as given (Section 4.2). Second, we derive the (symmetric) Pareto-efficient allocation (Section 4.3). Finally, we investigate the extent, in welfare terms, of the market failure, and analyze policy measures to internalize the externalities (Section 4.4).

4.1 The insurance value of conservative ecosystem management

In order to demonstrate how conservative ecosystem management acts as an insurance, consider a single ecosystem manager in isolation, i.e. the special case of $n = 1$. By choosing an action $x$, the ecosystem manager chooses a particular income lottery (Crocker and Shogren 2001, Shogren and Crocker 1999), which in our model is characterized by a normal distribution with mean $\mathbb{E}y = \mu(q(x,x)) - c(x)$ and variance $\text{var} y = \sigma^2(q(x,x))$ (Equations 7, 8). These are determined by $x$ and, therefore, one may speak of ‘the lottery $x$’. One standard method of valuing the riskiness of a lottery to a decision maker is to calculate the risk premium $R$ of the lottery $x$, which is defined by (e.g. Dasgupta and Heal 1979: 381, Kreps 1990: 84)\(^6\)

$$u(\mathbb{E}y - R) = \mathbb{E}u(y).$$  \hspace{1cm} (12)

\(^6\)If $y \in Y$ with $Y$ as an interval of $\mathbb{R}$, and if $u$ is continuous and strictly increasing, a risk premium $R$ uniquely exists for every lottery $x$ (Kreps 1990: 84).
The risk premium $R$ is the amount of money that leaves the decision maker equally well off, in terms of utility, between the two situations of (i) receiving for sure the expected pay-off from the lottery $E y$ minus the risk premium $R$, and (ii) playing the risky lottery with random pay-off $y$.

In general, the idea of an insurance is that it reduces the (income) risk to which one is exposed. In the extreme, under full insurance one does not have any income risk at all. For the sake of our analysis, we conceptualize this notion of insurance by employing the risk premium as a measure of riskiness. A change in the action $x$ such that, as a result, the risk premium $R$ is reduced, therefore has an insurance value equal to $-dR/dx$.

With utility function (11), the risk premium $R$ of a lottery with mean pay-off $E y$ and variance $\text{var} y$ is simply given by (see Appendix A.2):

$$R = \frac{\rho}{2} \text{var} y,$$

and the insurance value is given by

$$-\frac{dR}{dx} = -\frac{\rho}{2} \theta \sigma^2(q(x,x)) [q_x(x,x) + q_X(x,x)] > 0. \tag{14}$$

This insurance value captures (i) the ecosystem manager’s subjective valuation of risk, measured by the absolute risk-aversion $\rho$; (ii) the ecosystem’s response, in terms of reduced variance of ecosystem service provision, to an increased quality, given by the factor $\sigma^2$; and (iii) how ecosystem quality improves due to both individual and the aggregate management efforts (the ‘technology’ of ecosystem management), given by the factor $q_x + q_X$. Note that (i) captures a subjective aspect, while (ii) and (iii) capture objective aspects of the insurance value.

### 4.2 Laissez-faire equilibrium

As laissez-faire equilibrium, we consider the allocation which results as Nash-equilibrium without regulating intervention. Each ecosystem manager’s decision problem is to maximize his expected utility, taking the actions of all other ecosys-
tem managers as given. Formally, manager $i$’s decision problem is

$$\max_{x_i} \mu(q(x_i, X)) - c(x_i) - \frac{\rho}{2} \theta \sigma^2(q(x_i, X)),$$  \hspace{1cm} (15)

where $X = x_1 + \ldots + x_n$ and all $x_j$ for $j \neq i$ are treated as given. We assume (throughout the remainder of this paper) that an interior solution exists.\footnote{Ecosystem properties (1) and (4) and the cost function (5) do not exclude corner solutions. For instance, for very high marginal costs and low marginal benefits of management effort, the Nash equilibrium may be not to make any effort at all. On the other hand, for low marginal costs, the equilibrium could be to make the maximum possible effort, because ecosystem quality has the double benefit of increasing the mean and reducing the variance of ecosystem service provision.}

**Lemma 1**

The laissez-faire equilibrium has the following properties: (i) it is unique, (ii) all ecosystem managers choose the same level of ecosystem management, $x_i = x^*$ for all $i = 1, \ldots, n$, and (iii) it is characterized by the condition

$$\mu'(q(x^*, n x^*)) \left[ q_x(x^*, n x^*) + q_X(x^*, n x^*) \right] - \frac{\rho}{2} \theta \sigma^2(q(x^*, n x^*)) \left[ q_x(x^*, n x^*) + q_X(x^*, n x^*) \right] = c'(x^*).$$  \hspace{1cm} (16)

**Proof:** see Appendix A.3.

While the right hand side of Condition (16) captures the marginal costs of the effort to improve ecosystem quality, the left hand side contains its marginal benefits. They comprise two additive components: the marginal gain in the mean level of ecosystem service and the insurance value of improving ecosystem quality, i.e. the marginal reduction of the manager’s risk-premium due to a marginal increase in his individual management effort (cf. Section 4.1). Hence, the insurance value is a value component in addition to the value arguments which hold in a world of certainty. It leads to choosing a higher level of management effort than without taking the insurance value into account. How the equilibrium level of ecosystem management effort depends on the degree of uncertainty and on the managers’ risk aversion mainly depends on the properties of the insurance value.
Proposition 1

1. The equilibrium levels $x^*$ of ecosystem management effort and $q^*$ of ecosystem quality increase with uncertainty:

$$\frac{dx^*}{d\theta} > 0 \quad \text{and} \quad \frac{dq^*}{d\theta} > 0.$$  \hfill (17)

2. The equilibrium levels $x^*$ of ecosystem management effort and $q^*$ of ecosystem quality increase with the ecosystem managers’ degree $\rho$ of risk aversion:

$$\frac{dx^*}{d\rho} > 0 \quad \text{and} \quad \frac{dq^*}{d\rho} > 0.$$  \hfill (18)

**Proof:** see Appendix A.4.

The intuition behind the result is as follows. Since the individuals are risk-averse, the risk-premium increases if either the degree of risk-aversion or the uncertainty as such increase. As a consequence, the insurance value of improving ecosystem quality increases. The resulting higher marginal utility leads to a higher equilibrium level of management effort $x^*$ and to improved ecosystem quality $q^*$.

This corresponds to a result known from the literature on the use (or provision) of a public good under uncertainty (Bramoullé and Treich 2005: Propositions 4 and 8, Sandler et al. 1987, Sandler and Sterbenz 1990), according to which the condition $u'''(y) \geq 0$ is necessary for individual contributions to a public good to increase with uncertainty or risk-aversion. The utility function (10) employed here satisfies this condition. Our approach, being based on the concept of insurance value, points to additional conditions for the result. Besides the curvature of the marginal utility function ($\rho$), the insurance value (cf. Section 4.1) depends on the properties of the ecosystem ($\sigma^2$) and the ‘technology’ of ecosystem management ($q_x, q_X$). This suggests that there exist ecosystems or ecosystem management technologies for which this result does not hold.\(^8\)

\(^8\)For example, if $q_x < 0$ and $q_X > 0$ with $q_x + q_X < 0$, that is, if individual management effort causes a net private damage besides a public benefit, the Inequalities (17) and (18) are reversed.
4.3 Efficient allocation

The next step is to derive the efficient allocation. Since we are interested in comparing the efficient allocation to the laissez-faire equilibrium, we will concentrate on the symmetric Pareto-optimum in which all ecosystem managers make the same effort.\footnote{Conditions for a general Pareto-optimum are derived in Appendix A.5.} To derive this allocation we define social welfare as the sum of the utilities of all \( n \) ecosystem managers:

\[
W = \sum_{i=1}^{n} \left[ \mathcal{E} y_i - \frac{\rho}{2} \operatorname{var} y_i \right].
\] (19)

The efficient allocation is derived by choosing the individual levels of management effort, such that social welfare (19) is maximized subject to the constraints (7) and (8),

\[
\max_{x_1, \ldots, x_n} \sum_{i=1}^{n} \left[ \mu(q(x_i, X)) - \frac{\rho}{2} \sigma^2(q(x_i, X)) - c(x_i) \right].
\] (20)

The solution to this problem is characterized as follows.

**Lemma 2**

The efficient allocation has the following properties: (i) it is unique, (ii) all ecosystem managers make the same management effort \( \hat{x} \), and (iii) it is characterized by the condition

\[
\mu'(q(\hat{x}, n \hat{x})) \left[ q_x(\hat{x}, n \hat{x}) + n q_X(\hat{x}, n \hat{x}) \right] - \frac{\rho}{2} \theta \sigma^2(q(\hat{x}, n \hat{x})) \left[ q_x(\hat{x}, n \hat{x}) + n q_X(\hat{x}, n \hat{x}) \right] = c'(\hat{x}).
\] (21)

**Proof:** see Appendix A.6.

Like in the laissez-faire equilibrium, the insurance value of improving ecosystem quality plays an important role. The efficient insurance value in Condition (21) consists of similar components as the insurance value considered by the individual ecosystem managers in equilibrium (Condition 16), but the contribution of the aggregate effort on ecosystem quality is augmented by a factor \( n \). Because the positive externalities of individual management effort on the other ecosystem users’
risk premium are taken into account, the efficient insurance value is greater than
the equilibrium insurance value. Similarly, the marginal benefits with respect to
the mean level of ecosystem service provision are higher in the efficient allocation.
This implies that the efficient level $\hat{x}$ of individual management effort is greater
than the equilibrium level $x^*$, and the efficient level $\hat{q}$ of ecosystem quality is greater
than the equilibrium level $q^*$. The efficient allocation has the following properties.

Proposition 2

1. The efficient levels $\hat{x}$ of ecosystem management effort and $\hat{q}$ of ecosystem
   quality increase with uncertainty:

   $\frac{d\hat{x}}{d\theta} > 0 \quad \text{and} \quad \frac{d\hat{q}}{d\theta} > 0$.

   (22)

2. The efficient levels $\hat{x}$ of ecosystem management effort and $\hat{q}$ of ecosystem
   quality increase with the ecosystem managers’ degree $\rho$ of risk aversion:

   $\frac{d\hat{x}}{d\rho} > 0 \quad \text{and} \quad \frac{d\hat{q}}{d\rho} > 0$.

   (23)

Proof: see Appendix A.7.

The intuition behind the result is as follows. An increase in the uncertainty
or in the degree of the ecosystem managers’ risk-aversion increases the efficient
insurance value of ecosystem management effort. Hence, the marginal benefits of
management effort increase, leading to a higher efficient level $\hat{x}$ of effort. As a
consequence, ecosystem quality $\hat{q}$ increases. The effects go in the same direction
as in the laissez-faire equilibrium. However, they differ in their quantitative extent
because the positive externalities are taken into account.

4.4 Environmental policy

Due to the external effects of individual ecosystem management effort, the laissez-
faire equilibrium is not efficient. In equilibrium, ecosystem managers will spend
too little effort to improve ecosystem quality, because they do not take into consid-
eration the positive externality on other ecosystem users. In order to implement
the efficient allocation as an equilibrium, a regulator could impose a Pigouvian subsidy on individual management effort. Denoting the subsidy per unit $x_i$ with $\tau$, the optimization problem of ecosystem manager $i$ then reads

$$\max_{x_i} \mu(q(x_i, X)) - c(x_i) - \frac{\rho}{2} \theta \sigma^2(q(x_i, X)) + \tau x_i.$$  

(24)

Comparing the first order conditions for the efficient allocation (Equation 21) and for the regulated equilibrium (i.e. the first order condition of maximizing (24) with respect to $x_i$), we obtain the optimal subsidy $\hat{\tau}$.

**Lemma 3**

The efficient allocation is implemented as an equilibrium, if a subsidy $\hat{\tau}$ on individual ecosystem management effort is set with

$$\hat{\tau} = (n - 1) q_X(\hat{x}, n \hat{x}) \left[ \mu'(q(\hat{x}, n \hat{x})) - \frac{\rho}{2} \theta \sigma^2(q(\hat{x}, n \hat{x})) \right].$$  

(25)

Clearly, the optimal subsidy increases with $q_X(\hat{x}, n \hat{x})$, i.e. it is higher, the higher is the marginal benefit of aggregate effort in terms of ecosystem quality improvement. There are two contributions to the optimal subsidy rate, which are captured by the two terms in brackets. In the case of risk-neutrality, $\rho = 0$, only the first term in brackets remains. Then, the optimal subsidy is $(n - 1) q_X \mu'$, that is, it just internalizes the positive externality that an increase in individual management effort has on the expected payoff of the $n - 1$ other ecosystem managers. For $\rho > 0$, the second term in brackets captures the positive externality of an individual ecosystem manager’s contribution to ecosystem quality which is due to the insurance value that the higher ecosystem quality has for the $n - 1$ remaining ecosystem managers.

The optimal subsidy $\hat{\tau}$ can be interpreted as the extent of the regulation necessary in order to solve the public good problem. It has become clear from the discussion so far that the public good problem depends on the degree of uncertainty and of the ecosystem managers’ risk-aversion. The questions are whether more or less regulation is required if (i) the uncertainty of ecosystem services or (ii) the degree of risk-aversion increase.
Proposition 3

1. The optimal subsidy decreases/is unchanged/increases with uncertainty, i.e.

\[ \frac{d\hat{\tau}}{d\theta} \leq 0 , \tag{26} \]

2. and the optimal subsidy decreases/is unchanged/increases with the degree \( \rho \) of risk-aversion, i.e.

\[ \frac{d\hat{\tau}}{d\rho} \geq 0 , \tag{27} \]

if

\[ -\frac{\hat{x} \phi'(\hat{x})}{\phi(\hat{x})} \frac{q_x(\hat{x}, n \hat{x})}{n q_X(\hat{x}, n \hat{x})} \geq \frac{\hat{x} c''(\hat{x})}{c'(\hat{x})} , \tag{28} \]

where

\[ \phi(x) \equiv \frac{q_x(x, nx)}{q_x(x, nx) + n q_X(x, nx)} . \tag{29} \]

Proof: see Appendix A.8.

Although both increased uncertainty and increased risk-aversion have an unambiguously positive effect on the individual level of management effort to improve ecosystem quality (Proposition 1), the effect on the optimal regulation can go either way, depending on the ‘technology’ and the costs of ecosystem management as specified by Condition (28). On the left hand side of Condition (28), the expression \( \phi(x) \) is the share of marginal ecosystem quality improvement on the individual scale out of total marginal ecosystem quality improvement including the positive externalities on the aggregate scale. It is, in short, the individual share of marginal quality improvement. With this, the first factor on the left hand side of Condition (28) is the elasticity of the individual share of marginal quality improvement with respect to management effort. The second factor simply is the marginal rate of substitution between management efforts on the individual and on the aggregate scale. On the right hand side of Condition (28) is the elasticity of marginal costs. Hence, the Pigouvian subsidy decreases/is unchanged/increases with uncertainty (or risk-aversion), if the elasticity of the individual share of marginal quality improvement times the marginal rate of substitution between effort on the...
individual and aggregate scale is greater than/equal to/less than the elasticity of marginal costs. In particular, in the case of constant marginal costs, the Pigouvian subsidy decreases with uncertainty, if and only if the elasticity of the individual share of marginal quality improvement is positive. This does not need to be the case. Overall, whether the Pigouvian subsidy increases or decreases with uncertainty depends on how ecosystem processes operating at different scales influence ecosystem quality; it does not depend on how exactly ecosystem service provision is influenced by ecosystem quality.

Although the Pigouvian subsidy is an appropriate measure of the extent of regulation necessary to reach the efficient allocation in a decentralized economy, a different measure is required in order to determine the welfare loss due to the external effects. This welfare loss is the difference in welfare between the efficient allocation and the laissez-faire allocation. Employing the welfare function (19), it is given by

$$\hat{W} - W^* = n \left[ \mu(q(\hat{x}, n \hat{x})) - \frac{\rho}{2} \theta \sigma^2(q(\hat{x}, n \hat{x})) - c(\hat{x}) \right]$$

$$- n \left[ \mu(q(x^*, n x^*)) - \frac{\rho}{2} \theta \sigma^2(q(x^*, n x^*)) - c(x^*) \right] > 0 \quad (30)$$

**Proposition 4**

The welfare loss due to free-riding decreases/is unchanged/increases with uncertainty, i.e.

$$\frac{d}{d\theta} (\hat{W} - W^*) \lesssim 0 \quad (31)$$

if

$$\frac{\rho}{2} \left[ \sigma^2(q(x^*, n x^*)) - \sigma^2(q(\hat{x}, n \hat{x})) \right] - \tau^* \frac{dx^*}{d\theta} \lesssim 0 \quad (32)$$

where

$$\tau^* = (n - 1) q_x(x^*, n x^*) \left[ \mu'(q(x^*, n x^*)) - \frac{\rho}{2} \theta \sigma^2(q(x^*, n x^*)) \right] > 0 \quad (33)$$

**Proof:** This is proved by differentiating Equation (30) with respect to $\theta$, using the envelope theorem, $d\hat{W}/d\theta = \partial \hat{W}/\partial \theta$, and the equilibrium condition (16). □

Whether the welfare loss due to free-riding decreases or increases with uncertainty depends on the relative magnitude of two effects: on the one hand, the
difference between the variance of ecosystem service provision in the laissez-faire equilibrium and in the optimum is spread with increasing uncertainty. This effect worsens the market failure. On the other hand, uncertainty increases individual management effort in equilibrium (Proposition 1), which decreases the extent of market failure.\footnote{Sandler et al. (1987) and Bramoullé and Treich (2005) study exclusively this latter effect and, therefore, conclude that increasing risk reduces the public-good problem.} The second effect is weighted by a factor of $\tau^\star$, which is the external marginal benefit of an ecosystem manager’s individual effort for all $n - 1$ other ecosystem managers in equilibrium. This positive externality determines how valuable it is, in welfare terms, that individual ecosystem management effort increases with uncertainty.

The net effect of increased uncertainty on the welfare loss due to the externality is ambiguous. In the following, we will demonstrate that whether the welfare loss decreases or increases depends on the ecosystem’s properties. For this purpose, we consider an example which is simple enough to enable a closed-form solution. Let

$$q(x, X) = x^{1-\gamma} X^\gamma \quad \text{with} \quad 0 < \gamma < 1.$$  \hfill (34)

This specification is analytically very convenient, and it corresponds to the well established species-area-relationship at each of the two hierarchical levels, i.e. $q$ is a power function of both $x$ and $X$. We further assume constant marginal costs of management effort, $c(x) = c \cdot x$, which corresponds to a constant price of land set aside for biodiversity protection. In order to focus on the insurance effect we disregard that improved ecosystem quality increases the mean level of ecosystem services, i.e. $\mu(q) = \mu = \text{constant}$. Finally, the variance of ecosystem services depends on ecosystem quality as follows\footnote{For $\eta > 0$, we define $\sigma^2(q)$ by (35) for all $q \leq \delta/\eta$ and by $\sigma^2(q) \equiv 0$ for all $q > \delta/\eta$. In the case $\eta = 0$, the specification (35) becomes $\sigma^2(q) = \exp(-q/\delta)$.}

$$\sigma^2(q) = (\delta - \eta q)^{\frac{1}{\eta}} \quad \text{with} \quad \eta < 1 \quad \text{and} \quad \delta > 0.$$  \hfill (35)

This specification includes (for different $\eta$) large variety of functions satisfying Conditions (4). For $\eta > 0$, it is possible to obtain the ecosystem service at zero
variance, provided ecosystem quality is high enough. This is not possible for \( \eta < 0 \).

Whether the welfare losses due to the public good problem decreases, is unchanged, or increases with uncertainty depends on the type of ecosystem, as specified by the parameter \( \eta \).

**Proposition 5**

*Given the specifications of the example, with increasing uncertainty the welfare loss due to free-riding decreases if \( \eta > 0 \), is not affected if \( \eta = 0 \), and increases if \( \eta < 0 \).*

**Proof:** see Appendix A.9.

The case \( \eta > 0 \) corresponds to an ecosystem which is very effectively manageable: an increase of ecosystem quality strongly reduces the uncertainty of ecosystem service provision and can, eventually, completely remove the uncertainty. In this case (and given the other specifications of the example), increasing uncertainty reduces the welfare loss due to free-riding. However, even if \( \eta > 0 \) uncertainty does not necessarily increase welfare: in the efficient allocation, uncertainty unambiguously reduces welfare of risk-averse individuals; in the laissez-faire equilibrium, welfare can, in principle, increase with uncertainty. In our example, this is the case if \( \eta > 1 - \gamma \frac{n-1}{n} \) (cf. Appendix A.9), which is a stronger assumption than \( \eta > 0 \).

In the case \( \eta = 0 \), uncertainty plays no role for the extent of welfare loss. For \( \eta < 0 \), the effect that the difference in variance between the efficient allocation and the laissez-faire equilibrium increases with uncertainty outweighs the welfare-increasing effect of increased individual management effort. In that case, uncertainty increases the welfare loss due to free-riding.

## 5 Conclusions

We have analyzed how risk-averse ecosystem users manage an ecosystem for its services. The ecosystem model captures three stylized facts, as identified in the
ecological literature: (i) the mean level of ecosystem services increases with biodiversity; (ii) the variance of ecosystem services decreases with biodiversity; (iii) biodiversity is influenced by ecosystem processes operating at different hierarchical scales. We have considered two such scales: individual management action affects processes at the lower scale, while aggregate action affects processes at the higher scale. Thus, an individual management action has not only a private benefit, but also a positive external effect on other ecosystem users.

We have demonstrated that conservative biodiversity management has an insurance value, which depends on the ecosystem managers’ risk-aversion and on ecosystem properties. Because ecosystem managers, when choosing a management action under uncertainty, take into account the ecosystem’s insurance value, the level of individual effort to improve ecosystem quality increases with increasing uncertainty and risk-aversion. As a consequence, higher uncertainty and higher risk-aversion lead to a higher level of biodiversity. Thus, under uncertainty the ecosystem management is more conservative, and the resulting level of biodiversity is higher, than it would be in a world of certainty.

Due to the external effect of individual management effort, the laissez-faire equilibrium is not efficient. In order to study how the public good-problem is affected by uncertainty, we have analyzed how (i) the extent of regulation necessary to implement the efficient allocation and (ii) the welfare loss due to free-riding depend on the degree of uncertainty.

How the Pigouvian subsidy, as a measure of the extent of efficient regulation, is affected by uncertainty depends on how management effort at the individual scale and at the aggregate scale contribute to ecosystem quality. For constant marginal costs of management effort, the Pigouvian subsidy decreases with uncertainty if the elasticity of the individual share of marginal quality improvement is positive, and increases otherwise. Hence, the extent of the regulatory intervention necessary to implement the efficient allocation depends on the hierarchical structure of how ecosystem management affects biodiversity, but not on how exactly biodiversity influences the provision of ecosystem services. In contrast, the latter crucially de-
termines whether the welfare loss due to free-riding decreases or increases with uncertainty: if management measures translate effectively into reduced variance of ecosystem service provision, the welfare loss tends to decrease with uncertainty; if, on the other hand, the ecosystem is poorly manageable and variance can only be reduced partially, higher uncertainty tends to increase welfare losses due to free-riding. These results highlight that ecosystem properties determine how efficient environmental policy and welfare losses change with uncertainty. This is new to the literature on the provision of public goods under uncertainty, which generally focuses on consumer preferences and disregards the nature of ecosystem functioning.

Our analysis enables a number of extensions. Besides the insurance function of conservative ecosystem management, one could account for socio-economic institutions for risk-management, for example, commercial insurance markets (Quaas and Baumgärtner 2005). Also, other sources of risk (e.g. price risk, institutional or political risk) and other risk characteristics (e.g. thresholds or skewed distributions) could have interesting effects. An extension of the analysis to capture the dynamics of environmental quality, as well as savings and credits as mechanisms to cope with risk, could provide further insights. Finally, the conceptual ecological-economic framework developed here, in which both environmental management and ecosystem properties determine the stochasticity of ecosystem service provision, can be applied to other environmental problems, such as river floods, climate change, or the spread of genetically modified organisms.

Appendix

A.1 Expected utility function (11)

With

\[ f(y) = \frac{1}{\sqrt{2\pi \text{var} y}} e^{-\frac{(y - \bar{y})^2}{2\text{var} y}} \]  

(A.1)
as the probability density function of the normal distribution of income \( y \) with mean \( \mathcal{E}y \) and variance \( \text{var} \ y \), the von Neumann-Morgenstern expected utility from the Bernoulli utility function (10) is

\[
\tilde{U} = \mathcal{E}u(y) = -\int e^{-\rho y} f(y) dy = -e^{-\rho [\mathcal{E}y - \frac{\rho}{2} \text{var} \ y]}. \tag{A.2}
\]

Using a simple monotonic transformation of \( \tilde{U} \), one obtains the expected utility function \( U \) (Equation 11).

**A.2 Risk premium (13)**

The risk premium \( R \) has been defined in Equation (12) as

\[
u (\mathcal{E}y - R) = \mathcal{E}u(y). \tag{A.3}
\]

With the Bernoulli utility function (10) the left hand side of this equation is given by

\[
u (\mathcal{E}y - R) = -e^{-\rho [\mathcal{E}y - R]}, \tag{A.4}
\]

and the right hand side is given by Equation (A.2). Hence, we have

\[-e^{-\rho [\mathcal{E}y - R]} = -e^{-\rho [\mathcal{E}y - \frac{\rho}{2} \text{var} \ y]}. \tag{A.5}
\]

Rearranging yields the result stated in Equation (13).

**A.3 Proof of Lemma 1**

The first order condition of Problem (15) is

\[
\left[ \mu'(q(x_i, X)) - \frac{\rho}{2} \theta \sigma^2(q(x_i, X)) \right] \left[ q_x(x_i, X) + q_X(x_i, X) \right] = c'(x_i). \tag{A.6}
\]

We denote by \( \tilde{X} \) the aggregate effort of all ecosystem managers except for manager \( i \), i.e. \( \tilde{X} = X - x_i \). Hence, we can write

\[
\left[ \mu'(q(x_i, x_i + \tilde{X})) - \frac{\rho}{2} \theta \sigma^2(q(x_i, x_i + \tilde{X})) \right] \left[ q_x(x_i, x_i + \tilde{X}) + q_X(x_i, x_i + \tilde{X}) \right]
\]

\[= c'(x_i). \tag{A.7}
\]
We prove the lemma in three steps: (i) we prove that a solution $x^*$ to (16) is unique, (ii) we prove that $x_i = x^*$ for all $i = 1, \ldots, n$ is a Nash-equilibrium. This is done by showing that $x_i = x^*$ solves (A.7), if $\tilde{X} = (n-1)x^*$. And (iii) we prove that no asymmetric Nash-equilibrium exists.

Ad (i). A solution $x^*$ of (16) is unique, because, by assumption (5), the right hand side $c'(x^*)$ is increasing with $x^*$, while the left hand side is decreasing with $x^*$;

$$
\frac{d}{dx^*} \left[ \mu' - \frac{\rho}{2} \theta \sigma^2 \right] [q_x + q_X] \\
= \left[ \mu'' - \frac{\rho}{2} \theta \sigma^2 \right] [q_x + q_X] [q_x + q_X] \\
+ \left[ \mu' - \frac{\rho}{2} \theta \sigma^2 \right] [q_{xx} + 2q_{xX} + q_{XX}] \leq 0, \quad (A.8)
$$

where we omitted arguments for the sake of a clearer exposition. The sign of this expression is negative by assumptions (1) and (4).

Ad (ii). To show that the symmetric allocation $x_i = x^*$ for all $i = 1, \ldots, n$ is a Nash equilibrium, we assume $\tilde{X} = (n-1)x^*$ is given for manager $i$. In this case, the optimal effort for manager $i$ is $x^*$, because $x_i = x^*$ solves Condition (16) uniquely. By symmetry, $x_i = x^*$ for all $i = 1, \ldots, n$.

Ad (iii). Consider the two cases (i) $\tilde{X} > (n-1)x^*$ and (ii) $\tilde{X} < (n-1)x^*$. In case (i), the optimal effort for manager $i$ is $x_i < x^*$. To prove this, we differentiate Condition (A.7) w.r.t. $\tilde{X}$, which yields

$$
\frac{dx_i}{d\tilde{X}} = - \frac{\left[ \mu'' - \frac{\rho}{2} \theta \sigma^2 \right] [q_x + q_X] q_X + [\mu' - \frac{\rho}{2} \theta \sigma^2] q_{xx} + q_{XX}}{\left[ \mu'' - \frac{\rho}{2} \theta \sigma^2 \right] [q_x + q_X]^2 + [\mu' - \frac{\rho}{2} \theta \sigma^2] [q_{xx} + 2q_{xX} + q_{XX}] - c''},
$$

which is negative by assumptions (1) and (4). Since $x_i = x^*$ for $\tilde{X} = (n-1)x^*$, $x_i < x^*$ for $\tilde{X} > (n-1)x^*$. Due to the symmetry, this contradicts the assumption $\tilde{X} > (n-1)x^*$, since all ecosystem managers would choose $x_i < x^*$. Hence, there is no equilibrium where $\tilde{X} > (n-1)x^*$. With a similar argument, we can rule out case (ii). Hence, $x_i = x^*$ for all $i = 1, \ldots, n$ is the unique equilibrium.
A.4 Proof of Proposition 1

Ad 1. Differentiating Condition (16) with respect to $\theta$ yields:

$$
\left[ \mu'' - \frac{\rho}{2} \theta \sigma'' \right] [q_x + q] [q_x + n q] \\
+ \left[ \mu' - \frac{\rho}{2} \theta \sigma'' \right] [q_{xx} + (n + 1) q_{xX} + n q_{XX} - c'] \\
\frac{dx^*}{d\theta} = \frac{\rho \sigma''}{2} [q_x + q_x] . \quad (A.10)
$$

Rearranging, and using the equilibrium condition (16), we have

$$
\frac{dx^*}{d\theta} = -\frac{1}{\theta} \left[ \frac{\mu'' - \frac{\rho}{2} \theta \sigma''}{\mu' - \frac{\rho}{2} \theta \sigma''} [q_x + n q] + \frac{q_{xx} + (n + 1) q_{xX} + n q_{XX} - c'}{q_x + q_x} \right]^{-1} . \quad (A.11)
$$

Because the term in brackets is negative (by Assumptions (1), (4) and (5)), we conclude $dx^*/d\theta > 0$. $dq^*/d\theta > 0$ follows, because

$$
\frac{dq(x^*, n x^*)}{dx^*} = q_x + n q_x > 0 . \quad (A.12)
$$

Ad 2. Differentiating (16) with respect to $\rho$ yields (omitting arguments):

$$
\left[ \mu'' - \frac{\rho}{2} \theta \sigma'' \right] [q_x + q] [q_x + n q] \\
+ \left[ \mu' - \frac{\rho}{2} \theta \sigma'' \right] [q_{xx} + (n + 1) q_{xX} + n q_{XX} - c'] \\
\frac{dx^*}{d\rho} = \frac{\theta \sigma''}{2} [q_x + q_x] . \quad (A.13)
$$

The same arguments as in Part 1 of the proof lead to the conclusion $dx^*/d\rho > 0$ and $dq^*/d\rho > 0$.

A.5 Pareto-efficient allocations

We consider the social planner’s problem

$$
\max_{x_1, \ldots, x_n} \mathcal{E}u(y_1) \quad \text{s.t.} \ (1), (2), (4), (7), (8), \text{ and } \mathcal{E}u(y_i) \geq U_i \ \forall \ i \neq 1.
$$
The Lagrangian for this problem reads

\[
\mathcal{L} = \mu(q(x_1, X)) - c(x_1) - \frac{\rho}{2} \theta \sigma^2(q(x_1, X)) + \sum_{k=2}^{n} \lambda_i [E_u(y_i) - U_i]
\]

\[
= \sum_{i=1}^{n} \lambda_i \left[ \mu(q(x_i, X)) - c(x_i) - \frac{\rho}{2} \theta \sigma^2(q(x_i, X)) - U_i \right] + U_1,
\]

where \( \lambda_1 = 1 \). The first order conditions of this problem read for all \( i \in \{1, \ldots, n\} \):

\[
\lambda_i \left[ \mu'(q(\hat{x}_i, \hat{X})) - c'(\hat{x}_i) - \frac{\rho}{2} \theta \sigma'^2(q(\hat{x}_i, \hat{X})) \right] q_x(\hat{x}_i, \hat{X}) = -\sum_{k=1}^{n} \lambda_k \left[ \mu'(q(\hat{x}_k, \hat{X})) - \frac{\rho}{2} \theta \sigma'^2(q(\hat{x}_k, \hat{X})) \right] q_X(\hat{x}_k, \hat{X}).
\]

(A.14)

Dividing the \( i \)-th equation by the first one yields:

\[
\lambda_i \frac{\mu'(q(\hat{x}_1, \hat{X})) q_x(\hat{x}_1, \hat{X}) - c'(\hat{x}_1) - \frac{\rho}{2} \theta \sigma'^2(q(\hat{x}_1, \hat{X})) q_x(\hat{x}_1, \hat{X})}{\mu'(q(\hat{x}_1, \hat{X})) q_x(\hat{x}_1, \hat{X}) - c'(\hat{x}_1) - \frac{\rho}{2} \theta \sigma'^2(q(\hat{x}_1, \hat{X})) q_x(\hat{x}_1, \hat{X})} q_X(\hat{x}_k, \hat{X}).
\]

(A.15)

In the symmetric case, i.e. \( x_i = \hat{x} \) for all \( i \in \{1, \ldots, n\} \), it is \( c'(x_1) = c'(x_i) \), and Equation (A.15) simplifies to

\[
-c'(\hat{x}) + [q_x(\hat{x}, n \hat{x}) + n q_X(\hat{x}, n \hat{x})] \left[ \mu'(q(\hat{x}, n \hat{x})) - \frac{\rho}{2} \theta \sigma'^2(q(\hat{x}, n \hat{x})) \right] = 0.
\]

(A.16)

### A.6 Proof of Lemma 2

First, we show that it is optimal to choose the same management for all \( n \) ecosystem managers, i.e. that

\[
\frac{1}{n} \sum_{i=1}^{n} \mu(q(x_i, X)) - \frac{\rho}{2} \theta \sigma^2(q(x_i, X)) - c(x_i)
\]

\[
\leq \mu(q(\frac{X}{n}, X)) - \frac{\rho}{2} \theta \sigma^2(q(\frac{X}{n}, X)) - c(\frac{X}{n}),
\]

(A.17)

where \( X = \sum_{j=1}^{n} x_j \). This is true by Jensen’s inequality, because the welfare function is concave in \( x_i \) for any given \( X \).\(^{12}\) Hence, we have to find the level \( x \) of

\(^{12}\)The idea for this proof is taken from Bramouillé and Treich (2005).
effort to improve ecosystem quality, which maximizes

\[ n \left[ \mu(q(x, n x)) - \frac{\rho}{2} \theta \sigma^2(q(x, n x)) - c(x) \right]. \]  
(A.18)

This is a strictly concave function of \( x \), since

\[
\frac{d^2}{dx^2} \left[ n \left[ \mu(q(x, n x)) - \frac{\rho}{2} \theta \sigma^2(q(x, n x)) - c(x) \right] \right] \\
= \left[ \mu'' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_x + n q_x]^2 + \left[ \mu' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_{xx} + 2 n q_{xx} + n^2 q_{XX}] - c'' < 0.
(A.19)

Hence, if an interior solution exists, it is uniquely determined by the first order condition

\[
\left[ \mu' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_x + n q_X] = c'.
(A.20)

A.7 Proof of Proposition 2

Ad 1. Differentiating Condition (21) with respect to \( \theta \) yields:

\[
\left[ \left[ \mu'' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_x + n q_X]^2 + \left[ \mu' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_{xx} + 2 n q_{xx} + n^2 q_{XX}] - c'' \right] \frac{d\hat{x}}{d\theta} \\
= \frac{\rho \sigma'^2}{2} [q_x + n q_X].
(A.21)

Because the term in brackets on the left hand side of this equation is negative and because the right hand side of the equation is negative (both by assumptions (1), (4) and (5)), we conclude \( d\hat{x}/d\theta > 0 \). \( d\hat{q}/d\theta > 0 \) follows, because

\[
\frac{dq(\hat{x}, n \hat{x})}{d\hat{x}} = q_x + n q_X > 0.
(A.22)

Ad 2. Differentiating (21) with respect to \( \rho \) yields (omitting arguments):

\[
\left[ \left[ \mu'' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_x + n q_X]^2 + \left[ \mu' - \frac{\rho}{2} \theta \sigma'^2 \right] [q_{xx} + 2 n q_{xx} + n^2 q_{XX}] - c'' \right] \frac{d\hat{x}}{d\rho} \\
= \frac{\theta \sigma'^2}{2} [q_x + n q_X].
(A.23)

The same arguments as in Part 1 of the proof lead to the conclusion \( d\hat{x}/d\rho > 0 \) and \( d\hat{q}/d\rho > 0 \).
A.8 Proof of Proposition 3

Ad 1. In order to derive the comparative statics of $\hat{\tau}$ with respect to $\theta$, we differentiate (25) with respect to $\theta$. This yields (omitting arguments)

$$\frac{d\hat{\tau}}{d\theta} = (n-1) \left[ \left[ q_{xx} + n q_{XX} \right] \left[ \mu' - \frac{\rho}{2} \theta \sigma^2' \right] + q_X \left[ \mu'' - \frac{\rho}{2} \theta \sigma^2'' \right] \left[ q_x + n q_X \right] \frac{d\hat{x}}{d\rho} - q_X \frac{\rho}{2} \sigma^2' \right]$$

(A.24)

From Equation (A.23), we have

$$\frac{d\hat{x}}{d\theta} = \frac{\frac{\rho}{2} \sigma^2' \left[ q_x + n q_X \right]}{\left[ \mu'' - \frac{\rho}{2} \theta \sigma^2'' \right] \left[ q_x + n q_X \right]^2 + \left[ \mu' - \frac{\rho}{2} \theta \sigma^2' \right] \left[ q_{xx} + 2 n q_{xX} + n^2 q_{XX} \right] - c''}$$

(A.25)

Using this in (A.25) and simplifying yields

$$\frac{d\hat{\tau}}{d\theta} = \frac{(n-1) \frac{\rho}{2} \sigma^2' \left[ q_x + n q_X \right]}{\left[ \mu'' - \frac{\rho}{2} \theta \sigma^2'' \right] \left[ q_x + n q_X \right]^2 + \left[ \mu' - \frac{\rho}{2} \theta \sigma^2' \right] \left[ q_{xx} + 2 n q_{xX} + n^2 q_{XX} \right] - c''} \cdot$$

(A.26)

Since the denominator of this expression is negative and the first two factors of the numerator together are negative, too, the change of $\hat{\tau}$ following an increase in $\theta$ has the same sign as

$$\left[ \mu' - \frac{\rho}{2} \theta \sigma^2' \right] \left[ -q_X q_{xx} - n q_{xX} q_X + n q_x q_{XX} + q_x q_{xx} \right] + q_X c''$$

(A.27)

Rearranging, this expression has the same sign as

$$- \left[ \frac{\hat{x} q_{xx}}{q_x} + \frac{\hat{X} q_{XX}}{q_X} - \frac{\hat{x} q_{xX}}{q_X} - \frac{\hat{X} q_{XX}}{q_X} \right] \left[ \mu' - \frac{\rho}{2} \theta \sigma^2' \right] q_x + \hat{x} c''$$

(A.29)

which is equal to, using the efficiency condition (21),

$$- \left[ -\frac{\hat{X} q_{XX} - \hat{x} q_{xX}}{q_X} - \frac{\hat{X} q_{XX} - \hat{x} q_{xX}}{q_X} \right] \frac{c' q_x}{q_x + n q_X} + \hat{x} c''$$

(A.30)

Using the abbreviation (29) and rearranging leads to Condition (28).
Ad 2. Differentiating the optimal subsidy (25) with respect to $\rho$ leads to

$$
\frac{d\hat{\tau}}{d\rho} = (n-1) \left[ \left( q_{xx} + n q_{XX} \right) \left( \mu' - \frac{\rho}{2} \sigma^2' \right) 
+ q_X \left( \mu'' - \frac{\rho}{2} \theta \sigma^2'' \right) \left( q_x + n q_X \right) \right] \frac{d\hat{x}}{d\theta} - q_X \theta \frac{\rho}{2} \sigma^2'.
$$

(A.31)

Using (A.21) and rearranging yields

$$
\frac{d\hat{\tau}}{d\theta} = (n-1) \left[ \left( q_{xx} + n q_{XX} \right) \left( \mu' - \frac{\rho}{2} \theta \sigma^2 \right) 
+ q_X \left( \mu'' - \frac{\rho}{2} \theta \sigma^2'' \right) \left( q_x + n q_X \right) \right] 
\frac{d\hat{x}}{d\theta} - q_X \theta \frac{\rho}{2} \sigma^2'.
$$

(A.32)

which is negative, if and only if Condition (28) is fulfilled.

**A.9 Proof of Proposition 5**

With the specifications (34), $c(x) = c \cdot x$, and $\mu(q) = \mu$, we have (using A.11)

$$
\frac{d \hat{x}}{d\theta} = \frac{1}{\theta} \left[ \frac{1 - \eta}{\delta - \eta q(x^*, n, x^*)} \frac{q(x^*, n, x^*)}{x^*} \right]^{-1}.
$$

(A.34)

and

$$
\hat{\tau} = (n-1) \gamma \frac{q(x^*, n, x^*)}{n} \frac{\rho}{2} \theta \frac{\sigma^2(q(x^*, n, x^*))}{\delta - \eta q(x^*, n, x^*)}.
$$

(A.35)

Thus,

$$
\hat{\tau} \frac{d \hat{x}}{d\theta} = \frac{\rho}{2} \sigma^2(q(x^*, n, x^*)) \frac{\gamma}{n} \frac{n-1}{1 - \eta}.
$$

(A.36)

Using this in Equation (32), we have

$$
\frac{d}{d\theta} \left( \hat{W} - W^* \right) = n \frac{\rho}{2} \left[ \left( 1 - \gamma \frac{n-1}{n} \frac{1}{1 - \eta} \right) \left( \delta - \eta n^\gamma \hat{x} \right)^{\frac{1}{\gamma}} - \left( \delta - \eta n^\gamma \hat{x} \right)^{\frac{1}{\gamma}} \right].
$$

(A.37)

With the specifications of the example, the condition for the efficient allocation, (21), becomes

$$
\frac{\rho}{2} \theta \left( \delta - \eta n^\gamma \hat{x} \right)^{\frac{1}{\gamma} - 1} n^\gamma = c,
$$

(A.38)

i.e.,

$$
\left( \delta - \eta n^\gamma \hat{x} \right)^{\frac{1}{\gamma}} = \left( \frac{2 c}{\rho \theta n^\gamma} \right)^{\frac{1}{1-\eta}}
$$

(A.39)
The equilibrium condition (16) becomes

\[ \frac{\rho}{2} \theta (\delta - \eta n^n x^*)^{\frac{1}{\gamma} - 1} \left(1 - \gamma \frac{n - 1}{n}\right) n^\gamma = c, \quad (A.40) \]

such that

\[ (\delta - \eta n^n x^*)^{\frac{1}{\gamma}} = \left(\frac{2c}{\rho \theta n^n}\right)^{\frac{1}{1-\eta}} \left(1 - \gamma \frac{n - 1}{n}\right)^{\frac{1}{1-\eta}} n^{1-\eta}. \quad (A.41) \]

Hence,

\[ \frac{d}{d\theta} (\hat{W} - W^*) = n \frac{\rho}{2} \left(\frac{2c}{\rho \theta n^n}\right)^{\frac{1}{1-\eta}} \left[ \frac{1 - \gamma \frac{n-1}{n}}{(1 - \gamma \frac{n-1}{n})^{\frac{1}{1-\eta}}} - 1 \right]. \quad (A.42) \]

A Taylor-series expansion-argument yields the result that the expression in brackets is negative for \( \eta > 0 \), zero for \( \eta = 0 \) and positive for \( 0 < \eta < 1 \).

References


