Pollution-reducing infrastructure and urban environmental policy

MARTIN F. QUAASS∗

Abstract:

Pollution-reducing infrastructure is introduced in a general spatial equilibrium model of a monocentric city as a public good which serves to abate polluting emissions from household’s consumption. This is an innovative extension to an urban economics model and motivated by stylised facts observed at the case of Bombay. It allows to develop and analyse improved policy instruments to solve urban environmental problems.

We demonstrate how the optimal density of people, goods consumption and pollution-reducing infrastructure are interrelated and spatially distributed. The public-good character of infrastructure is shown to favour an increased infrastructural density all over the city in response to increased population size. In two settings of public and private infrastructural supply, we derive three interrelated and spatially differentiated policy instruments, by which the optimal allocation is implemented as a spatial market equilibrium.

∗Department of Ecological Modelling, UFZ-Centre for Environmental Research Leipzig-Halle in the Helmholtz Association, PO Box 500135, D-04301 Leipzig, Germany. Email: martin.quaas@ufz.de. The author thanks Klaus Conrad, Bouwe Dijkstra, Malte Faber, Frank Jöst, Maximilian Mihm, Irene Ring, three anonymous referees and an associate editor of this journal for helpful comments. Financial support from the Research Training Group “Environmental and Resource Economics” of the Universities Heidelberg and Mannheim, financed by the German Research Foundation DFG, and from the German Academic Exchange Service (DAAD) is gratefully acknowledged.
Summary

The paper is concerned with environmental problems in cities. We introduce pollution-reducing infrastructure into an urban economics model as a public good which serves to abate polluting emissions from household’s consumption. These features of the model are innovative and based on stylised facts observed at the case of Bombay, the largest urban agglomeration in India. There, (i) private households contribute considerably to environmental problems (due to sewage effluents, domestic waste, and individual traffic, etc.) and (ii) an improved infrastructural endowment (sewage systems, public sanitation facilities, waste collection and disposal, paved roads, etc.) could considerably reduce environmental pollution. The model assumptions and the analysis aim at general theoretical insights, which hold for other cities as well.

We show that the optimal supply of pollution-reducing infrastructure and the optimal allocation of goods consumption, which causes polluting emissions, are interrelated, such that infrastructural supply and Pigouvian taxes have to be determined simultaneously. Because consumption of goods and of living space are interrelated, too, the use of pollution-reducing infrastructure affects the spatial distribution of households over the city. In the decentralised economy, the rent for living space is no longer a sufficient incentive for households to locate at the optimal positions. Rather, transfer payments are needed in order to obtain the efficient spatial distribution of households.

Despite these interrelations an optimal allocation can be determined and implemented as a spatial market equilibrium with the help of three policy instruments. If infrastructure is supplied publicly, these are (i) the efficient provision of infrastructure, (ii) a Pigouvian
tax on consumption and (iii) income transfers. All of these instruments have to be spatially differentiated. For infrastructure, which can be provided privately, the three instruments are (i) a Pigouvian subsidy on infrastructure, (ii) a Pigouvian tax on emissions, and (iii) income transfers. In this setting, the subsidy on infrastructural supply and the income transfers have to be spatially differentiated.

Providing pollution-reducing infrastructure is most important in growing cities in developing countries. In a comparative static analysis we show that, due to the public-good character of infrastructure, the higher the population size, the higher is the efficient infrastructural supply all over the city.

Introducing pollution-reducing infrastructure as an instrument of environmental policy leads to comparatively complicated policy recommendations. Taking the interrelations between the efficient supply of pollution-reducing infrastructure and Pigouvian taxes into account has important advantages, though. First, it can lead to considerable welfare gains. Second, Pigouvian taxes are lower. Third, the spatial heterogeneities are lower.

In order to apply the model empirically, a variety of extensions should be taken into account, as discussed in the conclusions. Yet, the present analysis takes up issues, which are of major importance, as the case of Bombay shows, but which have not been studied yet in economics. Thereby, it opens the field for a more realistic description of urban environmental problems and improved policy options to solve these problems.
1 Introduction

This paper is concerned with environmental problems in cities. The urban economics literature on this issue typically assumes that the production of goods is the origin of pollution and deals with the questions of how an efficient (spatial) allocation of polluting firms looks like and how this allocation can be implemented by means of taxes and transfers.

While following this line of research in the methodological approach, we address a somewhat different subject, relevant in particular to large cities in developing countries: we consider household’s consumption as the source of environmental pollution and pollution-reducing infrastructure as a public means of abating emissions and, thus, as an important instrument of urban environmental policy in addition to Pigouvian taxes. This focus is new to the literature. It is motivated by the following observations at the case of Bombay, the largest urban agglomeration in India.\(^1\)

Bombay’s population of about 16 million people (Government of India 2001) suffers from a variety of serious environmental problems (e.g., Quaas 2004): (i) An inadequate sewage system exposes the population to sewage water contaminated with bacteria and to the pollution of rivers and coastal waters. In Bombay, more than 40% of total population has to rely on public sanitation services, which are often of poor quality (Palnitkar 1998, Government of India 2001). (ii) As a consequence of insufficient waste collection, much refuse remains at the road-side (Prabhavalkar 2002). Additionally, waste is frequently

---

\(^1\)In 1994, Bombay has been renamed Mumbai. However, internationally the name Bombay is still common.
burned without any form of protection, which further increases air pollution (Shah and Nagpal 1997, Tondwalkar and Phatak 1997). Similar problems are caused by inadequate disposal sites (Sharma et al. 1997, Tondwalkar and Phatak 1997). (iii) Bad roads contribute to noise pollution, but they also affect air pollution: in Bombay, about a third of the SPM (suspended particulate matter) load of the air, which is one of the most serious health threats, comes from roads dust (Shah and Nagpal 1997).

Hence, the stylised facts are: (i) private households contribute considerably to Bombay’s environmental problems due to, among others, household’s sewage effluents, domestic waste, and individual traffic. (ii) An improved infrastructural endowment (sewage systems, public sanitation facilities, waste collection and disposal, or paved roads) could considerably reduce the environmental pollution.

These observations lead to the questions addressed in this paper. The first question is, how can pollution-reducing infrastructure be provided efficiently? Since the polluting emissions from households have their origin from all places in the city, the question is in particular, how should pollution-reducing infrastructure be spatially distributed?

Second, we argue in this paper that it is necessary to consider the provision of infrastructure as an instrument of urban environmental policy. When designing an optimal environmental policy, infrastructural supply and other economic instruments (Pigouvian taxes and location-specific transfers) have to be combined in order to reach an efficient outcome. The question is, what implications does it have to consider and employ these instruments of urban environmental policy as a bundle of interrelated measures?

To answer these questions, we develop a spatial general equilibrium model of a mono-
-centric city, following the tradition of von Thünen-type models (see, e.g., Anas et al. 1998; Nijkamp 1999; Fujita and Thisse 2002). When studying the relationship between environmental problems and infrastructure from an economic point of view, two strands of the environmental economics and urban economics literature are relevant: the literature on environmental pollution in cities on the one hand and on urban infrastructural supply on the other hand.

Concerning the latter, infrastructure is defined in general as the capital stock owned by the public sector. As the example of Bombay shows, a large part of it has an immediate impact on environmental quality, since it helps either to mitigate pollution (e.g., paved roads) or to dispose of wastes so that they do less damage to urban environmental quality (e.g., waste collection and disposal). We call this part of urban infrastructure ‘pollution-reducing infrastructure’. Although the utilities belonging to pollution-reducing infrastructure have been explicitly recognised as being a part of the urban infrastructure (e.g., Conrad and Seitz 1994, Conrad 2001), a rigorous theoretical treatment of the particular implications of their pollution-reducing function is missing so far. Only for the case of transportation infrastructure (e.g. Lundqvist et al. 1998), the infrastructure’s impact on environmental quality has been studied. The questions posed in the transportation context (e.g. concerning the efficient modal split between public and private transportation) are however quite different from ours. Overall, the literature which considers infrastructural supply as an endogenous quantity is scare (cf. Haughwout 2002).

Infrastructure usually is considered to produce a public service, which contributes directly to the utility of private households or to the productivity of private firms (Haugh-
Thus, infrastructure has the character of a public good. Brueckner (1997) and Knaap et al. (2001) address the question of how this public good can be efficiently provided in a growing city when there are congestion effects. In contrast the public good-character of infrastructure, as studied in these articles, the public good-character of pollution-reducing infrastructure is twofold: first, it serves more than one household to abate polluting emissions, which is its direct public service (in this paper we disregard congestion effects, though). Second, pollution-reducing infrastructure produces a public good by improving environmental quality. Both public good aspects are important for the optimal supply of pollution-reducing infrastructure. In particular, we demonstrate in a comparative static analysis that due to this twofold public good character population growth requires increased infrastructural supply all over the city.

Concerning the optimal supply of infrastructure, not only the total quantity has to be determined, but also its spatial distribution. A related question is how public facilities should be located within cities (for an overview on this issue see Revelle 1998). While public facilities are usually treated as discrete units, pollution-reducing infrastructure is more realistically treated as continuously distributed over the city. Hence, we will determine the efficient density of infrastructural supply across the whole city rather than discrete optimal locations.

The efficient supply of pollution-reducing infrastructure is part of an overall efficient allocation, which additionally requires environmental policy by means of Pigouvian taxes and transfers. The literature on urban environmental policy considers only the latter instruments. It is mainly concerned with the trade-off, which arises because households
suffer from polluting firms in their neighbourhood on the one hand, but face the costs of commuting to work on the other hand.\textsuperscript{2} Henderson (1977) considers a circular city, where firms are located in a Central Business District and pollution declines with distance from its origin. He concludes that a Pigouvian tax on emissions is necessary and that the redistribution of tax revenues has to be such that location decisions remain undistorted. Verhoef and Nijkamp (2002) include positive Marshallian externalities promoting agglomeration in addition to negative environmental externalities. More recently, in a model without the assumption of a central business district (following Lucas 2001; Lucas and Rossi-Hansberg 2002), Dijkstra and Lange (2003) have shown that in order to implement an optimal allocation of inhomogeneously polluting firms it is necessary to spatially differentiate Pigouvian tax rates.

Without particular reference to an urban context, Kolstad (1987) has found that spatially differentiated environmental policies lead to higher welfare than homogenous policies, if marginal costs and marginal damage of pollution differ between the locations of polluting firms. They lead to particularly high benefits, if marginal cost and marginal damage curves are steep.

In contrast to Kolstad (1987) and Dijkstra and Lange (2003), we consider identical preferences of all households and spatially homogenous pollution, i.e. each unit of emissions affects all households in the city equally, irrespective of where it is emitted or where the suffering household lives. We demonstrate that the optimal environmental policy

\textsuperscript{2}An early contribution to this issue is Mirrlees 1972. Altogether, the urban economics literature on environmental problems in cities is comparatively meagre (Verhoef and Nijkamp 2002 159).
is nevertheless spatially differentiated, if pollution-reducing infrastructure is taken into account as an instrument of urban environmental policy. The spatial structure of population density, infrastructure, and Pigouvian taxes depends on general equilibrium effects and ultimately is a consequence of the underlying urban spatial structure, i.e. of the central industrial district in the city centre and the commuting costs of households.

Pollution-reducing infrastructure can either be supplied publicly or privately. Both settings may be adequate to a given problem. A public infrastructural provision is required, if households cannot observe their polluting emissions and only the government is able to do so. (For example, individual households may have difficulties to observe their contribution to urban water pollution, but possibly an urban authority may be able to assess this.) In this setting, the public good problem in the provision of infrastructure is solved by public supply of infrastructure, but the Pigouvian tax on consumption has to be spatially differentiated (section 4.2).

If, on the other hand, households can monitor their actual emissions, it may be better, if private households provide infrastructure themselves. (This seems quite reasonable in the case of solid waste.) Here, a subsidy on infrastructural supply is required in order to reach the efficient level. This subsidy has to be differentiated over space. But since households can choose emissions independently of consumption (by choosing infrastructural supply), the Pigouvian tax on emissions is spatially homogenous (section 4.3).

Thus, on the one hand, introducing pollution-reducing infrastructure as an instrument of environmental policy leads to more complicated policy recommendations, as compared to a standard model where only Pigouvian taxes are considered.
On the other hand, taking the interrelations between the efficient supply of pollution-reducing infrastructure and Pigouvian taxes into account has important advantages, as we demonstrate in this paper. First, it can lead to considerable welfare gains. Second, in the setting of public infrastructural provision, Pigouvian taxes are lower. Third, in the setting of private infrastructural supply, the spatial differences in subsidies on infrastructure are lower (due to general equilibrium effects).

The paper is organised as follows: section 2 presents the model of spatial equilibrium. The optimal allocation is determined and analysed in section 3; environmental policy options are the subject of section 4. In section 5, the analytical results of the previous sections are illustrated by a numerical example and some further implications of the model are discussed. Section 6 concludes.

2 The model

The analysis is based on a general spatial equilibrium model of a linear city and its hinterland. The model comprises four goods (an aggregate consumption commodity, living space, infrastructure, and environmental quality) and a continuum of identical households.

Space has one dimension, represented by $z \in \mathbb{R}^+$. The border $Z$ between the city and the hinterland is endogenously determined. Production in the city does not need space and is concentrated in the Central Industrial District (CID) at $z = 0$. Employment

---

3It is straightforward to extend the model to form a symmetric two-dimensional plane and describe a circular city in polar coordinates.
is at a competitive wage rate $w$. Commuting from the place of residence at $z \in [0, Z]$ to the CID takes $t_c(z)$ units of time, where $t_c(z)$ increases monotonically in the distance commuted, $t'_c(z) > 0$. A household living in the immediate neighbourhood of the CID has no commuting costs, $t_c(0) = 0$. No further commuting costs arise. Hence, opportunity costs of commuting are $w t_c(z)$.

There is a continuum of $N > 0$ identical households living in the city (i.e., we are considering a closed city). They have identical preferences on private consumption of goods (amount $x$), living space (size $s$), and environmental pollution $E$, which are represented by the utility function

$$u(x, s, E) = x^\alpha \cdot s^{1-\alpha} - d(E), \quad (1)$$

where $\alpha \in (0, 1)$, i.e. $u(x, s, E)$ is increasing and concave in the consumption of goods $x$ and of living space $s$. Environmental damage $d(E)$ is increasing and convex in the environmental pollution $E$.

Each individual household is endowed with one unit of time, i.e. the gross time being available for working and commuting is $N$. (Leisure is ignored for reasons of simplicity.) Households have their places of residence distributed over the city, such that

$$N = \int_0^Z n(z) \, dz, \quad (2)$$

where $n(z)$ is population density at place $z \in [0, Z]$, i.e. $n(z)$ people live in a unit of space at $z$.

The consumption good is produced by means of labour alone. The technology is described by the production function $F(\cdot)$, which is assumed to be increasing and concave.\(^4\)

\(^4\)For some calculations below, we assume a linear production technology, which implies constant
The hinterland of the city is big, such that the city may be considered as a small open economy which trades the consumption good at a competitive price \( p \); transportation of the consumption good is costless.

So far, the urban economic model is fairly standard. The extension is to introduce environmental pollution, which we assume to be caused by the consumption of goods only. This captures the observation at the case of Bombay that private consumption contributes considerably to environmental pollution. To include additional pollution stemming from goods production would be straightforward, but would also complicate the model without further insights to be expected. Polluting emissions \( e(z) \) of a household residing at location \( z \) are generated as a by-product of, i.e. proportional to, consumption \( x(z) \). Environmental pollution \( E \) is a ‘public bad’ in the city and equals aggregate emissions, i.e.

\[
E = \int_0^Z n(z) \cdot e(z) \, dz \quad \text{for } 0 \leq z \leq Z. \tag{3}
\]

Here, environmental pollution \( E \) is a pure public bad: it is the same for all urban residents independent of their place of residence. We further assume that the adjacent neighbourhood is affected by urban pollution to a considerable extent such that there is no incentive for an urban dweller to move into the hinterland just to avoid the environmental damage in the city.

In order to capture the second observation at the case of Bombay, we introduce pollution-reducing infrastructure as a public means of abating pollution. Emissions \( e(z) \) generated by a household residing at location \( z \) are assumed to decrease with the density returns to labour. This is justified, if positive Marshallian externalities exist.
of infrastructure \( i(z) \) provided there,

\[
e(z) = \gamma(i(z)) \cdot x(z),
\]

where \( \gamma(i) \) has the properties

\[
0 < \gamma(i) \leq 1 \quad \text{with} \quad \gamma(0) = 1, \quad \gamma'(i) < 0, \quad \gamma''(i) > 0 \quad \text{and} \quad \gamma''(i) \gamma(i) \geq \gamma'(i)^2.
\]

The interpretation of modelling infrastructure in this way is as follows: the by-products of each unit of consumption are the same with and without infrastructure. If infrastructure exists with density \( i(z) > 0 \) at place \( z \), however, only a fraction \( \gamma(i(z)) < 1 \) of these by-products is actually emitted into the environment, the remainder \( 1 - \gamma(i(z)) \) is disposed of ‘properly’ by means of the infrastructure and causes no environmental damage. In the following, only the damaging part of the by-products will be called ‘emissions’ (or ‘polluting emissions’).

Infrastructure \( i(z) \) available in a unit of space at \( z \) is assumed to be a local public good, that is, it serves to reduce the polluting emissions of all \( n(z) \) households residing there, so that total emissions are \( n(z) e(z) = \gamma(i(z)) n(z) x(z) \). Since infrastructure removes a fraction of local emissions rather than an absolute amount, the same quantity of infrastructure is proportionally more useful in areas where total consumption of goods is higher. This means that infrastructure is a pure public good, i.e. we abstract from congestion.

The curvature properties of \( \gamma(\cdot) \) imply that an increased provision of infrastructure lowers the emissions generated by each unit of consumption, but the marginal benefits of additional infrastructure decrease. The last property in (5) requires that the relative
decrease in marginal benefits is not too small. This assumption will be discussed in section 3, where it becomes relevant.

Building infrastructure comes at two kinds of costs: first, the physical infrastructure has to be bought at a (‘world market’) price $p_i$, and second, operating one unit of infrastructure requires one unit of labour input. By reducing emissions at the location where it is installed, infrastructure generates a public good. Environmental pollution, which is given by

$$E = \int_0^Z n(z) e(z) \, dz = \int_0^Z n(z) \gamma(i(z)) x(z) \, dz \quad \text{for} \quad 0 \leq z \leq Z,$$

(6)

decreases with the density of infrastructure at any point in the city, i.e. $dE/di(z) < 0$ for all $z \in [0, Z]$.

The model is closed by the following assumptions. All land within the city is owned by an urban government, which buys the land at a given rural rent $r$, and converts it into living space at zero costs in such a way that one unit of land equals one unit of living space (i.e. the ‘height’ of the buildings is fixed). Moving within the city is costless. Hence, in the residential equilibrium, the utility of each and every individual household is the same across locations in the city. Otherwise, there would be an incentive to move for at least one individual. In the following, we will call this condition for the residential equilibrium with costless possibility of relocation the ‘spatial equilibrium condition’.
3 Optimal allocation

In order to derive the Pareto optimal allocations, we maximise the utility of one household given a minimum utility level of all others and subject to the constraints, which result from the model specification as described in section 2. Without loss of generality, we maximise the utility of a household residing at \( z = 0 \), given that all others enjoy at least a level \( U(z) \) of utility, which is allowed to differ between different places of residence \( z \in [0, Z] \). (A spatial equilibrium, however, requires \( U(z) = U \) for all \( z \in [0, Z] \).) Formally, this condition reads

\[
u(x(z), s(z), E) = U(z) \quad \text{for all} \quad 0 \leq z \leq Z. \tag{7}\]

This equation describes a continuum of constraints, since we require it to hold for each \( z \in [0, Z] \). Hence, there is a continuum of Lagrangian multipliers \( \lambda(z) \) associated with (7).

We now turn to the economic constraints of the optimisation. The spatial distribution of population in the city determines total labour supply \( \hat{L} \), which equals total endowment with time less total time spent for commuting, i.e.

\[
\hat{L} = N - \int_0^Z n(z) t_c(z) \, dz. \tag{8}
\]

Labour supply is divided into labour input \( L \) in the production sector in the CID and the amount of labour required to operate the infrastructure (i.e. one unit of labour for each unit of infrastructure). Using condition (2) in equation (8) yields the constraint (Lagrangian multiplier \( \omega \))

\[
L + \int_0^Z i(z) \, dz = \int_0^Z n(z) [1 - t_c(z)] \, dz. \tag{9}
\]
The output $F(L)$ of the production sector is used for aggregate consumption of goods and net exports, i.e. exports minus imports, $\Delta$ (Lagrangian multiplier $\pi$)

$$F(L) = \int_{0}^{Z} n(z) x(z) \, dz + \Delta.$$  

(10)

The consumption of goods generates environmental pollution, as described by equation (6). The Lagrangian multiplier for this constraint is $\eta$.

We finally require the governmental budget is balanced, i.e. the value of net exports less the value of goods acquired from the hinterland (these are land which is rented by the urban government and physical infrastructure bought from abroad) plus any net transfers $^5 \Theta(z)$ to a household residing at $z$ sum up to zero (Lagrangian multiplier $\mu$),

$$p \cdot \Delta - x \int_{0}^{Z} n(z) s(z) \, dz - p_i \int_{0}^{Z} i(z) \, dz + \int_{0}^{Z} n(z) \Theta(z) \, dz = 0.$$  

(11)

In the above constraints to the optimisation, $x(z)$, $s(z)$, $n(z)$ and $i(z)$ for all $z \in [0, Z]$ are variables which can be chosen independently of each other. In particular $x(z)$ and $x(z')$, for $z' \neq z$, can be chosen independently, and consumption need not to be continuous over space in the first place. The same is true for the other variables.

Except for these variables, the distance $z$ from the CID occurs in condition (9). This variable, however, is not independent from the lot sizes $s(z)$ and population densities $n(z)$. Consider the following thought experiment: to determine the optimal allocation, households get assigned lot sizes $s(z)$, starting at $z = 0$ and then continuing to places further outside. If the first $n_0 = \int_{0}^{z_0} n(\tilde{z}) \, d\tilde{z}$ households already have their homes, the

$^5$Transfers may be needed to re-distribute incomes in order to reach a particular Pareto-optimum.
next one will live at a distance

\[ z = \int_{0}^{z_0} n(\tilde{z}) s(\tilde{z}) d\tilde{z} \quad (12) \]

from the CID, as one unit of living space occupies one unit of land and, hence, increases the distance from the CID by one unit. Since no spaces between buildings will be left in the optimum (otherwise commuting costs would increase without any gain), population density at each place \( z \) is \( n(z) = 1/s(z) \), i.e. \( z = z_0 \) in equation (12). This argument hold for all places in the city, which yields the condition

\[ z = \int_{0}^{z} n(\tilde{z}) s(\tilde{z}) d\tilde{z} \quad \text{for all} \quad 0 \leq z \leq Z. \quad (13) \]

Using this constraint, we can eliminate \( z \) from condition (9) and have

\[ L + \int_{0}^{Z} i(z) \, dz = \int_{0}^{Z} n(z) \left[ 1 - t_c \left( \int_{0}^{z} n(\tilde{z}) \cdot s(\tilde{z}) \, d\tilde{z} \right) \right] \, dz. \quad (14) \]

The Pareto optimal allocation consists of the consumption of goods \( x(z) \) and flat size \( s(z) \) of all households, the density of infrastructure \( i(z) \) and population density \( n(z) \) at each place in the city, as well as labour input \( L \) in production, pollution \( E \), and net exports \( \Delta \).

It is found by solving the following problem:

\[ \max_{\{x(z), s(z), n(z), i(z)\}, \Delta, L, E} u(x(0), s(0), E) \text{ subject to (6), (7), (10), (11) and (14).} \quad (15) \]

In appendix A.1, we derive the conditions for an optimal allocation by applying the Lagrangian formalism.\(^6\) They lead to the following conditions on the optimal allocation

\(^6\)The appendix is available from the author upon request or can be downloaded from the internet at http://www.ufz.de/data/Quaas_Infrastructure_Appendix3740.pdf
of consumption goods $x(z)$ and living space $s(z)$,

$$\frac{(1 - \alpha)}{\alpha} \frac{x(z)}{s(z)} = \left[ \mu r + \omega \int_{z}^{Z} n(\tilde{z}) t'(\tilde{z}) d\tilde{z} \right] / [\pi + \gamma(i(z)) \eta]$$

(16)

and for the optimal allocation of pollution-reducing infrastructure,

$$-\gamma'(i(z)) n(z) x(z) \eta = \mu p_i + \omega.$$  

(17)

Condition (16) states that the marginal rate of substitution between living space and goods consumption equals the ratio of social marginal costs of living space and goods consumption at the optimum. The social marginal costs of living space (the numerator on the right hand side of equation 16) consist of two parts: the shadow value of undeveloped land, which is the rural land rent times the shadow price $\mu$ of government income, plus the marginal increase in opportunity costs of commuting for all households living further away from the CID (where $\omega$ is the shadow wage rate). Also, the social marginal costs of goods consumption (the denominator on the right hand side of equation 16) have two parts: they are composed of the shadow price $\pi$ of goods production and the social costs $\eta$ of environmental damage due to the $\gamma(i(z))$ units of emissions, which are caused by a unit of consumption at $z$.

Condition (17) states that marginal utility from infrastructural provision at $z$ equals marginal costs at the optimum. Marginal costs consist of the price of installing the infrastructure plus the shadow price of labour needed to operate that infrastructure. Marginal utility equals the decrease in social marginal costs of goods consumption. Here, the character of infrastructure as a public means of emission abatement appears: increasing infrastructural supply decreases environmental costs caused by the consumption of
all \( n(z) \) households living in the unit of space under consideration.

To simplify notation, we choose government income as numerâtre and define the following abbreviations,

\[
D' := \eta / \mu,
\]

which is social marginal damage of pollution and

\[
r(z) := r + p F'(L) \int_z^Z n(\tilde{z}) t'_c(\tilde{z}) d\tilde{z},
\]

which are the social marginal costs of living space, both in terms of government income.

With these abbreviations, equations (16) and (17) simplify to

\[
\frac{(1 - \alpha)}{\alpha} \frac{x(z)}{s(z)} = \frac{r(z)}{[p + \gamma(i(z)) D']} - \frac{\gamma''(i(z)) n(z) x(z) D'}{\gamma'(i(z))} < 0 \quad (22)
\]

for \( 0 \leq z \leq Z \).

**Proof:** see appendix A.2.

To illustrate, how this result arises, consider condition (21) for the efficient level of infrastructural supply. Anticipating a result of corollary 1, population density declines
from the CID towards the periphery. The question is, how should infrastructure density change with distance from the CID, in order to keep marginal benefits equal to the (spatially constant) marginal costs?

On the one hand, a decrease of $i(z)$ rises marginal benefits (since $\gamma(i(z))$ is concave), compensating for the decline in population density. On the other hand, also an increase of infrastructural supply has a positive effect on marginal benefits of infrastructure: it makes consumption of goods cleaner, such that the efficient level of consumption increases. This, in turn, increases marginal benefits of infrastructural supply. Assumption (5) assures that this somewhat perverse effect is dominated by the effect of decreasing marginal benefits.

However, it is worthwhile noting that this second (general equilibrium) effect exists, which tends to lessen the decrease in infrastructural supply from the CID to the periphery. In particular, in a partial equilibrium setting, where the effect of infrastructural supply on the efficient level of consumption would not be considered, spatial inequalities in infrastructural supply would be stronger.

The main conclusion of proposition 1 is that the supply of infrastructure has to be spatially differentiated in an adequate manner. Specifically, the optimal density of infrastructure decreases from the CID to the periphery. This result is driven, on the one hand, by the way how infrastructure works to reduce environmental damages from goods consumption. On the other hand, the spatial distribution depends on the spatial structure of the city, which is reflected by the factor $r'(z)/r(z)$ in (22). This factor is negative, because living space is relatively scarcest in the neighbourhood of the CID. As a consequence of proposition 1, the further outside an individual lives, the more she substitutes
consumption of goods by consumption of living space in the optimum:

**Corollary 1**

If the utility level is the same for all urban residents, \( U(z) = U \), from the CID to the periphery,

- consumption of goods decreases, \( dx(z)/dz < 0 \),
- consumption of living space increases, \( ds(z)/dz > 0 \),
- population density decreases, \( dn(z)/dz < 0 \), and
- total emissions decrease \( d[n(z)e(z)]/dz < 0 \).

**Proof:** see appendix A.3.

These results are in line with intuition, only the last one needs some discussion. Three effects affect the spatial distribution of emissions: population density and consumption are decreasing, leading *ceteris paribus* to decreasing emissions, but infrastructural density is also decreasing, leading to higher emissions per unit of consumption. Given assumption (5), i.e., strongly decreasing marginal benefits of infrastructure, the former effects dominate the latter, such that total emissions decrease with the distance from the CID.

One question of interest is, how optimal infrastructural supply changes, if the population \( N \) of the city grows. We address this question in a comparative static analysis by considering an exogenous change of \( N \). It seems obvious that if the city expands, newly inhabited areas should be supplied with infrastructure. But also in other areas infrastructure supply has to be adjusted to the change in population size. In particular, if some
additional assumptions are met, it can be shown that the optimal supply of infrastructure increases everywhere in the city.

**Proposition 2 (Adjustment of infrastructure to increasing population)**

1. If the urban population increases, the optimal supply of infrastructure has to be adjusted everywhere in the city, $\frac{dI(z)}{dN} \neq 0$ for all $z \in [0, Z]$.

2. If $t_c(z) = t_c \cdot z$, $F(L) = f \cdot L$ and $d(E) = \delta \cdot E$, the optimal supply of infrastructure increases everywhere in the city, if the urban population increases, $\frac{dI(z)}{dN} > 0$ for all $z \in [0, Z]$.

**Proof:** see appendix A.4

If population increases, marginal benefits of infrastructure increase due to two effects related to the twofold public good character of pollution-reducing infrastructure. First, social marginal damage of pollution increases – with a growing population more people suffer from environmental pollution, which is a public bad. Second, population density increases, because opportunity costs of living space increase due to higher competition for housing sites near the CID. This leads to a higher marginal benefit of the (local) public good infrastructure. Both effects favour an increase in infrastructural supply all over the city.

In the second part of the proposition, we have assumed constant returns to labour input in goods production (which might be justified by positive Marshallian externalities). This assumption is important for the unambiguous result. With decreasing returns to labour two opposing effects would prevail. On the one hand, the marginal costs of operating infrastructure would decrease, since wages decline if population grows. This effect favours
a higher endowment with infrastructure. On the other hand, goods consumption per head would decrease due to decreased productivity. This effect would decrease the marginal benefit of infrastructure, favouring a lower infrastructural supply.

The general results of propositions 1 and 2 are illustrated by the example shown in figure 1:

[ Figure 1 about here ]

the density of infrastructure decreases with the distance from the CID. It is higher all over the city in the case of the higher population; also the city is larger in this case. More details about the example are given in section 5.

4 Urban environmental policy

Now we turn to the problem of how to implement the socially optimal allocation in a decentralised economy. We consider three settings: first, we determine the laissez-faire allocation without governmental intervention (section 4.1). Second, we consider a situation where the urban government supplies pollution-reducing infrastructure and imposes a Pigouvian tax on consumption (section 4.2). Third, we consider a setting, where the government imposes a Pigouvian tax on emissions and households provide pollution-reducing infrastructure themselves. The government has to subsidise infrastructure in order to achieve an optimal provision of this public good (section 4.3).

Whether the setting investigated in section 4.2 or in section 4.3 is relevant to a specific context depends on whether the households can observe their consumption of goods
only or if they can monitor their polluting emissions: if households cannot monitor their emissions, only the urban government can supply infrastructure efficiently. Since households cannot choose emissions independently of consumption, the Pigouvian tax is on consumption rather than on emissions. In the case of Bombay, the World Bank funded a large project of improving sewage disposal, where outfalls for sewage water into the coastal sea have been built (World Bank 2000; the project closed in 2003). This project could, for example, be continued by providing improved canalisation throughout the city, which most likely would be publicly financed. According to the analysis in section 4.2, the supply of sewage pipelines should take into account the spatial structure of the city in an adequate manner. In addition, a Pigouvian tax on freshwater-use, which is the source of possibly polluting effluents, should be spatially differentiated according to the infrastructural supply.

If, on the other hand, households can monitor their emissions, infrastructure can be provided by private households. In this case, the Pigouvian tax is on emissions, because households can choose the emission level per unit of consumption by choosing the infrastructure density $i(z)$. An example for pollution-reducing infrastructure in Bombay, which would best be provided privately, is the supply of public sanitation facilities (Palnitkar 1998, Government of India 2001). Since in this case local residents are probably well-informed about the effluents, private provision of this infrastructure could be the best option to reach an efficient supply. As shown in section 4.3, in such a situation the subsidy per unit of infrastructure has to be spatially differentiated and combined with a transfer scheme to avoid the introduction of (additional) distortions in the housing
market.

4.1 Laissez-faire

The laissez-faire equilibrium is the allocation in which households maximise utility, given their income $y(z)$, by choosing the consumption of goods and living space and their place of residence, and firms maximise profits. In equilibrium all markets clear, and the spatial equilibrium condition holds, i.e. all households enjoy the same utility. In the laissez-faire case, the urban government’s role is to rent out living space to urban residents and to redistribute revenues (net of expenditures to rent undeveloped land) equally among the inhabitants of the city. Let $w$ be the wage rate and $r(z)$ be the rent for living space. The income $y(z)$ of a household living at place $z \in [0, Z]$ is the sum of wage earnings and an equal share $\Theta$ of redistributed rents, $y(z) = w (1 - t_c(z)) + \Theta$.

Profit maximisation of firms implies that the value of the marginal product of labour equals the wage rate $w$, i.e. $p F'(L) = w$. Utility optimisation of a household living at $z$ yields the demand functions $x(z) = \alpha y(z)/p$ for goods and $s(z) = (1 - \alpha) y(z)/r(z)$ for living space. The spatial equilibrium requires that utility is the same at all locations $z \in [0, Z]$, i.e. none of the identical households has an incentive to move. In equilibrium, rents $r(z)$ for living space adjust such that this condition is fulfilled. Using the demand functions for living space and goods consumption in the utility function, and plugging in the condition $n(z) = 1/s(z)$, we arrive at the following condition (see appendix A.5):

$$r'(z) = -n(z) w t'_c(z).$$

Exactly the same condition is derived by differentiating equation (19), which gives the
marginal costs of living space in the Pareto optimum, with respect to \( z \) and using the condition for the firm’s profit maximum, \( w = p F'(L) \). This observation reflects the fact that the market for living space is undistorted. However, that both conditions are formally identical does not imply that the allocation of living space is the same in the Pareto optimum and the laissez-faire, nor that the rent for living space is the same. In the Pareto optimum, compared to the laissez-faire, consumption of goods is substituted by clean consumption of living space. This leads to a different population density in both settings and thus, according to equation (23), also to a different rent for living space.

### 4.2 Public provision of infrastructure

Turning to urban environmental policy, we start with the case that the urban government provides pollution-reducing infrastructure. It is assumed to do this optimally, i.e. according to the optimality condition (21). In addition, we assume that the government imposes a Pigouvian tax with rate \( \tau(z) \) on consumption, and pays net transfers \( \Theta(z) \) to the households. Both of these instruments are allowed to be spatially differentiated.

Hence, the income of a household residing at \( z \) is \( y(z) = w (1 - t_c(z)) + \Theta(z) \), where \( w = p F'(L) \) is the competitive wage rate. Denoting the rent for living space with \( r(z) \), the household’s optimisation problem reads

\[
\max_{x(z), s(z)} x(z)^\alpha s(z)^{1-\alpha} - d(E) \text{ subject to } w (1 - t_c(z)) + \Theta(z) = (p + \tau(z)) x(z) + r(z) s(z).
\]

(24)

The first order conditions for this problem yield:

\[
(1 - \alpha)/\alpha \ x(z)/s(z) = r(z)/ [p + \tau(z)] .
\]

(25)
By comparing this equation to (20), we find that the condition for the household’s optimum is equal to the condition for the Pareto optimum, if the rent $r(z)$ for living space equals social marginal costs of living space, as given by equation (19), and if the tax rate $\tau(z)$ on consumption is

$$\tau(z) = \gamma(i(z)) D', \quad (26)$$

where $D'$ is the social marginal damage from pollution (equation 18). Since each unit of consumption generates $\gamma(i(z))$ units of emissions, which add to pollution (cf. equations 4 and 6), the Pigouvian tax rate just captures the marginal environmental damage of consumption. This is a standard result. However, considering pollution-reducing infrastructure qualifies this result. The tax rate $\tau(z)$ on consumption depends on the amount of infrastructure provided at the place where the respective unit of goods is consumed. Hence, the Pigouvian tax rate on consumption differs over space, although the marginal damage from pollution is the same throughout the city. At places where infrastructure density is high, marginal damage is comparatively low and vice versa. Hence, the tax rate is lowest in the city centre, where, according to proposition 1, infrastructure density is highest, and increases towards the periphery, where infrastructure is is provided at a lower level. This result is obtained formally by differentiating (26) with respect to space, using $\gamma'(i(z)) < 0$ (assumption 5) and $i'(z) < 0$ (condition 22), which yields the result that the optimal tax rate increases from the CID to the periphery, i.e. $\tau'(z) > 0$.

To implement the optimum as a spatial equilibrium, we also need to ensure that the spatial equilibrium condition holds, i.e. that the utility of a representative household everywhere is the same. Because, in contrast to the laissez-faire allocation, the consumer
price of the consumption good increases from the CID to the periphery (as $\tau'(z) > 0$), households have an incentive to re-locate to the city centre, unless a spatially differentiated transfer is paid to compensate for this.

Together with the results derived above, this determines the optimal environmental policy in the case of publicly supplied infrastructure.

**Proposition 3 (Environmental policy with public provision of infrastructure)**

Three policy instruments are needed to reach a first best in a decentralised economy with public provision of infrastructure:

1. a spatially differentiated supply of infrastructure according to (22),

2. a spatially differentiated tax on consumption with rate $\tau(z) = \gamma(i(z)) D'$, where $\tau(z)$ increases monotonically with the distance from the CID, $d\tau(z)/dz > 0$ for $0 \leq z \leq Z$,

3. a spatially differentiated transfer $\Theta(z)$ of incomes, where $\Theta'(z) = \tau'(z) x(z) > 0$.

**Proof:** see appendix A.6.

Hence, the optimal environmental policy requires three instruments, and all of them have to be spatially differentiated: the density of infrastructure declines from the CID to the periphery, the Pigouvian tax rate increases with the distance from the CID, and there is a redistribution of incomes from the centre to the periphery. All these spatial differentiations – which are jointly endogenously determined – result from including pollution-reducing infrastructure in the model. If the possibility of a spatially distributed supply of infrastructure is neglected, i.e. if $\gamma(i(z)) \equiv \text{const}$, the tax rate $\tau$ is uniform all over the city and a spatially differentiated redistribution of incomes would be unnecessary.
Using the example described in section 5 below, the results of proposition 3 are illustrated in figure 2, where the spatial differentiation of the tax on consumption and the net transfer between households is depicted (the differentiation of infrastructural supply is illustrated in figure 1).

[ Figure 2 about here ]

4.3 Private provision of infrastructure

In this section, we shall consider the setting in which infrastructure is provided by private households. We introduce three policy instruments, (i) a subsidy $\sigma(z)$ on the private supply of pollution-reducing infrastructure, (ii) a tax $\theta(z)$ on polluting emissions, and (iii) net transfers $\Theta(z)$ of incomes. All three instruments are allowed to be spatially differentiated in the first place. In this setting, two decisions of the household determine emissions $e(z)$: consumption of goods and infrastructural supply $i(z)$. The optimisation problem of a household residing at $z$ is

$$\max_{x(z), s(z), i(z)} x(z)^\alpha s(z)^{1-\alpha} - d(E) \quad \text{subject to}$$

$$w(1-t_c(z)) + \Theta(z) = p x(z) + r(z) s(z) + (p_i + w - \sigma(z)) i(z) + \theta(z) e(z)$$

$$e(z) = \gamma(i(z)) x(z).$$

The first order conditions for the household’s optimum lead to the following equations:

$$x(z)/(\alpha s(z)) = r(z)/(p + \theta(z) \gamma'(i(z)))$$

$$\sigma(z) - \theta(z) \gamma'(i(z)) x(z) = p_i + w. \quad (29)$$
The comparison of (28) with the condition (20) for the optimal allocation yields:

\[ \theta(z) = D', \]  

(30)

i.e. the Pigouvian tax rate \( \theta(z) \) on emissions equals the social marginal damage of pollution, which is constant throughout the city. Hence, it is the same for all households, independently of their place of residence.

Substitution of equation (30) into (29) and comparing this equation to the condition (21) for the optimal supply of pollution-reducing infrastructure yields

\[ \sigma(z) = (n(z) - 1)/n(z) (p_i + w). \]  

(31)

The necessity of subsidising infrastructure results from the fact that infrastructure is a public good. The subsidy (31), which ensures its optimal supply, varies within the city. As stated in corollary 1, the optimal population density declines from the city centre to the periphery. Thus, the subsidy declines as well. As a consequence, in the case of privately supplied infrastructure, a similar redistribution of incomes is necessary as in the case of public supply of infrastructure (cf. proposition 3).

The three instruments of environmental policy in the case of private supply of infrastructure are summarised in the following proposition.

**Proposition 4 (Environmental policy with private provision of infrastructure)**

Three policy instruments are needed to achieve a first best allocation in a decentralised economy with a private provision of infrastructure:

1. a spatially differentiated subsidy on infrastructure (equation 31),

2. a spatially homogenous tax on emissions (equation 30), and
3. a spatially differentiated redistribution $\Theta(z)$ of incomes, where $\Theta'(z) = -(p_i + w)/n(z) i(z) n'(z)/n(z) > 0$.

**Proof:** see appendix A.7.

The spatial differentiation of the subsidy on infrastructural supply and the transfers of income are illustrated in figure 3 using the example described in the following section.

[ Figure 3 about here. ]

The amount of transfers differs in the setting of public and private infrastructural supply (cf. figures 2 and 3), because what the government receives and pays is different in both settings.

## 5 An example

By employing an example we shall illustrate the general results of the previous sections and discuss some further implications of the model. We therefore specify the commuting time $t_c(Z) = t_c \cdot z$, the production function $F(L) = f \cdot L$, environmental damage $d(E) = \delta \cdot E$, and how emissions per unit of consumption depend on pollution-reducing infrastructure,

$$\gamma(i) = \exp \left(-i/\epsilon\right).$$

(32)

This specification allows an analytical solution of the model, which is performed in appendix A.8. For the parameters given in table 1, the allocation of private goods is as shown in figure 4.

[ Figure 4 about here. ]
In order to assess the implications of using pollution-reducing infrastructure as a tool for environmental policy, the resulting first-best allocation (given by the solid lines) is compared to (i) the second-best allocation, where only Pigouvian taxes are available as an instrument for environmental policy (the dashed lines) and (ii) the laissez-faire case without environmental policy (the dotted lines).

Per capita utility is $u = 0.389 - 0.023 = 0.366$ in the first-best case (i.e. utility from private goods consumption is 0.389 and environmental damage is 0.023), $u_P = 0.382 - 0.467 = -0.085$ in the second-best and $u_0 = 0.451 - 1 = -0.549$ in the laissez-faire case. Obviously, it is highest in the first-best case and higher with some environmental policy (in the second-best) than without (in the laissez-faire). In the laissez-faire case, consumption of private goods is highest, but also, environmental damage is highest, which yields the lowest total utility. Total utility is higher in the case of environmental policy by means of Pigouvian taxes only. But environmental damage is considerably lower, if additionally pollution-reducing infrastructure is used. In addition, utility from private consumption is higher than if only Pigouvian taxes are employed. This is the result of two opposing effects: on the one hand, infrastructural supply comes at costs, but on the other hand, it allows for more consumption of goods without affecting the environment too much. In the current example, the second effect outweighs the first one,

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\bar{r}$</th>
<th>$p_i$</th>
<th>$\alpha$</th>
<th>$f$</th>
<th>$t_e$</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 1: The parameters used to construct the figures.
making pollution-reducing infrastructure an even more favourable instrument of urban environmental policy.

One observation in figure 4 is that in the second-best case where Pigouvian taxes but no infrastructure are used for environmental policy, consumption of goods is usually lower and consumption of living space is usually higher compared to the laissez-faire case. This is because environmentally harmful consumption of goods is substituted by the ‘clean’ consumption of living space. The higher overall consumption of living space also leads to a larger city, i.e. the city’s border is further outside, \( Z_P > Z_0 \).

In the first-best, where also pollution-reducing infrastructure is used as a means of environmental policy, this effect still prevails, but in the current example, it is offset by another effect: since infrastructure is a public good, it is profitable to increase the number of people enjoying the services of each unit of infrastructure, i.e. to increase population density. This effect leads to a comparatively smaller city.

An interesting observation is that with pollution-reducing infrastructure, the allocation of private goods is ‘closer’ to the laissez-faire allocation compared to the case where only a Pigouvian tax is used. Hence, it could be even easier to implement the optimal environmental policy than the second-best without pollution-reducing infrastructure.

### 6 Conclusions and discussion

In this paper, pollution-reducing infrastructure has been introduced into an urban economics model as a public good which serves to abate polluting emissions from household’s consumption. These innovative features of the model are motivated by stylised facts ob-
served at the case of Bombay, and decide it from standard urban economics models. The model assumptions and the analysis were coined to general theoretical insights, which hold for other cities as well.

We have shown that the efficient supply of pollution-reducing infrastructure and the efficient allocation of (polluting) consumption goods are interrelated. Increased infrastructural supply lessens the environmental costs of consumption, and, the other way around, increased consumption increases the marginal benefit of pollution-reducing infrastructure. Because consumption of goods and of living space are interrelated, too, the introduction of pollution-reducing infrastructure affects the spatial distribution of households across the city. In the decentralised economy, the rent for living space is no longer a sufficient incentive for households to locate at the optimal positions. Rather, transfer payments are needed, in order to obtain the efficient spatial distribution of households.

Despite these intricacies, an efficient allocation can be determined and implemented as a spatial market equilibrium with the help of three policy instruments. If infrastructure is supplied publicly, these are (i) the efficient supply of infrastructure, (ii) a Pigouvian tax on consumption and (iii) income transfers. All of these instruments have to be spatially differentiated, as determined in section 4.2. If infrastructure is provided privately, the three instruments are (i) a Pigouvian subsidy on infrastructure, (ii) a Pigouvian tax on emissions, and (iii) income transfers. In this setting, while the subsidy on infrastructural supply and the income transfers have to be spatially differentiated, the Pigouvian tax on emissions is the same all over the city reflecting the spatially homogenous environmental pollution.
Providing pollution-reducing infrastructure is most important in growing cities in developing countries. In a comparative static analysis, we have shown that, in response to increased population size, infrastructural provision has to be changed throughout the city, not just in newly inhabited areas. The twofold public-good character of infrastructure (which serves several households to abate emissions and thereby generates higher environmental quality) favours an increased infrastructural supply throughout the city. In order to investigate the question of how infrastructural supply should be adapted to a growing population in more detail, however, an extension to a dynamic model would be necessary, which would then also be capable of describing infrastructure – more realistically – as a capital good.

In order to apply the model empirically, it should be extended furthermore to include the specific characteristics of different kinds of infrastructure (in particular, congestion effects should be included) and to map a more realistic spatial structure of the city, by, e.g., dropping the assumption of a central industrial district, including several income classes, or inhomogeneous environmental pollution. Despite these challenges for future research, the present analysis has taken up issues of major importance, as the case of Bombay shows, but which have not been studied yet in economics. Thereby, it opens the field for a more realistic description of urban environmental problems and improved policy options to solve these problems.
References


Figure 1: The optimal density of infrastructure over space for two different population sizes. The example is specified in section 5.
Figure 2: The tax rate $\tau(z)$ and the transfer $\Theta(z)$ for the setting of public infrastructural supply for the example described in section 5.
Figure 3: The subsidy rate $\sigma(z)$ and the transfer $\Theta(z)$ for the setting of private infrastructural supply for the example described in section 5.
Figure 4: The rent for living space, lot size and per capita consumption over space, for three different scenarios described in the text.