Revisitation of the dipole tracer test for heterogeneous porous formations

Alraune Zech[a,*], Claudia D'Angelo[b], Sabine Attinger[a,c], Aldo Fiori[b]

[a] *Department of Computational Hydrosystems, Helmholtz Centre for Environmental Research - UFZ, Leipzig, Germany*
[b] *Dipartimento di Ingegneria, Universita di Roma Tre, Rome, Italy*
[c] *Institute for Geosciences, Friedrich Schiller University, Jena, Germany*

Abstract

In this paper, a new analytical solution for interpreting dipole tests in heterogeneous media is derived by associating the shape of the tracer breakthrough curve with the log-conductivity variance. It is presented how the solution can be used for interpretation of dipole field test in view of geostatistical aquifer characterization on three illustrative examples.

The analytical solution for the tracer breakthrough curve at the pumping well in a dipole tracer test is developed by considering a perfectly stratified formation. The analysis is carried out making use of the travel time of a generic solute particle, from the injection to the pumping well. Injection conditions are adapted to different possible field setting. Solutions are presented for resident and flux proportional injection mode as well as for an instantaneous pulse of solute and continuous solute injections.

The analytical form of the solution allows a detailed investigation on the impact of heterogeneity, the tracer input conditions and ergodicity conditions at the well. The impact of heterogeneity manifests in a significant spreading of solute particles that increases the natural tendency to spreading induced by the dipole setup. Furthermore, with increasing heterogeneity the number of layers needed to reach ergodic conditions become larger. Thus, dipole test in highly heterogeneous aquifers might take place under non-ergodic conditions giving that the log-conductivity variance is underestimated. The method is a promising geostatistical analyzing tool being the first analytical solution

*Corresponding Author
Email address: alraune.zech@ufz.de (Alraune Zech)

for dipole tracer test analysis taking heterogeneity of hydraulic conductivity into account.

Keywords:

- Analytical solution for breakthrough curve (BTC) of dipole tracer tests in heterogeneous media
- Interpretation of dipole field tests and estimation of aquifer heterogeneity (log-conductivity variance)
- Main features of dipole tests (strong preferential flow and persistent tail) are further strengthened by heterogeneity

1. Introduction

Groundwater is an important natural resource for drinking water supply. Hence, the investigation and predictive modeling of flow and transport in porous media are of broad relevance in particular for water quality aspects and methods for contaminated site treatment like remediation, risk assessment, and natural attenuation. Tracer tests are thereby the foremost used observation method for inferring hydrogeological, structural and transport parameters of the subsurface. Most important, the hydraulic conductivity K, which determines the velocity of groundwater flow, typically exhibits a large spatial heterogeneity with values varying over orders of magnitudes (Gelhar, 1993).

A bunch of different tracer test types exist which all have their merits but also limitations. Tracer test under ambient flow conditions are either limited to short travel distances, because groundwater flow is usually very slow, or the test operation suffers high costs due to a long test duration and the need for a large observation network. More efficient in time and size of observation network are tracer tests under forced flow conditions due to higher velocities and directed flow. However, the analysis of test types under well flow conditions are more complex due to the non-uniform flow field. Especially taking aquifer heterogeneity into account requires sophisticated analyzing methods.

In dipole tracer tests, also named two-well test or doublet tests, a tracer is introduced at a recharge well and the breakthrough curve (BTC) is measured at a pumping well. The pumped water can optionally be used for recharge in a recirculation. The test setting has the advantage of circumventing the problem of removal of waste water and the need for an additional water source. However, the complex flow pattern causes the interpretation of the transport behavior to be more complicated. The dipole shape of the flow field gives that the observed concentration at the pumping well to be a superposition of tracer transport along different streamlines with different tracer arrival times. The foremost aim in this work is to present an alternative interpretation method for dipole tests in heterogeneous media. It will be given a simple solution for interpretation of dipole test in view of geostatistical aquifer characterization.

The first analytical analysis of dipole test was performed by Hoopes and Harleman (1967). They presented a mathematical description of the flow field, provided equations for streamlines and the travel time of a tracer particle along a streamline. They gave analytical expressions for the temporal and spatial distribution of tracer in a homogeneous aquifer. By analyzing an approximate solutions for the concentration in the presence of convection, dispersion and diffusion, Hoopes and Harleman (1967) analyzed the impact of these processes. They found that dispersion impacts only the very early part of the BTC, afterwards the arrival of different streamlines is dominant. For constant tracer injection, the BTC is almost insensitive to dispersion.

Shortly after, Grove and Beetem (1971) presented a method for analyzing BTC of dipole tests in homogeneous aquifers in order to evaluate the porosity and the longitudinal dispersivity. The method focuses on tests with pulse tracer injection and recirculation being constructed as superposition of 1D solutions for individual streamlines based on calculations of the streamline length and particle travel time. The approach took the crucial assumption that velocity is constant along flow lines. The work of Grove and Beetem (1971) also included an illustrative example of an analysis of a dipole test in the fractured carbonate aquifer near Carlsbad, New Mexico.

Gelhar (1982) derived a semi-analytical solution and provided type curves for the BTC measured at the pumping well in a dipole flow system. The type curves where numerically derived based on the theoretical work of Gelhar and Collins (1971) about longitudinal dispersion along streamlines in nonuniform flow. For the solution, transverse dispersion is assumed to be negligible. The work of Gelhar (1982) included the analysis of a dipole test at the Hanford Site for illustrating the method. Welty and Gelhar (1989) reanalyzed several field tests using the method of Gelhar (1982) in order to estimate dispersivity.

The first approach to numerically interpret dipole tests was presented by Huyakorn et al. (1986a). They developed a problem adapted simulation software making use of a curvilinear FEM. The method was applied to the dipole field test at the Chalk River site (Pickens and Grisak, 1981). More recently, Bianchi et al. (2011) presented a numerical model to interpret the dipole test at the MADE site with explicitly including aquifer heterogeneity. The authors performed simulations with conditioned log-normal hydraulic conductivity fields.

Several examples for dipole field tests in consolidated media ca be found in literature, mostly as illustrative examples along analytical method development,e.g. in the previously mentioned papers of Grove and Beetem (1971); Gelhar (1982); Welty and Gelhar (1989). Some early works on tracer tests report large distance dipole tests. Examples are the test at the Savannah River Plant (Webster et al., 1970) where wells are 538 m (1,765 feet) apart with a duration of 2 years or the test at Amargosa (Claasen and Cordes, 1975) over a distance of 122 m (400 feet). There are also several examples for tests in consolidated media, mostly over shorter ranges which a well distance maximally a few tenth of meters, for instance Tucson, Arizona (Wilson, 1971); Chalk River Site, Canada (Pickens and Grisak, 1981); Kesterson Aquifer, California (Hyndman and Gorelick, 1996); Rocky Mountain Arsenal, Colorado (Thorbjarnarson and Mackay, 1997). Tests of particular interest within this work are the dipole tests at MADE (Bianchi et al., 2011), Barstow (Robson, 1974), and Mobile (Molz et al., 1986). They will act as illustrative examples for the method developed herein and will be described in detail later.

Dipole tests in the presence of significant spatial heterogeneity of hydraulic conductivity have rarely been studied. Though, the actual streamline structure of dipole tests is strongly impacted by aquifer heterogeneity. The analysis of dipole test with an explicit representation of aquifer heterogeneity has been done only using numerical models, e.g. Bianchi et al. (2011). Analytical models for dipole analysis including heterogeneity parameters are not available. The methods of Hoopes and Harleman (1967); Grove and Beetem (1971); Gelhar (1982); Welty and Gelhar (1989) are conceptualized for homogeneous conductivity, taking the effect of aquifer heterogeneity only implicitly onto account by the lumped parameter of macrodispersivity.

In this paper, we present an alternative concept for dipole test analysis by taking heterogeneity of hydraulic conductivity explicitly into account. A geostatistical approach is used due to the limited data availability in subsurface hydrology in combination with high uncertainty in values. The characteristics of hydraulic conductivity K are captured by the one-point statistical parameters of mean conductivity K_{G} and the log-conductivity variance σ_Y^2, with $Y = \ln K$ the log-conductivity. An analytical solution for the BTC at the pumping well in a dipole tracer test is developed by considering a stratified heterogeneous hydraulic conductivity structure, thus associating shape of BTC with statistical properties of the conductivity field. The stochastic framework and part of the analysis have some similarities with the model presented by Pedretti and Fiori (2013) for convergent flow. The sensitivity of dipole tests to spatial heterogeneity might be used to determine statistical parameters, especially the log-conductivity variance σ_Y^2 offering an alternative test method for geostatistical aquifer analysis.

The plan of the paper is given as following: the mathematical framework sets the background of the method providing the derivation of the analytical solutions for different test configurations; it is followed by an illustration of results and then a discussion on the impact of parameters and test conditions on the BTC; the method is then used for conductivity characterization on illustrative examples with a re-interpretation of three dipole field tests with the newly developed method. The paper ends with a summary and conclusions.

2. Mathematical framework

We consider a confined aquifer of thickness L; the heterogeneous hydraulic conductivity K field is modeled by considering the aquifer as a perfectly stratified formation, i.e. made up from N layers of vertical thickness $2I$, with I the vertical integral scale of hydraulic conductivity. Each layer is of random and independent conductivity K_i $(i = 1, .., N)$.

The justification of the stratified model is in the relative short distance between pumping and injecting wells in dipole tests which is often found in recent applications, of the order of the horizontal integral scale of K or even less. The perfectly stratified formation has been often used in the past for modeling flow and transport in heterogeneous porous formations, often leading to useful analytical solutions (Mercado, 1967; Matheron and De Marsily, 1980; Dagan, 1990).

A dipole is created by injecting and extracting a discharge Q in two fully penetrating wells at relative distance $2a$. Adopting head boundary conditions in the wells, the piezometric head does not depend on the vertical coordinate; thus, the distribution of head is the same for all layers. A sketch of the conceptual model is provided in Figure 1.

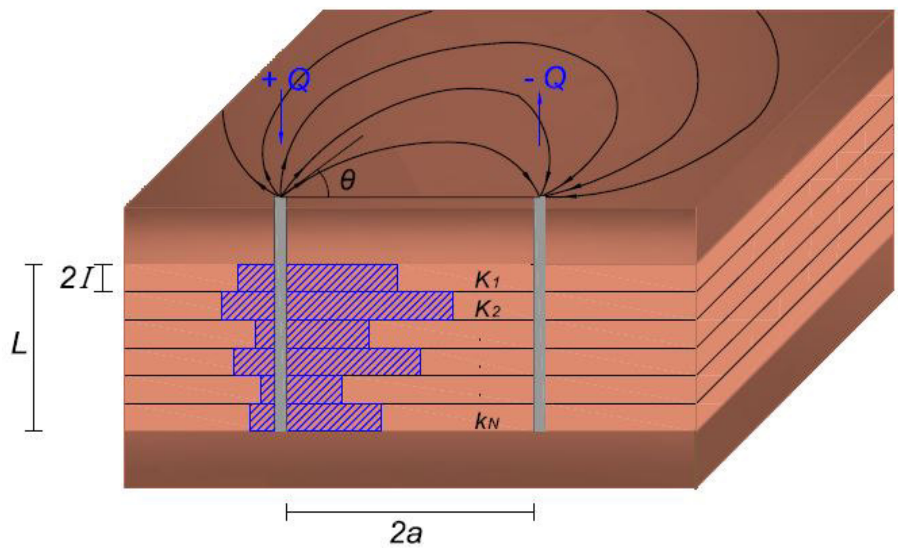


Figure 1: Illustration sketch of the conceptual model.

The discharge per unit thickness q_i for each layer i is equal to

$$q_i = \frac{K_i}{\bar{K}} q \tag{1}$$

with $\bar{K} = N^{-1} \sum_{i=1}^{N} K_i$ the arithmetic mean of K_i and $q = Q/L$ the aquifer discharge per unit depth.

At a given initial time, a pulse of solute of initial concentration C_0 and duration Δ is introduced in the injection well. The solute travels in the porous medium and it is collected downstream in the pumping well, resulting in a breakthrough curve (BTC) at the same well. The BTC, which corresponds to the temporal behavior of solute flux at the pumping well, depends on both the dipole setup (e.g. the distance $2a$, the discharge Q, the duration Δ etc) and the medium configuration, i.e. the vertical distribution of hydraulic conductivities K_i and their porosity, which in the following is assumed as constant in the entire domain.

Scope of the present analysis is to calculate the BTC in dependence on both the dipole setting and the aquifer configuration. The analysis is carried out by considering the travel time of a generic solute particle, from the injection to the pumping well. It is well known, that the probability density function (PDF) of such travel time is identical to the BTC of a solute instantaneous pulse. In the following we focus on advection only, which is the most significant source of spreading due to the non-uniform flow configuration. Local dispersion mechanisms like hydrodynamic dispersion or molecular diffusion are neglected.

The present solution is based on the analysis of Hoopes and Harleman (1967). They analyzed the travel time t of a particle, from the injection to the extraction well, pertaining to the generic streamline departing from the injection well at an angle θ with respect to the line joining the wells (see Figure 1). They derived an analytical solution for travel time in homogeneous formations under the assumption that the aquifer is indefinite, homogeneous and isotropic and confined between two horizontal planes, in absence of natural flow. The flow and piezometric head were obtained from the superposition of the flow fields of a line source and a line sink, assuming negligible well radii. The flow fields are obtained by the solution of water continuity equations and Darcy's law. The travel time t was obtained by integration of the flow field along the streamlines originating form the injection well. The resulting formula for the travel time in an arbitrary layer i is

$$t = \frac{4\pi n a^2}{q_i \sin^2\theta}\left(1 - \theta\cot\theta\right) \qquad \left(-\pi \leq \theta \leq \pi\right) \tag{2}$$

with being n the constant porosity, q_i is the layer's unit discharge, θ is the angle and $2a$ is the distance between injection and pumping well.

Hereinafter we work with the dimensionless travel time τ, defined as

$$\tau = \frac{qt}{4\pi n a^2} = g\left(\theta\right)\frac{\bar{K}}{K_i} \tag{3}$$

where

$$g\left(\theta\right) = \frac{1}{\sin^2\theta}\left(1 - \theta\cot\theta\right) \tag{4}$$

The form of the dimensionless travel time result from the by definition $\tau = v_c t/s_c$ with $v_c = q/n$ being the characteristic velocity and $s_c = 4\pi a$ being a characteristic length scale. τ is symmetrical with respect to $\theta = 0$, and the half space $\theta = [0;\pi]$ can be safely considered in the analysis.

The travel time (3) is a random variable, that depends on the hydraulic conductivity K_i of each layer and the angle of attack θ, which is uniformly distributed in the interval $[0;\pi]$. From (3) one can calculate the travel time distribution for the entire aquifer formation, considering the dependence of τ on the random variables K and θ, which distributions are $f_K(K)$ and $f_\theta = 1/\pi$, respectively. From basic statistics (see, e.g. Papoulis (1991)), the cumulative density function (CDF) P_τ of τ follows as

$$P_\tau(\tau) = \int\int_D f_K(K) f_\theta(\theta)\, dK d\theta = \frac{1}{\pi}\int_0^\pi \int_{g(\theta)\bar{K}/\tau}^\infty f_K(K)\, dK d\theta \qquad (5)$$

where D is the region in the (θ, K) space such that $g(\theta)\bar{K}/K \le \tau$.

Assuming ergodicity gives $\bar{K} = K_A$, with K_A being the arithmetic (ensemble) mean. The integration of (5) over K yields

$$P_\tau(\tau) = \frac{1}{\pi}\int_0^\pi \{1 - P_K(g(\theta) K_A/\tau)\}\, d\theta \qquad (6)$$

with P_K being the CDF of K. From the above expression, the travel time PDF is calculated as

$$f_\tau^{\mathrm{R}}(\tau) = \frac{1}{\pi}\frac{K_A}{\tau^2}\int_0^\pi g(\theta) f_K(g(\theta) K_A/\tau)\, d\theta \quad \text{(resident injection mode)} \qquad (7)$$

If we assume a log-normal distribution for K, the above specializes as follows

$$f_\tau^{\mathrm{R}}(\tau) = \frac{1}{\pi\tau\sqrt{2\pi\sigma_Y^2}}\int_0^\pi \exp\left[-\frac{(\sigma_Y^2/2 + \ln g(\theta) - \ln\tau)^2}{2\sigma_Y^2}\right] d\theta \qquad (8)$$

Expression (7) is the PDF of the dimensionless travel time τ. In its calculation we have assumed that the mass of solute entering in each layer from the injection well is constant for all layers, i.e. the injection condition is of *resident concentration*. Instead, the typical injection condition in applications is of *flux proportional*, i.e. the mass of solute entering each layer is proportional to the local velocity at the injection well, which is variable in the vertical and is proportional to the hydraulic conductivity K_i of each layer. The different injection conditions and their impact on the BTC is deeply discussed in Jankovic and Fiori (2010) for transport in mean uniform flow and in Pedretti and Fiori (2013) for transport in convergent flow.

The travel time PDF for flux proportional injection mode, along the above lines, can be calculated by weighting each solute particle by the layer conductivity K_i. This way, the resulting travel time PDF is obtained by averaging the PDF (7) by the weight

$$\frac{K_i}{K_A} = \frac{g(\theta)}{\tau} \tag{9}$$

obtaining

$$f_\tau^{\mathrm{F}}(\tau) = \frac{1}{\pi}\frac{K_A}{\tau^3}\int_0^\pi g^2(\theta)\, f_K\left(g(\theta)\, K_A/\tau\right) d\theta \quad \text{(flux-proportional injection mode)} \tag{10}$$

Assuming a log-normal distribution for K the latter becomes

$$f_\tau^{\mathrm{F}}(\tau) = \frac{1}{\pi\tau^2\sqrt{2\pi\sigma_Y^2}}\int_0^\pi g(\theta)\exp\left[-\frac{\left(\sigma_Y^2/2 + \ln g(\theta) - \ln\tau\right)^2}{2\sigma_Y^2}\right] d\theta \tag{11}$$

Summarizing, expression (10) represents the travel time PDF under flux proportional injection mode, that is the one to be used in applications.

The above solutions correspond to the BTC for an instantaneous pulse of solute. The extensions for continuous solute injections of initial concentration C_0 and duration Δ, which are typically employed in the tracer tests, are calculated by the convolution

$$C(\tau) = \int_0^{t^*} C_0(t')\, f_\tau(\tau - t')\, dt' \tag{12}$$

where

$$t^* = \begin{cases} \tau & \text{when } \tau \le \delta \\ \delta & \text{when } \tau \ge \delta \end{cases} \qquad \left(\delta = \frac{q\Delta}{4\pi n a^2}\right) \tag{13}$$

In the simple but relevant case when C_0 is constant and K is log-normally distributed (i.e. when f_τ is equal to 11), (12) becomes

$$\frac{C(\tau)}{C_0} = \begin{cases} \Phi(\tau) & \text{when } \tau \le \delta \\ \Phi(\tau) - \Phi(\tau - \delta) & \text{when } \tau > \delta \end{cases} \tag{14}$$

where

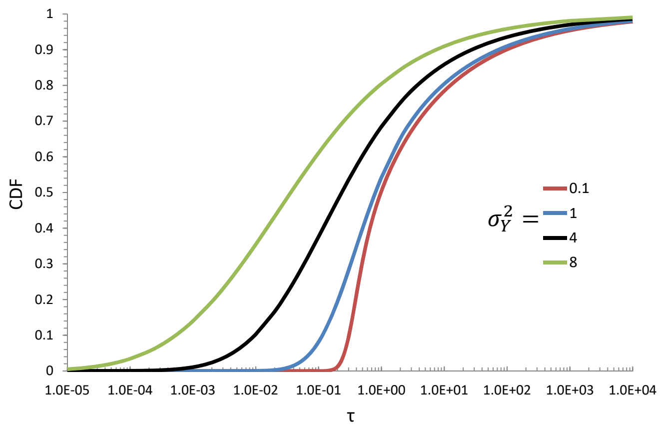


Figure 2: The travel time CDF P_τ^{F} as function of the dimensionless travel time $\tau = qt/(4\pi n a^2)$ for a few values of the log-conductivity variance σ_Y^2.

$$\Phi\left(\tau\right) = \frac{1}{2\pi}\int_0^\pi \left\{1 + \mathrm{erf}\left(\frac{\sigma_Y^2/2 - \ln g\left(\theta\right) + \ln\left(\tau\right)}{\sqrt{2\sigma_Y^2}}\right)\right\} d\theta \tag{15}$$

Formula (14) is the final result of the present analysis and provides the solute BTC for the dipole configuration examined here. The formula can be conveniently applied to the interpretation of dipole tests. We remind that the above solutions consider that the vertical distribution of conductivity is fully sampled, i.e. ergodicity is assumed; such condition is typically met when $L/I \gg 1$. The issue shall be further discussed in the next sections.

3. Illustration of results

In this section we illustrate the main results related to the proposed analytical solutions for the dipole tracer test. The PDF of the dimensionless

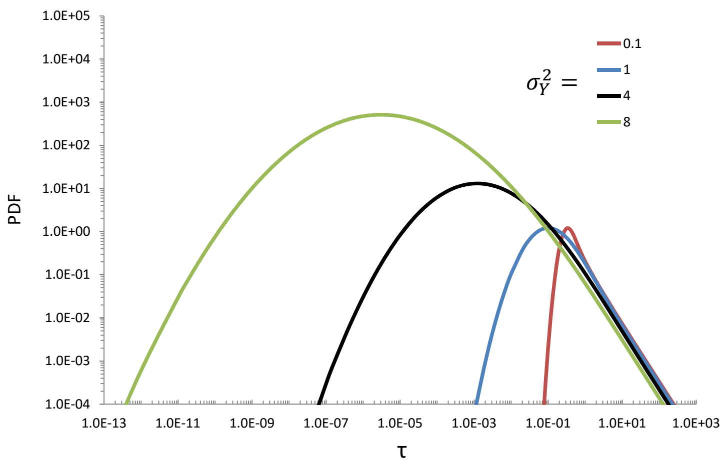


Figure 3: The travel time PDF f_τ^{F} as function of the dimensionless travel time $\tau = qt/(4\pi n a^2)$ for a few values of the log-conductivity variance σ_Y^2.

travel time $\tau = qt/(4\pi n a^2)$ for the flux -proportional injection condition and a log-normal distribution of K is given by (11); the latter depends only on the log-conductivity variance σ_Y^2, which represents the degree of heterogeneity of the aquifer system.

Figures 2 and 3 display the travel time PDF (P_τ^{F}) and CDF (f_τ^{F}) for a few values of σ_Y^2, respectively. Starting from an almost homogeneous formation ($\sigma_Y^2 = 0.1$), it is seen that the distribution of τ is characterized by a rising limb, which is determined by the first "fast" arrivals of solute moving along the most connected paths between the injection and pumping wells (small angle θ). In turn, the tail of the distribution is typically long and persistent, being determined by the slow arrivals of solute particles that move along the longer and slower path lines (large θ). Thus, the dipole setup always determines a wide variety of paths in the medium, causing a similar variability of arrival times and hence dispersion; this is a well known feature which has already been described in past work (Hoopes and Harleman, 1967; Grove and

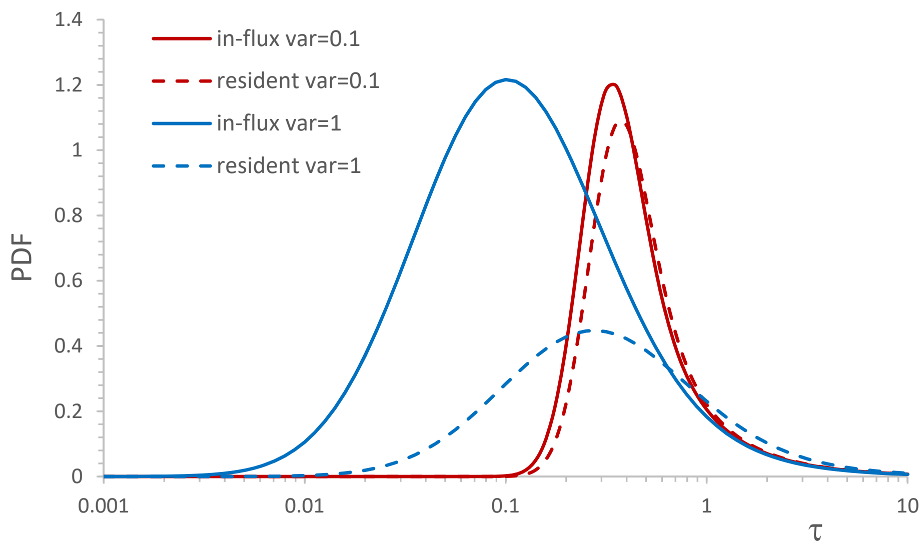


Figure 4: Comparison between the travel time PDFs pertaining to flux-proportional ("in-flux"; solid lines) and resident modes ("resident", dashed lines) injection conditions, for $\sigma_Y^2 = 0.1$ (red lines) and $\sigma_Y^2 = 1$ (blue lines).

Beetem, 1971; Koplik, 2001).

The spatial distribution of conductivity present in the aquifer system further enhances the above dispersion of solute, as visible in the curves of Figures 2 and 3 for $\sigma_Y^2 > 0$. It is seen that, for increasing degree of heterogeneity σ_Y^2, the travel time distributions depart from the solution for homogeneous formations in two ways: (i) a stronger preferential flow and (ii) a more persistent tail. Hence, the two main features of f_τ^{F} discussed before are further strengthened by the medium heterogeneity. The increase of preferential flow, and hence a faster rising limb of the PDF and a peak higher and closer to $\tau = 0$, is the results of the availability of highly conductive layers in the system, the relative numbers of which increases with heterogeneity σ_Y^2.

The second feature observed in heterogeneous systems is also a stronger and more persistent tail as compared to the homogeneous case. Such behavior is determined by the combination of long and slow solute paths in the low conductive layers that are present in heterogeneous systems; again, the number of such low-K elements increases with σ_Y^2. Altogether, the impact of heterogeneity manifests in a significant spreading of solute particles that increases the natural tendency to spreading induced by the dipole setup.

We emphasize that the injection mode, flux proportional or resident concentration, has a strong impact on the travel time distribution, especially for highly heterogeneous formations. In Figure 4, we show the PDF of τ for the two injection conditions above for a log-normally distributed K, i.e. formula (11) and (8), respectively, for a low ($\sigma_Y^2 = 0.1$) and mild heterogeneous formation ($\sigma_Y^2 = 1$). Clearly, the effect of the injection condition is small to negligible when heterogeneity is small, while it is important when heterogeneity increases. In particular, the resident concentration injection condition generally leads to a less pronounced preferential flow and a heavier tail.

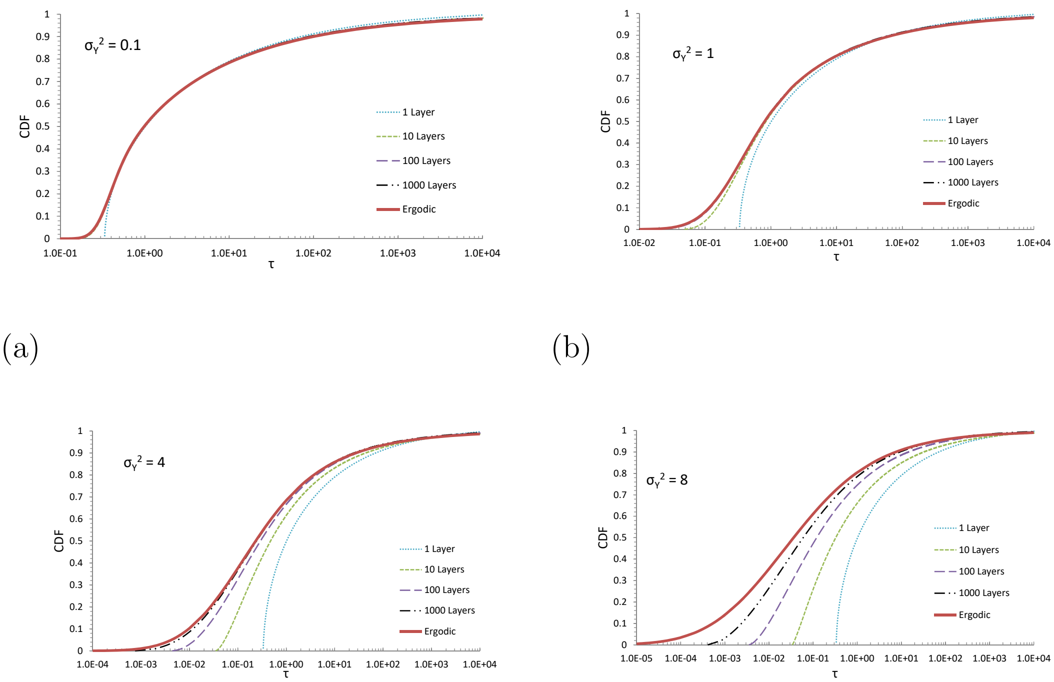
We note that similar results were observed by Pedretti and Fiori (2013) for the convergent tracer test, in particular regarding the emergence of fast, preferential flows when in presence of strongly heterogeneous formations. Such feature mostly affects the early limb of the BTC. Instead, the BTC tail is in the present case mostly affected by the particular flow configuration determined by the dipole, which is very much different from the convergent flow considered by Pedretti and Fiori (2013).

4. Impact of aquifer thickness (non-ergodicity)

As previously mentioned, the solutions derived in Section 2 are formally valid for an ergodic system, i.e. when the number of layers is large enough that the conductivity distribution f_K is adequately sampled over the aquifer depth L; such conditions require $L/I \gg 1$. Since the vertical integral scale of conductivity I is usually of the order of $0.1m$ (Rubin, 2003, Table 2.1), the latter condition may be met in applications.

Still, it is worth exploring the travel time PDF under non-ergodic conditions, i.e. for moderate or small L/I, and check the conditions for the departure of the travel time PDF from the ergodic solutions derived in Section 2. Once again we assume in the following f_τ^{F} for a log-normal K as reference for the ergodic solution, i.e. formula (11).

The travel time PDF for a finite number of layers $N = L/(2I)$ can be easily obtained by a simple Monte Carlo numerical procedure, along the following lines: (i) a vector of N random conductivities drawn from a given



(a) (b)

(c) (d)

Figure 5: The mean travel time CDF for a few values of the number of layers N of the porous formation, for $\sigma_Y^2 = 0.1, 1, 4, 8$ (panels a,b,c,d).

CDF for K (f_K log-normal in the following examples) is generated, i.e. K_i ($i = 1, .., N$); (ii) for each K_i a set of travel times is calculated from equation (2) after discretization of θ in the interval $[0; \pi]$; (iii) the whole set of travel times for all layers are sorted and a cumulative frequency distribution is obtained. The entire procedure (i)-(iii) is repeated N_{MC} times, generating N_{MC} cumulative frequency distributions that are averaged in order to obtain f_τ^{F} for a formation of given thickness $L = 2NI$. The procedure is very simple and can be easily coded. Although it can also provide the bands of uncertainty of f_τ^{F}, in the following we shall focus for brevity on f_τ^{F} only, i.e. the ensemble average.

The principal results of the above procedure are represented in Figure 5 that display f_τ for four degrees of heterogeneity σ_Y^2 (four panels) and a few values of N. The limiting cases are the case $N = 1$ corresponding to the solution for a homogeneous formations, and the ergodic solution (formula 11) which is represented as a red thick line.

Starting from low heterogeneous formations ($\sigma_Y^2 = 0.1$, panel a), most of the curves collapse to the ergodic solution when $N \approx 10$ and above. Hence, a relatively small aquifer thickness is needed in order to reach ergodic conditions in the test, and the ergodic solution can be safely applied in aquifers of low heterogeneity. A similar situation happens for mild heterogeneity ($\sigma_Y^2 = 1$, panel b), for which, however, the number of layers needed to reach ergodic conditions are larger than the previous case. Such increase is more consistent for highly heterogeneous systems ($\sigma_Y^2 = 4$, panel c), and more so for $\sigma_Y^2 = 8$ (panel d), for which a relatively high number of layers ($N \geq 1000$) needs to be sampled to get to ergodic conditions. The reasons for this behavior is simple: when the variance σ_Y^2 grows, the log-conductivity distribution becomes broad and a larger sample size is needed in order to capture it. We note again that a similar result was observed by Pedretti and Fiori (2013) for the convergent tracer test, although such sampling problems always occurs when dealing with highly heterogeneous random fields.

Thus, under non-ergodic conditions the ergodic solutions developed in this work may overestimate the BTC spreading when applied to a tracer test. In particular, if such solutions are employed for the aquifer characterization, as discussed in the next Section, the inferred σ_Y^2 may be underestimated when in presence of highly heterogeneous aquifers.

5. Conductivity characterization by the dipole test with application examples

The solution developed in this work can be used for the characterization of hydraulic conductivity K. In particular, the results of a dipole test can be interpreted through the analytical solution for f_τ developed in Section 2 to determine the log-conductivity variance σ_Y^2, which epitomizes the degree of heterogeneity in the aquifer. For instance, assuming a log-normal K, expressions (14,15) can be fitted to the experimental BTC in order to obtain σ_Y^2. We emphasize, however, that the solutions (10,12) are for a generic distribution f_K, and in principle even a numerical one could be employed. The elements of f_τ that are mostly impacted by the conductivity heterogeneity is the first segment, i.e. the rising limb and the peak, and the fitting procedure is typically influenced by that part. Instead, the effects of σ_Y^2 on the tail are less relevant because the tail is anyway influenced by the later arrivals of the particles characterized by the longest (and slowest) path lines.

In the following we show a few application examples. In all cases a log-normal f_K is assumed, and hence the solution (11) is employed for the fitting of the experimental data. The fitting of tracer tests depends on several different factors, like e.g. the choice of the objective function, the weight given to data points, the part of the BTC of interest, to mention some; thus, the fitting method typically depends on the experience and preference of the modeler. For the sole purpose of illustration we have adopted here a simple least-square fit method, with an a-posteriori visual check that the fitted curves have a reasonable behavior.

The parameters employed in the applications, as well as the fitted log-conductivity variance, are reproduced in Table 1. The results are represented in terms of either concentration C, relative concentration C/C_0 or the CDF of travel time, depending on the presentation of the results in the source papers.

In order to support the novelty of our approach, the equivalent homogeneous solution for advective-dispersive flow to a pumping well in a dipole test was calculated for all three examples. Therefore, the model by Hoopes and Harleman (1967) (their Eq. 26), similar to the one developed by Gelhar (1982), was implemented here. The model accounts for mixing along streamlines but neglects mixing between streamlines and adopts a few analytical approximations, such that the solution is limited to small values of α/a, with α the equivalent longitudinal dispersivity. We anticipate that the homogeneous model can reasonably capture the experimental data only for small heterogeneity of the porous medium, but fails for heterogeneous formations. Furthermore, the fitted dispersivities α are relatively large compared to the well distance a, beyond the range of validity of the solution, as also visible by the non-zero concentration for $t = 0$. This shows that the application of the equivalent homogeneous solution can be problematic for mild/high heterogeneity since it is formally only valid for small dispersivities α.

5.1. MADE

A dipole tracer experiment (referred to as MADE-5) was performed in 2008 at the Columbus Air Force Base in Columbus, Mississippi, commonly known as the MADE (MAcro Dispersion Experiment) site. The main aim of the test was to investigate the influence of small-scale mass-transfer and dispersion processes on well-to-well transport. Test settings and results are

Table 1: Parameters of the study cases.

Symbol	Parameter	MADE	Mobile	Barstow
Q	discharge [m^3/h]	0.34	56.76	12.5
n	porosity [-]	0.32	0.35	0.30
$2a$	distance between wells [m]	6.0	38.3	6.4
L	aquifer thickness [m]	8.1	21.6	27.5
C_0	initial concentration [mg/L]	1000	169	-
Δ	duration of pulse [h]	6	76.6	84
σ_Y^2	estimated log-conductivity variance [-]	4.1	0.24	0.5

presented in detail in line with a numerical model of the experiment by Bianchi et al. (2011). Details on aquifer characteristics and parameters of the dipole test are summarized in Table 1.

Four wells have been installed for the dipole test. The injection and extraction well are located 6 m apart and two multi-level sampling wells were installed in between, at distances of 1.5 m and 3.75 m from the injection well. The BTCs were measured at the extraction well as well as at seven different depths in the two multi-level sampling wells.

The test was performed in 3 phases: Initially clean water was injected for 48 h at a rate of 5.68 l/min. After a relative steady state flow field was established, 2078 l of bromide solution with a concentration of 1000 mg/l was introduced into the aquifer within 366 min. Clean water was injected again during the third phase until the experiment was finished after 32 days from injection.

Multiple hydrogeological, geophysical investigations as well as tracer tests have been performed since the MADE site was established with the motivation to gain new insights into transport in highly heterogeneous aquifers (for details see e.g. Zheng et al. (2011)). Thus, for this site detailed geostatistical information is available for comparison. The following values for geometric mean of hydraulic conductivity and variance of log-conductivity as the one-point statistical parameters were reported: Interpretation of the flowmeter measurements resulted in $K_\mathrm{G} = 4.3 \cdot 10^{-5}$m/s and $\sigma_Y^2 = 4.4 \pm 1$ whereas the DPIL observations deliver values of $K_\mathrm{G} = 6.7 \cdot 10^{-6}$m/s and $\sigma_Y^2 = 5.9 \pm 1.5$ (Bohling et al., 2012, 2016).

A fit of the analytical curve provided in this work with the experimental data is given in Figure 6, and the estimated log-conductivity variance is about $\sigma_Y^2 = 4$. Such value is lower than the recent DPIL-based estimate by

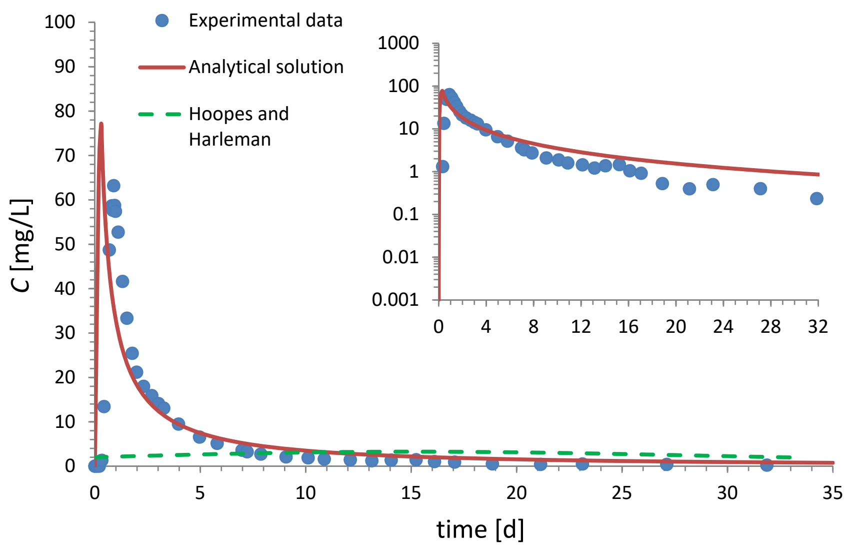


Figure 6: Experimental results of the dipole tracer test at the MADE site and its interpretation by the proposed analytical model as well as the equivalent homogeneous solution of Hoopes and Harleman (1967); experimental data from Bianchi et al. (2011).

Bohling et al. (2016), that is $\sigma_Y^2 = 5.9$ with a 95% confidence interval of [4.4; 7.4], being close to the lower bound; instead, the inferred value is closer to the one obtained by flowmeter measurements. The differences between the two estimates might be explained by either the different areas of the MADE site explored by the two methods, where the DPIL analysis covered a much larger domain than the one related to the dipole test, or, more likely, by non-ergodic effects, as discussed at the end of Section 4.

The good fit of the analytical solution to the experimental data not only for the peak (although with some temporal anticipation), but also for the tailing behavior can be nicely seen in the insert semi-log plot in Figure 6. A direct comparison to Figure 7 of Bianchi et al. (2011) shows that the analytical curve based on an ADE approach can reproduce the concentration distribution similarly good as the dual domain model fitted by Bianchi et al. (2011).

It can be seen that an equivalent homogeneous transport model (solution of Hoopes and Harleman (1967), with optimal fit of dispersivitiy $\alpha = 2.73\,\mathrm{m}$) can neither reproduce the heavy peak behavior nor the tailing which is obviously strongly impacted by the strong aquifer heterogeneity.

5.2. Mobile

Molz et al. (1986) and Huyakorn et al. (1986b) presented and analyzed the results of a two well tracer test with pulse input of bromide at a site near Mobile, Alabama. The sites formation is composed of a low-terrace deposit of Quaternary age consisting of interbedded sands and clays which have been deposited along the western edge of the Mobile River. The sandy confined aquifer section is about 20 m thick and located between 40 m and 60 depth.

The two-well tracer test was performed making use of two fully penetrating wells in a distance of 38.3 m. Equal injection and withdrawal rate average to $0.946\,\mathrm{m}^3/\mathrm{min}$. A slug of bromide as tracer was added to the injection water during the first 76.6 hours of the experiment which persisted in total 32.5 days. Since the withdrawn water was re-injected the tracer recirculated. Details on aquifer characteristics and parameters of the dipole test are summarized in Table 1.

Investigation of hydraulic conductivity distribution have been performed by Molz et al. (1990) based on impeller meter measurements and small scale pumping tests. Results lead to the conclusion that the study aquifer is fairly homogeneous. They reported a mean value of hydraulic conductivity of

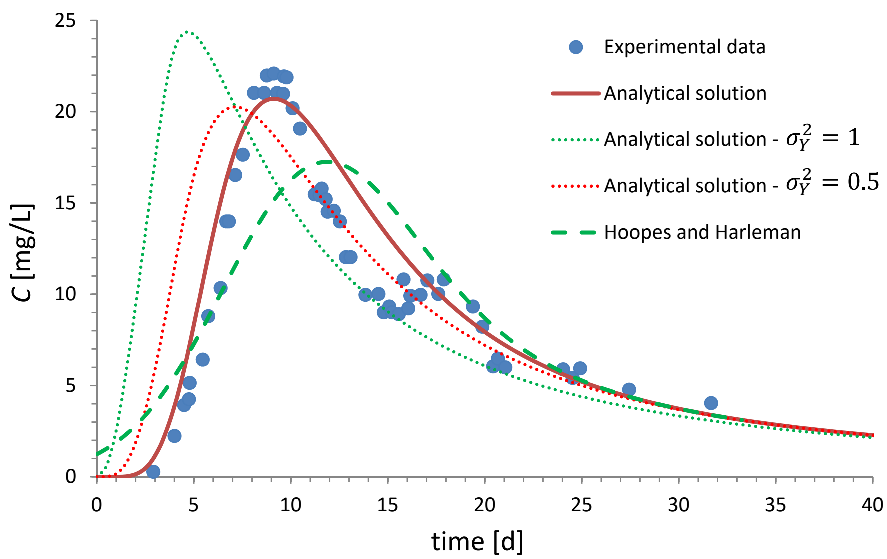


Figure 7: Experimental results of the dipole tracer test at the Mobile site and its interpretation by the proposed analytical model as well as the equivalent homogeneous solution of Hoopes and Harleman (1967); experimental data from Molz et al. (1986).

54.9 m/day with a standard deviation of only 2.4 m/day. Further geostatistical analysis for the hydraulic conductivity at that site are not known to the authors.

Figure 7 shows a fit of the analytical curve provided in this work with the experimental data. The very small value of $\sigma_Y^2 = 0.24$ inferred for the estimated log-conductivity variance supports the findings that the Mobile aquifer is weakly heterogeneous. In addition, Figure 7 gives a simple sensitivity analysis with the analytical curve for two higher variances of $\sigma_Y^2 = 0.5$ and $\sigma_Y^2 = 1$ showing that the solution is typically quite sensitive towards the log-conductivity variance but mostly for the early-time behavior and the peak. The tail is mostly dominated by the arrival times of the different flow paths giving a diminishing impact of the heterogeneity on late time BTC behavior. Furthermore, the best fit of the equivalent homogeneous solution of Hoopes and Harleman (1967) (dispersivity $\alpha = 4.06$ m) to the data indicates that although the aquifer is mildly heterogeneous a purely homogeneous solution is not able to adequately reproduce the concentration distribution.

5.3. Barstow

Robson (1974) reported the results of a small scale dipole tracer test in the Barstow's aquifer which consist of very permeable younger alluvium of Holocene age deposited by the Mojave River and alluvial fans. The injection and withdrawal wells are located in a distance of 6.4 m, both were perforated through most of the 27.45 m aquifer thickness. A recharge/discharged rate of 55 gallons per minute was reported. The tracer solution of sodium chloride was constantly injected during the 84-hour span of the test. Since the withdrawn water was re-injected the tracer recirculated.

Concentration was measured at temporal intervals of more than 4 h, giving a sparse database in particular for the early time of the BTC. The test was originally analyzed making use of the method of Grove and Beetem (1971) to estimate values of macrodispersivity. As pointed out by Robson (1974), the late time behavior of the concentration curve should be taken with caution due to the tracer recirculation that was carried out during the test. Details on aquifer characteristics and parameters of the dipole test are summarized in Table 1. Geostatistical analysis for the hydraulic conductivity at that site are not known to the authors.

A fit of the analytical curve with the experimental data is given in Figure 8 with an estimated log-conductivity variance of $\sigma_Y^2 = 0.5$. along with the best fit for the equivalent homogeneous solution of Hoopes and Harleman

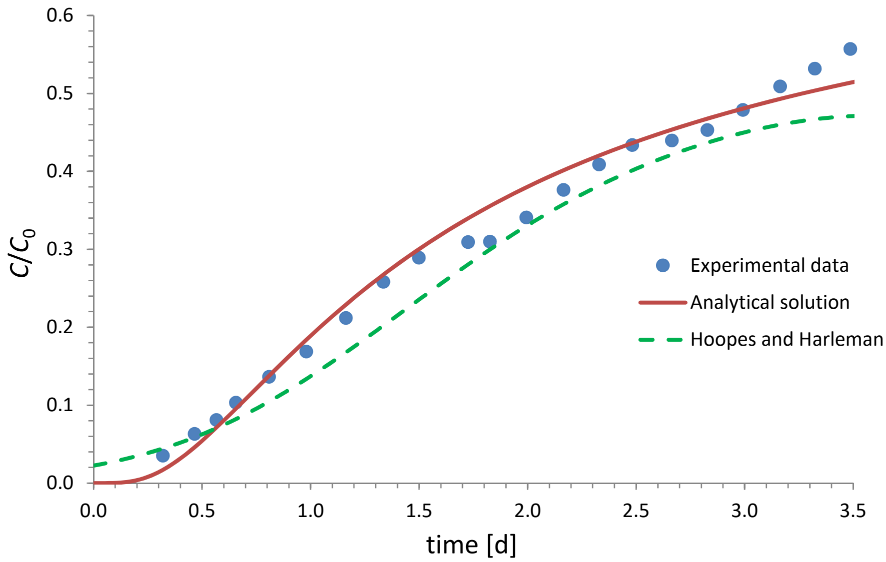


Figure 8: Experimental results of the dipole tracer test at the Barstow site and its interpretation by the proposed analytical model as well as the equivalent homogeneous solution of Hoopes and Harleman (1967); experimental data from Robson (1974).

(1967) (fitted dispersivity of $\alpha = 0.98$). Since no geostatistical analysis of the hydraulic conductivity in the aquifer is known, there is no reference for comparison. However, the fit of the analytical curves with the measured data shows nicely how the shape of the BTC can be related to the aquifer heterogeneity by the log-conductivity variance rather than dispersivity.

6. Summary and Conclusions

In this paper, we derived a new analytical solution for interpreting dipole tests in heterogeneous media. Furthermore, it was presented how the solution can be used for interpretation of dipole field test in view of geostatistical aquifer characterization. The work was motivated by the lack of methods for dipole tests taking the strong spatial heterogeneity of hydraulic conductivity into account, although tracer tests are central tools for inferring hydrogeological, structural and transport parameters of the subsurface.

In dipole tracer tests, also two-well test or doublet tests, a tracer is introduced at a recharge well and the breakthrough curve (BTC) is measured at a pumping well. The analytical solution for the BTC at the pumping well was developed by considering a stratified heterogeneous hydraulic conductivity structure. The analysis of the BTC is this kind of media was carried out by considering the travel time of a generic solute particle, from the injection to the pumping well. The derivation of the analytical solutions was performed for two different injection conditions: (i) resident concentration, assuming that the mass of solute entering each layer from the injection well is constant for all layers; and (ii) flux proportional injection mode where the entering mass is assumed proportional to layer conductivity. The solution was derived for an instantaneous pulse of solute and extended to a formula for continuous solute injections.

The illustration of results focused on different aspects of the solution: (i) the impact of heterogeneity; (ii) the impact of the injection condition; and (iii) the impact of non-ergodic conditions at the injection well. The analysis lead to the following conclusions:

- The impact of heterogeneity manifests in a significant spreading of solute particles that increases the natural tendency to spreading induced by the dipole setup. For a log-normal conductivity distribution an increasing degree of heterogeneity leads to a stronger preferential flow and a more persistent tail.

- The injection mode has a strong impact on the travel time distribution, especially for highly heterogeneous formations. The resident concentration injection condition generally leads to a less pronounced preferential flow and a heavier tail.

- With increasing heterogeneity the number of layers needed to reach ergodic conditions become larger. Under non-ergodic conditions the solutions developed in this work may overestimate the BTC spreading. In particular, if such solutions are employed for the aquifer characterization, the inferred log-conductivity variance σ_Y^2 may be underestimated when in presence of highly heterogeneous aquifers.

In final step, the derived method was used for conductivity characterization at three dipole field tests as illustrative examples. Thereby, the log-conductivity variance war inferred from the shape of the observed BTCs. The analysis of dipole tests at two sites indicated a mild heterogeneity, being in line with other observations at these sites. The dipole test performed at the heterogeneous MADE site was analyzed resulting in a high value of variance being in the same range as an geostatistical interpretation of flowmeter measurements but smaller then the variance resulting from a geostatistical interpretation of DPIL measurements. This results can be associated to the assumption of ergodicity in the analytical solution with might not be present at the heterogeneous field site giving an underestimation of variance by the analytical solution.

The presented method is the first fully analytical tool for dipole tracer test analysis taking heterogeneity of hydraulic conductivity into account. Assumptions in the derivation of the analytical solutions have been taken to be in line with the conditions encountered in the field. It could be shown that the method is easily applicable to measured BTCs for inferring the degree heterogeneity, namely the log-conductivity variance. The method is a promising geostatistical analyzing tool as addition to other geostatistical investigations methods, often being time- and cost-intensive.

Acknowledgements

We are grateful to Marco Bianchi for providing the data of the MADE-5 experiments. Furthermore, we would like to thank Fred Molz for providing background information of the Mobile test site and Sebastian Mller for support with computations.

References

Bianchi, M., Zheng, C., Tick, G. R., Gorelick, S. M., 2011. Investigation of Small-Scale Preferential Flow with a Forced-Gradient Tracer Test. Ground Water 49 (4), 503–514.

Bohling, G. C., Liu, G., Dietrich, P., Butler, J. J., 2016. Reassessing the MADE direct-push hydraulic conductivity data using a revised calibration procedure. Water Resour. Res. 52 (11), 8970–8985.

Bohling, G. C., Liu, G., Knobbe, S. J., Reboulet, E. C., Hyndman, D. W., Dietrich, P., Butler, J. J., 2012. Geostatistical analysis of centimeter-scale hydraulic conductivity variations at the MADE site. Water Resour. Res. 48, W02525.

Claasen, H., Cordes, E., 1975. Two-Well Recirculating Tracer Test in Fractured Carbonate Rock, Nevada. Hydrological Sciences Bulletin 20 (3), 367–382.

Dagan, G., 1990. Transport in heterogeneous porous formations: Spatial moments, ergodicity, and effective dispersion. Water Resour. Res. 26 (6), 1281–1290.

Gelhar, L., 1993. Stochastic Subsurface Hydrology. Prentice Hall, Englewood Cliffs, N. Y.

Gelhar, L. W., 1982. Analysis of two-well tracer tests with a pulse input. Tech. Rep. RHO-BW-CR-131 P, Rockwell Int., Richland, Wash.

Gelhar, L. W., Collins, M. A., 1971. General analysis of longitudinal dispersion in nonuniform flow. Water Resour. Res. 7 (6), 1511–1521.

Grove, D. B., Beetem, W. A., 1971. Porosity and dispersion constant calculations for a fractured carbonate aquifer using the two well tracer method. Water Resour. Res. 7 (1), 128–134.

Hoopes, J. A., Harleman, D. R. F., 1967. Waste water recharge and dispersion in porous media. Journal of the Hydraulics Division 93 (5), 51–71.

Huyakorn, P. S., Andersen, P. F., Güven, O., Molz, F. J., 1986a. A curvilinear finite element model for simulating two-well tracer tests and transport in stratified aquifers. Water Resour. Res. 22 (5), 663–678.

Huyakorn, P. S., Andersen, P. F., Molz, F. J., Güven, O., Melville, J. G., 1986b. Simulations of two-well tracer tests in stratified aquifers at the Chalk River and the Mobile Sites. Water Resour. Res. 22 (7), 1016–1030.

Hyndman, D. W., Gorelick, S. M., 1996. Estimating lithologic and transport properties in three dimensions using seismic and tracer data: The Kesterson aquifer. Water Resour. Res. 32 (9), 2659–2670.

Jankovic, I., Fiori, A., 2010. Analysis of the impact of injection mode in transport through strongly heterogeneous aquifers. Adv. Water Resour. 33 (10), 1199–1205.

Koplik, J., 2001. The tracer transit-time tail in multipole reservoir flows. Transp. Porous Media 42 (1-2), 199–209.

Matheron, G., De Marsily, G., 1980. Is transport in porous media always diffusive? a counterexample. Water Resour. Res. 16 (5), 901–917.

Mercado, A., 1967. The spreading pattern of injected water in a permeability stratified aquifer. IAHS Press, pp. 23–36.

Molz, F. J., Güven, O., Melville, J. G., Crocker, R. D., Matteson, K. T., 1986. Performance, analysis, and simulation of a two-well tracer test at the Mobile Site. Water Resour. Res. 22 (7), 1031–1037.

Molz, F. J., Gven, O., Melville, J. G., Cardone, C., 1990. Hydraulic Conductivity Measurements at Different Scales and Contaminant Transport Modelling. In: Dynamics of Fluids in Hierarchical Porous Media. Academic Press, London.

Papoulis, A., 1991. Probability, Random Variables, and Stochastic Processes. McGraw-Hill.

Pedretti, D., Fiori, A., 2013. Travel time distributions under convergent radial flow in heterogeneous formations: Insight from the analytical solution of a stratified model. Advances in Water Resources 60, 100–109.

Pickens, J. F., Grisak, G. E., 1981. Scale-dependent dispersion in a stratified granular aquifer. Water Resour. Res. 17 (4), 1191–1211.

Robson, S. G., 1974. Feasibility of digital water-quality modeling illustrated by application at Barstow, California. Tech. Rep. WRI - 73-46, U. S. Geol. Surv.

Rubin, Y., 2003. Applied Stochastic Hydrogeology. Oxford Univ. Press, New York.

Thorbjarnarson, K. W., Mackay, D. M., 1997. A field test of tracer transport and organic contaminant elution in a stratified aquifer at the Rocky Mountain Arsenal (Denver, Colorado, U.S.A.). J. Contam. Hydrol. 24 (3–4), 287–312.

Webster, D. S., Proctor, J. F., Marine, I. W., 1970. Two-well tracer test in fractured crystalline rock. Tech. Rep. WSP - 1544-I, United States Geological Survey.

Welty, C., Gelhar, L. W., 1989. Evaluation of longitudinal dispersivity from tracer test data. Tech. Rep. 320, M.I.T. Press, Cambridge, Mass.

Wilson, L. G., 1971. Investigations on the subsurface disposal of waste effluents at inland sites. Res. Dev. Prog. Rep. 650, U.S. Dep. of the Inter., Washington, D.C.

Zheng, C., Bianchi, M., Gorelick, S. M., 2011. Lessons learned from 25 years of research at the MADE site. Ground Water 49 (5), 649–662.