

Hydroinformatik II: Grundlagen der Kontinuumsmechanik

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Dresden, 17. April 2015

Vorlesungsplan Hydroinformatik II WS2014/15

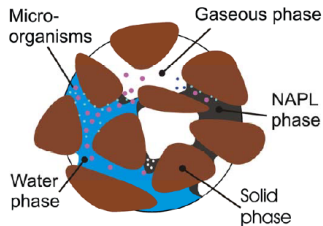
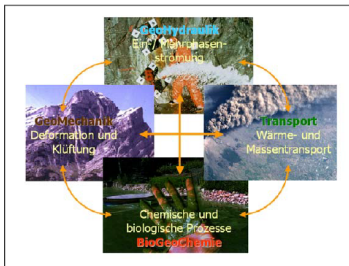
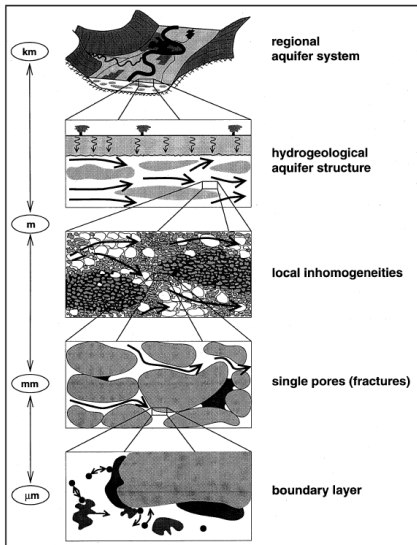
#	Datum	Thema
1	17.10.2014	Einführung, Grundlagen: Kontinuumsmechanik
2	24.10.2014	Grundlagen: Kontinuumsmechanik/Hydromechanik
-	31.10.2014	Reformationstag
3	07.11.2014	Grundlagen: Partielle Differentialgleichungen / T _E X
3a	07.11.2014	Einführung in Qt (Installation)
4	14.11.2014	Grundlagen: Numerische Methoden
6	28.11.2014	Numerik: Finite Differenzen Methode
7	05.12.2014	Grundlagen: Diffusionsprozesse
8	12.12.2014	Numerik: Übung explizite FDM
9	12.12.2014	Numerik: Implizite FDM
10a	19.12.2014	Numerik: Übung implizite FDM - die Erste
10b	09.01.2015	Numerik: Übung implizite FDM - die Zweite
11	16.01.2015	Gerinnehydraulik: Theorie - Grundlagen
12	23.01.2015	Gerinnehydraulik: Saint-Venant Gleichung (\implies HSA)
13a	30.01.2015	Gerinnehydraulik: Programmierung, Übung
13b	30.01.2015	Gerinnehydraulik: Programmierung, Übung
14	06.02.2015	Kurs-Zusammenfassung und Abschluss

Fahrplan für heute ...

- ▶ Motivation
- ▶ Lagrange Konzept
- ▶ Euler Konzept
- ▶ Reynolds Transport Theorem
- ▶ Fluxes
- ▶ Bilanzgleichungen
- ▶ Erhaltungsgrößen

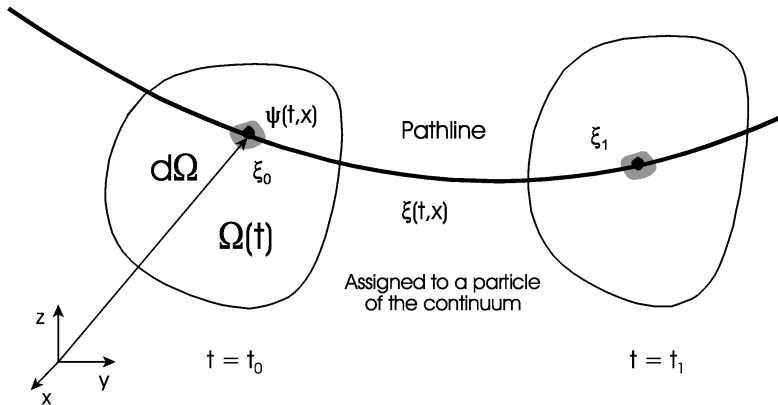
⇒ viel Theorie - vor allem die mathematische Schreibweise verstehen "zu lesen"

Skalen

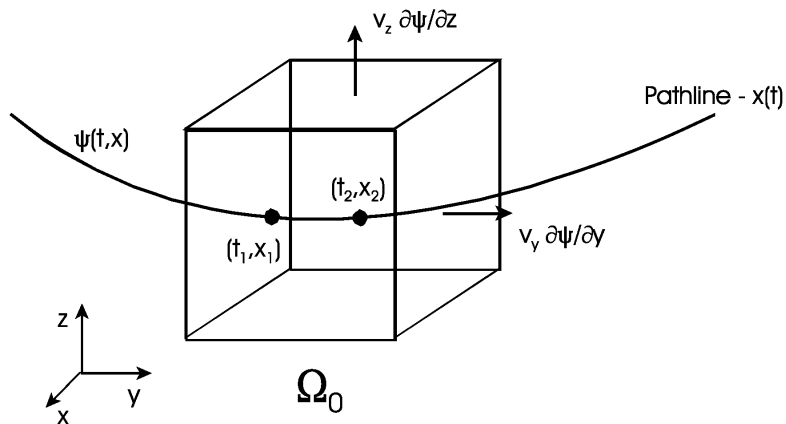


Quellen: Kobus et al. (1995), Kolditz (2002)

Lagrange Konzept (1.1.1)

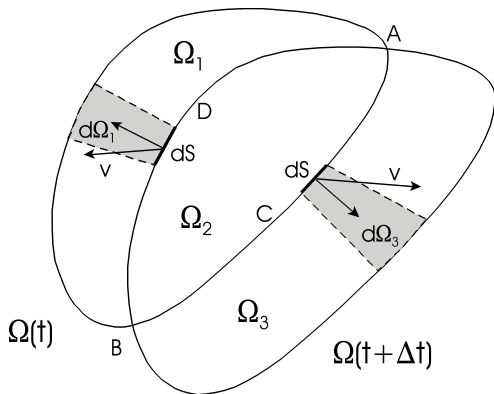


Euler Konzept (1.1.1)

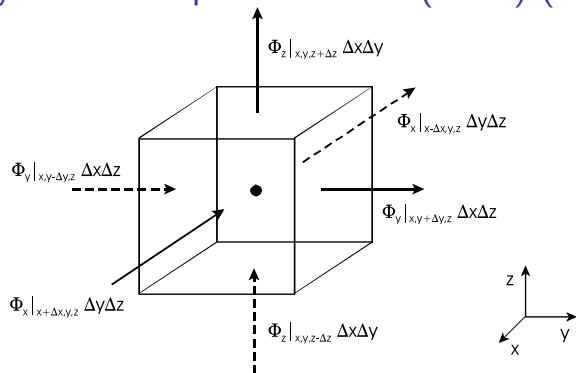


Reynolds Transport Theorem (Lagrange) (1.1.3)

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \oint_{\partial \Omega} \psi(t) \mathbf{v} \cdot d\mathbf{S} = \int_{\Omega} q^{\psi} d\Omega \quad (1)$$



Reynolds Transport Theorem (Euler) (1.1.3)



$$\oint_{\partial\Omega} \mathbf{\Phi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \mathbf{\Phi} \, d\Omega \quad (2)$$

$$\nabla \cdot \mathbf{\Phi} = \lim_{\Omega \rightarrow 0} \frac{1}{\Omega} \oint_{\partial\Omega} \mathbf{\Phi} \cdot d\mathbf{S} \quad (3)$$

Reynolds Transport Theorem (Euler) (1.1.3)

$$\underbrace{\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega}_1 = - \underbrace{\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S}}_2 + \underbrace{\int_{\Omega} q^{\psi} d\Omega}_3 \quad (4)$$

with:

1. Rate of change of total amount of quantity ψ in the control volume,
2. Net rate of increase / decrease of ψ due to fluxes,
3. Rate of increase / decrease of ψ due to sources.

Reynolds Transport Theorem (Euler) (1.1.3)

$$\underbrace{\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega}_1 = - \underbrace{\oint_{\partial\Omega} \mathbf{\Phi}^{\psi} \cdot d\mathbf{S}}_2 + \underbrace{\int_{\Omega} q^{\psi} d\Omega}_3 \quad (5)$$

using

$$\oint_{\partial\Omega} \mathbf{\Phi}^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \mathbf{\Phi}^{\psi} d\Omega \quad (6)$$

$$\int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega = - \int_{\Omega} \nabla \cdot \mathbf{\Phi}^{\psi} d\Omega + \int_{\Omega} q^{\psi} d\Omega \quad (7)$$

Reynolds Transport Theorem

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \left(\frac{\partial \psi}{\partial t} + \nabla \cdot \Phi^{\psi} \right) d\Omega = \int_{\Omega} q^{\psi} d\Omega$$

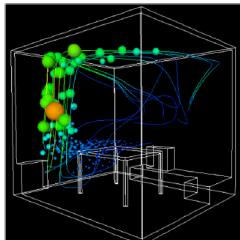
$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^E \nabla \psi$$



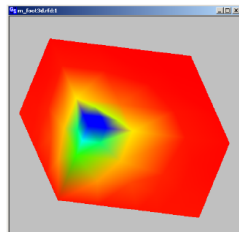
Lagrange



Euler



<http://www.cscs.ch/~mvalle/Libro/>



Reynolds Transport Theorem

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega + \oint_{\partial\Omega} \nabla \cdot \Phi \partial\Omega = \int_{\Omega} q^{\psi} d\Omega \quad (8)$$

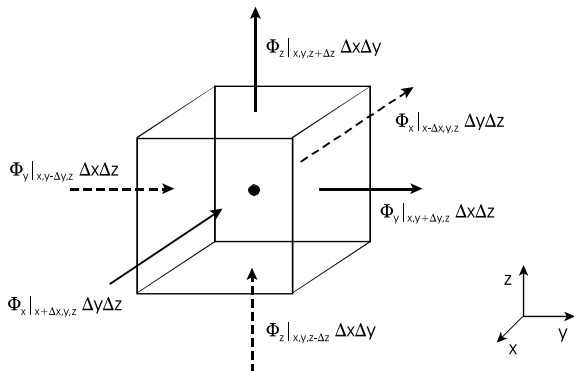
$$\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \nabla \cdot \oint_{\partial\Omega} \Phi \partial\Omega = \int_{\Omega} q^{\psi} d\Omega \quad (9)$$

$$\forall \Omega : \frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot \Phi = q^{\psi} \quad (10)$$

Fluxes (1.1.6)

The total flux Φ^ψ of a quantity ψ is defined as

$$\Phi^\psi = \mathbf{v}^E \psi \quad (11)$$



Fluxes (1.1.6)

$$\Phi^\psi = \mathbf{v}^E \psi = \underbrace{\mathbf{v} \psi}_{\Phi_A^\psi} + \underbrace{(\mathbf{v}^E - \mathbf{v}) \psi}_{\Phi_D^\psi} \quad (12)$$

and, therefore, decomposed into two parts: an advective flux Φ_A^ψ and a diffusive flux Φ_D^ψ relative to the mass-weighted velocity:

- ▶ advective flux of quantity ψ

$$\Phi_A^\psi = \mathbf{v} \psi \quad (13)$$

- ▶ diffusive flux of quantity ψ (Fick's law)

$$\Phi_D^\psi = -\alpha \nabla \psi \quad (14)$$

General Balance Equation (1.1.7)

- ▶ Integral form

$$\int_{\Omega} \frac{d\psi}{dt} = \int_{\Omega} \frac{\partial\psi}{\partial t} + \int_{\Omega} \nabla \cdot (\mathbf{v}\psi) - \int_{\Omega} \nabla \cdot (\mathbf{D}^{\psi} \nabla\psi) = \int_{\Omega} \mathbf{Q}^{\psi} \quad (15)$$

- ▶ Differential form

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^{\psi} \nabla\psi) = \mathbf{Q}^{\psi} \quad (16)$$

Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume Ω is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \quad (17)$$

where Ψ is an extensive conservation quantity (i.e. mass, momentum, energy) and ψ is the corresponding intensive conservation quantity such as mass density ρ , momentum density $\rho\mathbf{v}$ or energy density e .

Extensive quantity	Symbol	Intensive quantity	Symbol
Mass	M	Mass density	ρ
Linear momentum	\mathbf{m}	Linear momentum density	$\rho\mathbf{v}$
Energy	E	Energy density	$e = \rho i + \frac{1}{2}\rho v^2$