# Hydroinformatik II - SoSe 2024

# HyBHW-S2-01-V03: Grundlagen der Kontinuumsmechanik

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# Zeitplan: Hydroinformatik II - SoSe 2024

| Datum | HI | II | Thema | Typ |
|---|---|---|---|---|
| 14.06.2024 | 14 | 2-01 | Einführung in die Lehrveranstaltung - Teil 2 | L |
| 14.06.2024 | 15 | 2-02 | Werkzeuge \| Tools | L |
| 14.06.2024 | 16 | 2-03 | Grundlagen: Kontinuumsmechanik | L |
| 21.06.2024 | 17 | 2-04 | Grundlagen: Hydromechanik | L |
| 21.06.2024 | 18 | 2-05 | Grundlagen: Partielle Partialgleichungen | L |
| 21.06.2024 | 19 | 2-06 | Übung: Analytische Lösungen | E |
| 28.06.2024* | 20 | 2-07 | Grundlagen: Näherungsverfahren | L |
| 28.06.2024* | 21 | 2-08 | Übung: Jupyter Diffusionsprozess | E |
| 02.07.2024* | 22 | 2-09 | Numerik: Finite-Differenzen-Methode (explizit) | L |
| 02.07.2024* | 23 | 2-10 | Numerik: Finite-Differenzen-Methode (implizit) | L |
| 12.07.2024 | 24 | 2-11 | Übung: Finite-Differenzen-Methoden | E |
| 12.07.2024 | 25 | 2-12 | Grundlagen: Gerinnehydraulik | L |
| 12.07.2024 | 26 | 2-13 | Übung: Gerinnehydraulik | E |
| 19.07.2024 | 27 | 2-14 | Ausblick: Grundwassermodellierung | E |
| 19.07.2024 | 28 | 2-15 | Klausur/Beleg: Besprechung zur Vorbereitung | L |

*online Vorlesung

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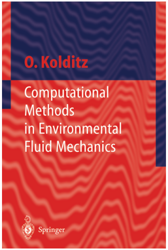
# Fahrplan für heute ...

- Motivation
- Lagrange Konzept
- Euler Konzept
- Reynolds Transport Theorem
- Fluxes
- Bilanzgleichungen
- Erhaltungsgrößen




>> Skript

$\Longrightarrow$ viel Theorie - vor allem die mathematische Schreibweise verstehen "zu lesen"

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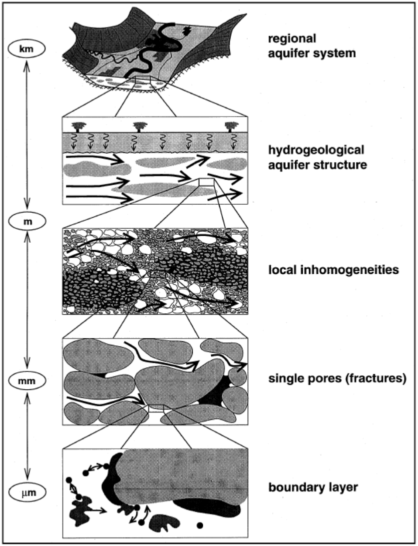
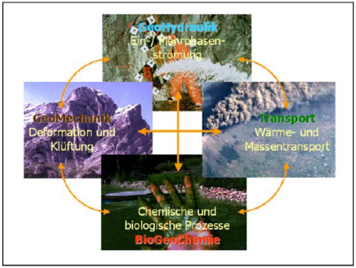
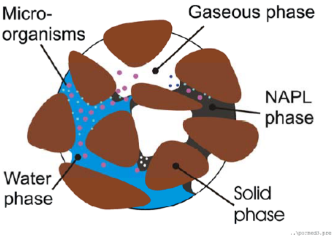
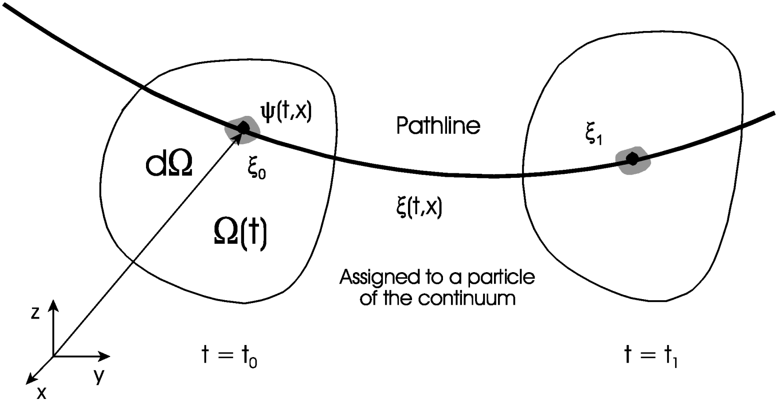


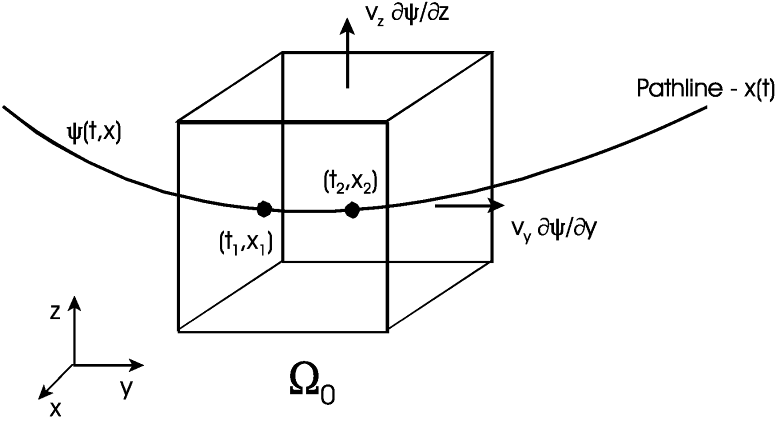




Quellen: Kobus et al. (1995), Kolditz (2002)

# Lagrange Konzept (1.1.1)



# Euler Konzept (1.1.1)



- Volumenintegral (Zeichnung)

$$\int_{\Omega} d\Omega \tag{1}$$

$$\int_a^b f(x)dx = \lim_{(x_{k+1}-x_k)\to 0} \sum_{k=1}^{\infty} \left(f(x_{k+1} - f(x_k)\right)\left(x_{k+1} - x_k\right) \tag{2}$$

- Oberflächen-(Ring)-Integral (Zeichnung)

$$\oint_{\partial\Omega} \mathbf{n} \cdot d\mathbf{S} \tag{3}$$

- Materielle Ableitung

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v} \cdot \nabla\psi \tag{4}$$

- Gradient (Vektor)

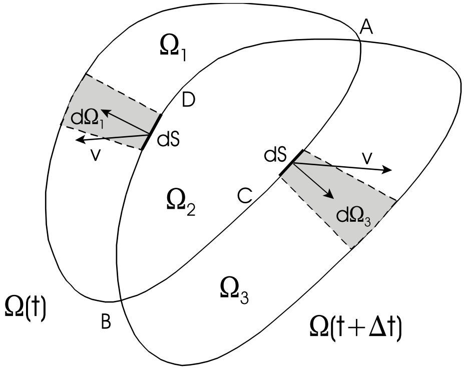
$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \tag{5}$$

- Divergenz (Skalar)

$$\nabla \cdot \mathbf{v} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z \tag{6}$$

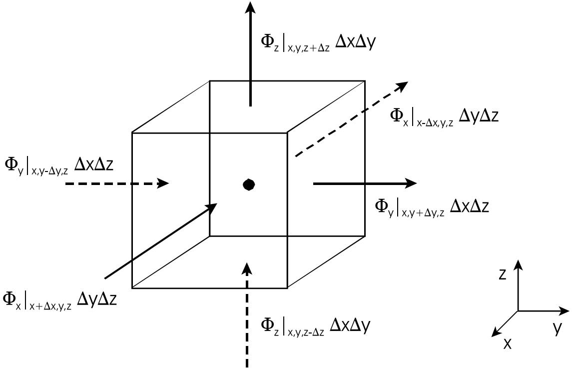
Aufgabe: Was ist $\mathbf{v} \cdot \nabla\psi$?

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \oint_{\partial\Omega}\psi(t)\mathbf{v}\cdot d\mathbf{S} = \int_{\Omega} q^{\psi} d\Omega \qquad (7)$$




Beweisführung:
Siehe Skript
Abschn. 1.1.3

# Reynolds Transport Theorem (Euler) (1.1.3)




$$\Phi_x^{\psi} = \partial\psi/\partial x \quad , \quad \Phi^{\psi} = \nabla\psi$$

Frage: Ist $\Phi^{\psi}$ eine skalare oder vektorielle Größe ?

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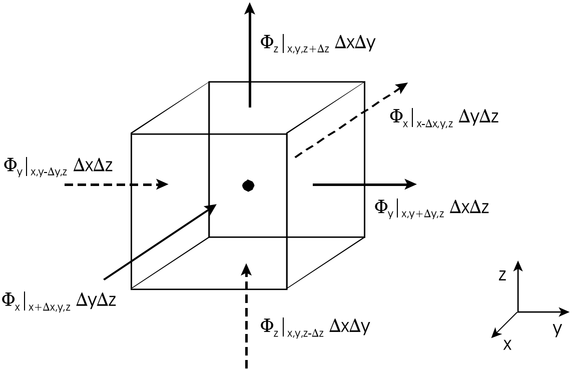
# Reynolds Transport Theorem (Euler) (1.1.3)




$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \qquad (8)$$

# Reynolds Transport Theorem (Euler) (1.1.3)

## Divergence

$$\oint_{\partial\Omega} \mathbf{\Phi}^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \mathbf{\Phi}^{\psi} d\Omega \tag{9}$$

$$\lim_{\Omega \to 0} \frac{1}{\Omega} \oint_{\partial\Omega} \mathbf{\Phi} \cdot d\mathbf{S} = \nabla \cdot \mathbf{\Phi} \tag{10}$$

**Point-Flux**

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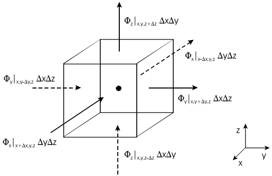
# Reynolds Transport Theorem (Euler) (1.1.3)

## General Balance Equation




$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int_{\Omega}\psi d\Omega}_{1} + \underbrace{\oint_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S}}_{2} = \underbrace{\int_{\Omega} q^{\psi} d\Omega}_{3} \tag{11}$$

with:

1. Rate of change of total amount of quantity $\psi$ in the control volume,
2. Net rate of increase / decrease of $\psi$ due to fluxes,
3. Rate of increase / decrease of $\psi$ due to sources.

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# Reynolds Transport Theorem (Euler) (1.1.3)

$$\frac{d}{dt}\int\limits_{\Omega} \psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int\limits_{\Omega} \psi d\Omega}_{1} + \underbrace{\oint\limits_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S}}_{2} = \underbrace{\int\limits_{\Omega} q^{\psi} d\Omega}_{3} \qquad (12)$$

using

$$\oint\limits_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int\limits_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \qquad (13)$$

$$\frac{d}{dt}\int\limits_{\Omega} \psi d\Omega = \int\limits_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \int\limits_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega = \int\limits_{\Omega} q^{\psi} d\Omega \qquad (14)$$

# Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \int\limits_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \int\limits_{\Omega}\nabla\cdot\Phi^{\psi}d\Omega = \int\limits_{\Omega}q^{\psi}d\Omega \tag{15}$$

$$\forall\Omega : \frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla\cdot\Phi^{\psi} = q^{\psi} \tag{16}$$

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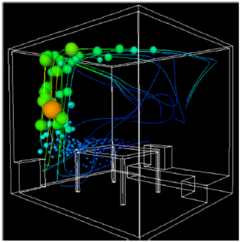
# Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega} \psi \, d\Omega = \int\limits_{\Omega} \left( \frac{\partial \psi}{\partial t} + \nabla \cdot \mathbf{\Phi}^{\psi} \right) d\Omega = \int\limits_{\Omega} q^{\psi} \, d\Omega$$

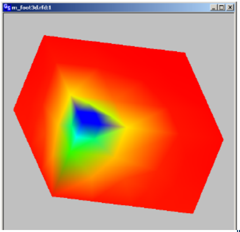
$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^E \nabla \psi$$

Lagrange

Euler



http://www.cscs.ch/~mvalle/Libro/



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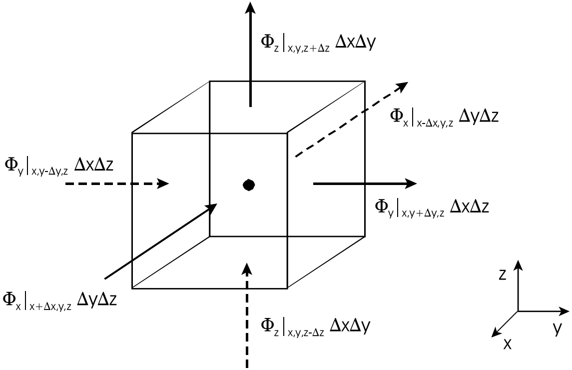
# Fluxes (1.1.6)

The total flux $\Phi^{\psi}$ of a quantity $\psi$ is defined as

$$\Phi^{\psi} = \mathbf{v}^{E}\psi \tag{17}$$

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# Fluxes (1.1.6)

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot \Phi^{\psi} = q^{\psi} \tag{18}$$

$$\Phi^{\psi} = \mathbf{v}^{E}\psi \tag{19}$$

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}^{E}\psi) = q^{\psi} \tag{20}$$

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# Fluxes (1.1.6)

$$\Phi^{\psi} = \mathbf{v}^{E}\psi = \underbrace{\mathbf{v}\psi}_{\Phi_A^{\psi}} + \underbrace{(\mathbf{v}^{E} - \mathbf{v})\psi}_{\Phi_D^{\psi}} \tag{21}$$

and, therefore, decomposed into two parts: an advective flux $\Phi_A^{\psi}$ and a diffusive flux $\Phi_D^{\psi}$ relative to the mass-weighted velocity:

- advective flux of quantity $\psi$

$$\Phi_A^{\psi} = \mathbf{v}\psi \tag{22}$$

- diffusive flux of quantity $\psi$ (Fick's law)

$$\Phi_D^{\psi} = -\mathbf{D}^{\psi}\nabla\psi \tag{23}$$

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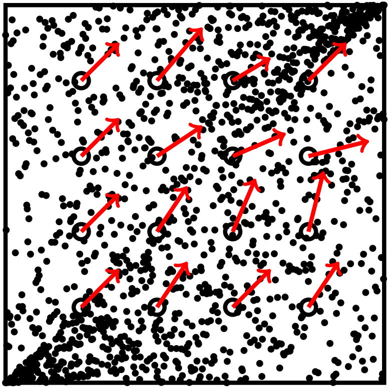
# Fluxes (1.1.6)

## Velocities



$$\mathbf{v}^E = \cup \mathbf{v}_i$$

$$\mathbf{v}^E = \mathbf{v} + \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \mathbf{v}^E - \mathbf{v}$$

- Integral form

$$\int_\Omega \frac{d\psi}{dt} = \int_\Omega \frac{\partial \psi}{\partial t} + \int_\Omega \nabla \cdot (\mathbf{v}\psi) - \int_\Omega \nabla \cdot (\mathbf{D}^\psi \nabla \psi) = \int_\Omega Q^\psi \tag{24}$$

- Differential form

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^\psi \nabla \psi) = Q^\psi \tag{25}$$

The amount of a quantity in a defined volume $\Omega$ is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{26}$$

where $\Psi$ is an extensive conservation quantity (i.e. mass, momentum, energy) and $\psi$ is the corresponding intensive conservation quantity such as mass density $\rho$, momentum density $\rho\mathbf{v}$ or energy density $e$.

| Extensive quantity | Symbol | Intensive quantity | Symbol |
|---|---|---|---|
| Mass | $M$ | Mass density | $\rho$ |
| Linear momentum | $\mathbf{m}$ | Linear momentum density | $\rho\mathbf{v}$ |
| Energy | $E$ | Energy density | $e = \rho i + \frac{1}{2}\rho v^2$ |

# Übung
# Git: Übungen klonen

- EX01: Compiler installieren
- EX02: Python installieren (Pfadvariable!)
- EX03: Git installieren
- EX04: Git: Übungen klonen

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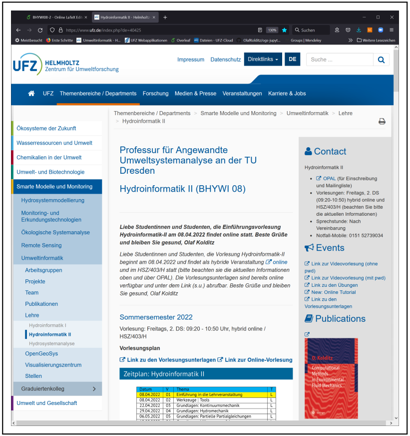
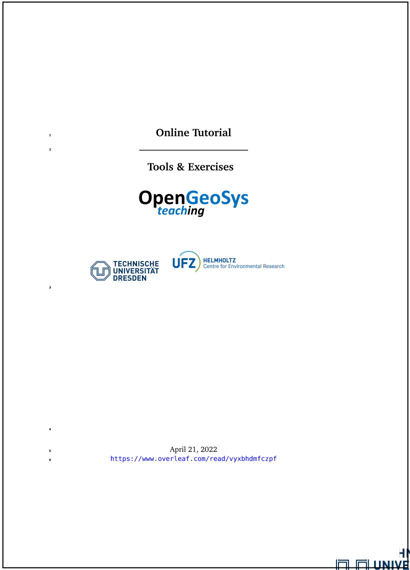
# OGS-Teaching Tutorial

## Git: Übungen klonen




- https://www.overleaf.com/read/vyxbhdmfczpf



Online Tutorial

Tools & Exercises

OpenGeoSys teaching

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UFZ HELMHOLTZ Centre for Environmental Research

April 21, 2022

https://www.overleaf.com/read/vyxbhdmfczpf

## 1.1.3 Cloning sources from a `git` repository

Eingabeaufforderung

```
Microsoft Windows [Version 10.0.18363.1443]
(c) 2019 Microsoft Corporation. Alle Rechte vorbehalten.

C:\Users\okolditz>git clone https://github.com/OlafKolditz/Hydroinformatik-II.git
```

## 1.1.4 Updating sources from a `git` repository

Eingabeaufforderung

```
C:\Users\okolditz>cd C:\User\15_REP\Hydroinformatik-II

C:\User\15_REP\Hydroinformatik-II>git fetch --all
Fetching origin

C:\User\15_REP\Hydroinformatik-II>git pull
Already up to date.

C:\User\15_REP\Hydroinformatik-II>
```

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