

Hydroinformatik II

”Prozesssimulation und Systemanalyse”

Einführung in die Lehrveranstaltung

Olaf Kolditz

*Helmholtz Centre for Environmental Research – UFZ

¹Technische Universität Dresden – TUDD

²Centre for Advanced Water Research – CAWR

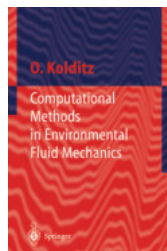
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23.04.2021 - WebLecture

Zeitplan: Hydroinformatik II

Datum	V	Thema	T
16.04.2021	01	Einführung in die Lehrveranstaltung Tools	L
23.04.2021	02	Grundlagen: Kontinuumsmechanik	L
30.04.2021	03	Grundlagen: Hydromechanik	L
07.05.2021	04	Grundlagen: Partielle Partialgleichungen	L
14.05.2021	05a	Tools: Compiler, Python, Jupyter	E
14.05.2021	05b	Übung: Elliptische PDG	E
21.05.2021	06	Übungen: Übersicht und Werkzeuge	E
28.05.2021	–	Pfingsten	
04.06.2021	07	Grundlagen: Näherungsverfahren	L
11.06.2021	08	Numerik: Finite-Differenzen-Methode (explizit)	L
18.06.2021	09	Numerik: Finite-Differenzen-Methode (implizit)	L
25.06.2021	10	Reserve	L/E
02.07.2021	11	Übung: Diffusionsprozesse	E
09.07.2021	12	Übung: Gerinnehydraulik	E
16.07.2021	13	Übung: Grundwassermodellierung	E
23.07.2021	14	Beleg: Besprechung zur Vorbereitung	L

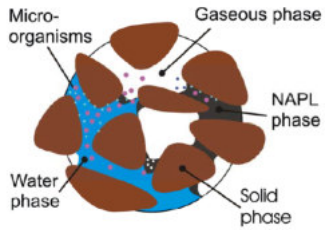
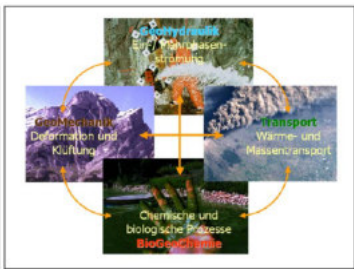
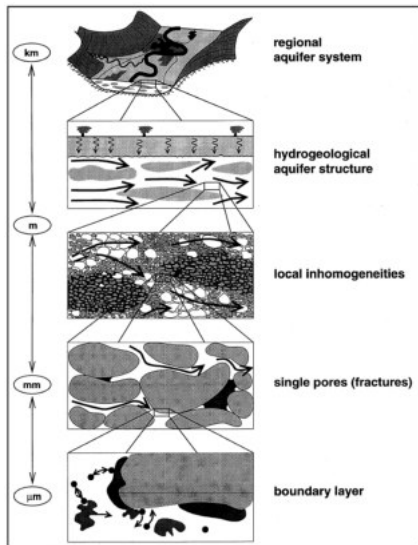
- ▶ Motivation
- ▶ Lagrange Konzept
- ▶ Euler Konzept
- ▶ Reynolds Transport Theorem
- ▶ Fluxes
- ▶ Bilanzgleichungen
- ▶ Erhaltungsgrößen



>> Skript

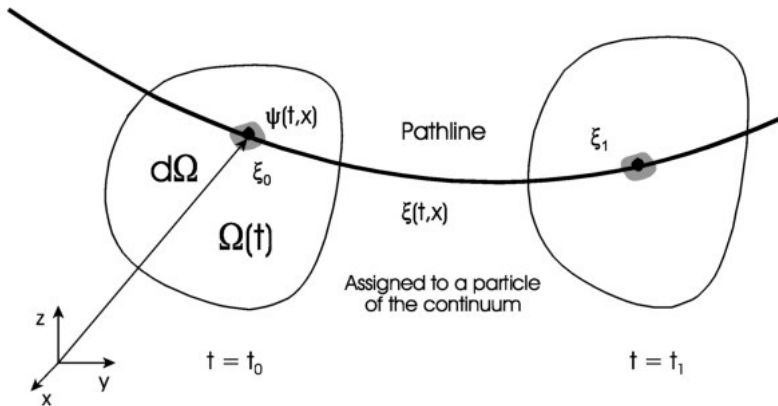
⇒ viel Theorie - vor allem die mathematische Schreibweise verstehen "zu lesen"

Skalen

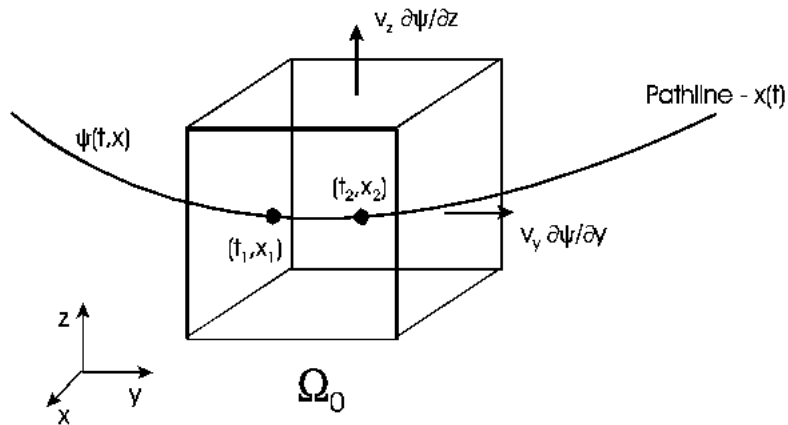


Quellen: Kobus et al. (1995), Kolditz (2002)

Lagrange Konzept (1.1.1)



Euler Konzept (1.1.1)



- ▶ Volumenintegral (Zeichnung)

$$\int_{\Omega} d\Omega \quad (1)$$

$$\int_a^b f(x) dx = \lim_{(x_{k+1}-x_k) \rightarrow 0} \sum_{k=1}^{\infty} (f(x_{k+1}) - f(x_k)) (x_{k+1} - x_k) \quad (2)$$

- ▶ Oberflächen-(Ring)-Integral (Zeichnung)

$$\oint_{\partial\Omega} \mathbf{n} \cdot d\mathbf{S} \quad (3)$$

- ▶ Materielle Ableitung

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v} \cdot \nabla\psi \quad (4)$$

- ▶ Gradient (Vektor)

$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \quad (5)$$

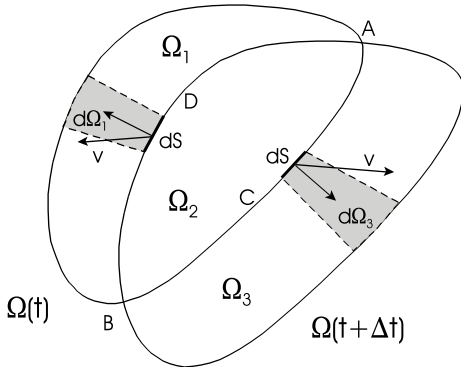
- ▶ Divergenz (Skalar)

$$\nabla \cdot \mathbf{v} = \partial v_x / \partial x + \partial v_y / \partial y + \partial v_z / \partial z \quad (6)$$

Aufgabe: Was ist $\mathbf{v} \cdot \nabla\psi$?

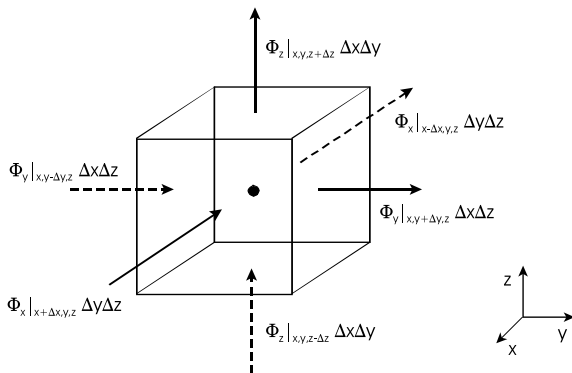
Reynolds Transport Theorem (Lagrange) (1.1.3)

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \oint_{\partial \Omega} \psi(t) \mathbf{v} \cdot d\mathbf{S} = \int_{\Omega} q^{\psi} d\Omega \quad (7)$$



Beweisführung:
Siehe Skript
Abschn. 1.1.3

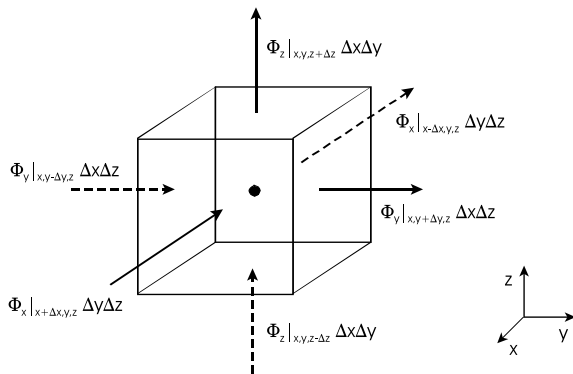
Reynolds Transport Theorem (Euler) (1.1.3)



$$\Phi_x^\psi = \partial\psi/\partial x \quad , \quad \Phi^\psi = \nabla\psi$$

Frage: Ist Φ^ψ eine skalare oder vektorielle Größe ?

Reynolds Transport Theorem (Euler) (1.1.3)



$$\oint_{\partial\Omega} \Phi^\psi \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^\psi d\Omega \quad (8)$$

Reynolds Transport Theorem (Euler) (1.1.3)

Divergence

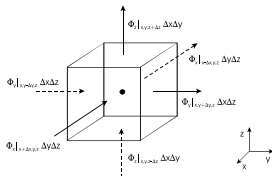
$$\oint_{\partial\Omega} \Phi^\psi \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^\psi d\Omega \quad (9)$$

$$\lim_{\Omega \rightarrow 0} \frac{1}{\Omega} \oint_{\partial\Omega} \Phi \cdot d\mathbf{S} = \nabla \cdot \Phi \quad (10)$$

Point-Flux

Reynolds Transport Theorem (Euler) (1.1.3)

General Balance Equation



$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \underbrace{\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega}_1 + \underbrace{\oint_{\partial\Omega} \Phi \psi \cdot d\mathbf{S}}_2 = \underbrace{\int_{\Omega} q \psi d\Omega}_3 \quad (11)$$

with:

- 1 Rate of change of total amount of quantity ψ in the control volume,
- 2 Net rate of increase / decrease of ψ due to fluxes,
- 3 Rate of increase / decrease of ψ due to sources.

Reynolds Transport Theorem (Euler) (1.1.3)

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \underbrace{\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega}_1 + \underbrace{\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S}}_2 = \underbrace{\int_{\Omega} q^{\psi} d\Omega}_3 \quad (12)$$

using

$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \quad (13)$$

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial\psi}{\partial t} d\Omega + \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega = \int_{\Omega} q^{\psi} d\Omega \quad (14)$$

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega = \int_{\Omega} q^{\psi} d\Omega \quad (15)$$

$$\forall \Omega : \frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot \Phi^{\psi} = q^{\psi} \quad (16)$$

Reynolds Transport Theorem

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \left(\frac{\partial \psi}{\partial t} + \nabla \cdot \Phi^{\psi} \right) d\Omega = \int_{\Omega} q^{\psi} d\Omega$$

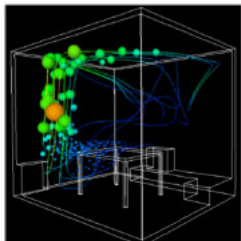
$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^E \nabla \psi$$



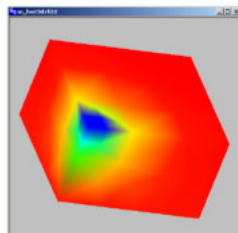
Lagrange



Euler



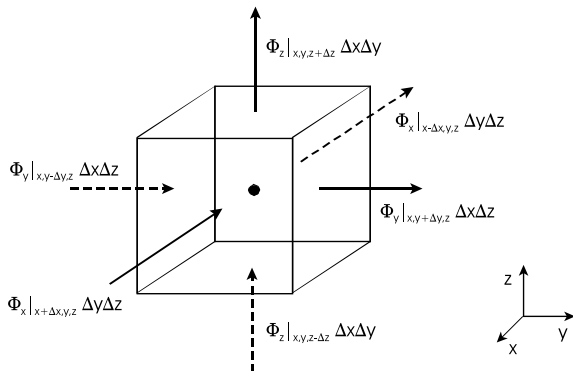
<http://www.cscs.ch/~mvalle/Libro/>



Fluxes (1.1.6)

The total flux Φ^ψ of a quantity ψ is defined as

$$\Phi^\psi = \mathbf{v}^E \psi \quad (17)$$



$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot \Phi^\psi = q^\psi \quad (18)$$

$$\Phi^\psi = \mathbf{v}^E \psi \quad (19)$$

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}^E \psi) = q^\psi \quad (20)$$

$$\Phi^\psi = \mathbf{v}^E \psi = \underbrace{\mathbf{v}\psi}_{\Phi_A^\psi} + \underbrace{(\mathbf{v}^E - \mathbf{v})\psi}_{\Phi_D^\psi} \quad (21)$$

and, therefore, decomposed into two parts: an advective flux Φ_A^ψ and a diffusive flux Φ_D^ψ relative to the mass-weighted velocity:

- ▶ advective flux of quantity ψ

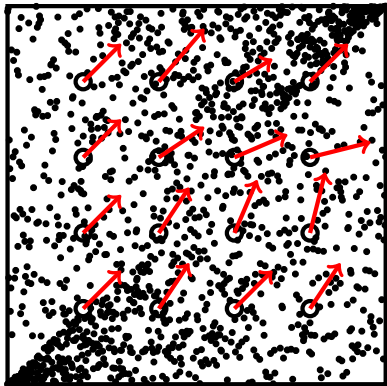
$$\Phi_A^\psi = \mathbf{v}\psi \quad (22)$$

- ▶ diffusive flux of quantity ψ (Fick's law)

$$\Phi_D^\psi = -\mathbf{D}^\psi \nabla \psi \quad (23)$$

Fluxes (1.1.6)

Velocities



$$\mathbf{v}^E = \cup \mathbf{v}_i$$

$$\mathbf{v}^E = \mathbf{v} + \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \mathbf{v}^E - \mathbf{v}$$

General Balance Equation (1.1.7)

- ▶ Integral form

$$\int_{\Omega} \frac{d\psi}{dt} = \int_{\Omega} \frac{\partial\psi}{\partial t} + \int_{\Omega} \nabla \cdot (\mathbf{v}\psi) - \int_{\Omega} \nabla \cdot (\mathbf{D}^{\psi} \nabla\psi) = \int_{\Omega} Q^{\psi} \quad (24)$$

- ▶ Differential form

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^{\psi} \nabla\psi) = Q^{\psi} \quad (25)$$

Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume Ω is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \quad (26)$$

where Ψ is an extensive conservation quantity (i.e. mass, momentum, energy) and ψ is the corresponding intensive conservation quantity such as mass density ρ , momentum density $\rho\mathbf{v}$ or energy density e .

Extensive quantity	Symbol	Intensive quantity	Symbol
Mass	M	Mass density	ρ
Linear momentum	\mathbf{m}	Linear momentum density	$\rho\mathbf{v}$
Energy	E	Energy density	$e = \rho i + \frac{1}{2}\rho v^2$