

# Hydroinformatik II

## ”Prozesssimulation und Systemanalyse”

### Einführung in die Lehrveranstaltung

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[www.ufz.de/cawr](http://www.ufz.de/cawr)

24.04.2020 - WebLecture

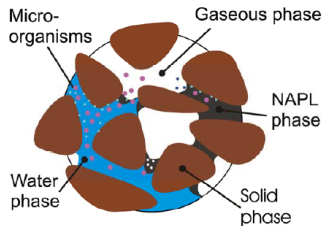
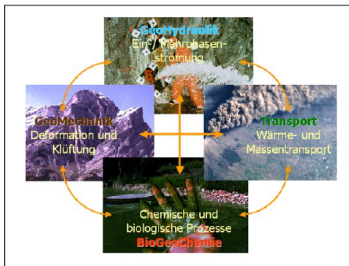
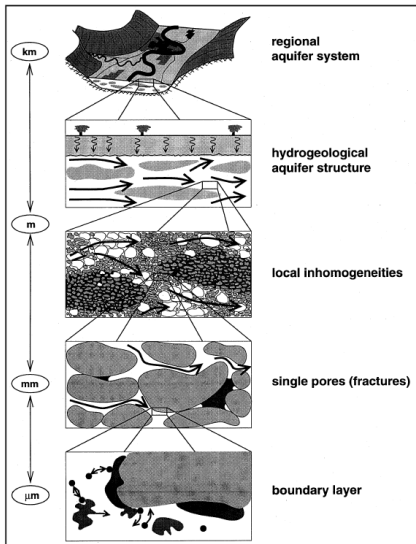
# Zeitplan: Hydroinformatik II

Datum	V	Thema	.
17.04.2020	01	Einführung in GoToMeeting (Web-Conferencing)	
17.04.2020	02	Einführung in die Lehrveranstaltung	
24.04.2020	03	Grundlagen: Kontinuumsmechanik	
08.05.2020	04	Grundlagen: Hydromechanik	
15.05.2020	05	Grundlagen: Partielle Partialgleichungen	
22.05.2020	06	Installation: Python, Qt C++	
29.05.2020	07	Programmierung: Einführung in Python	
05.06.2020	08	Numerik: Finite-Differenzen-Methode (explizit)	
12.06.2020	09	Numerik: Finite-Differenzen-Methode (implizit)	
19.06.2020	10	Anwendung: Diffusion (Übung)	
26.06.2020	11	Anwendung: Gerinnehydraulik (Theorie)	
03.07.2020	12	Anwendung: Gerinnehydraulik (Übung)	
10.07.2020	13	Anwendung: Grundwassermodellierung (datenbasierte Methoden)	
17.07.2020	14	Beleg: Besprechung zur Vorbereitung	

- ▶ Motivation
- ▶ Lagrange Konzept
- ▶ Euler Konzept
- ▶ Reynolds Transport Theorem
- ▶ Fluxes
- ▶ Bilanzgleichungen
- ▶ Erhaltungsgrößen

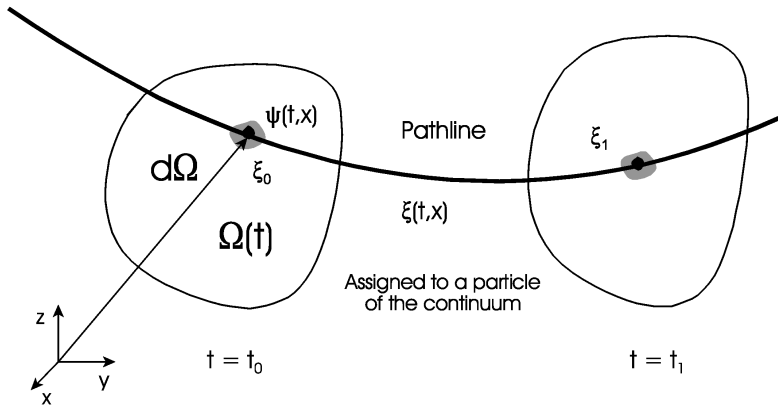
⇒ viel Theorie - vor allem die mathematische Schreibweise verstehen "zu lesen"

# Skalen

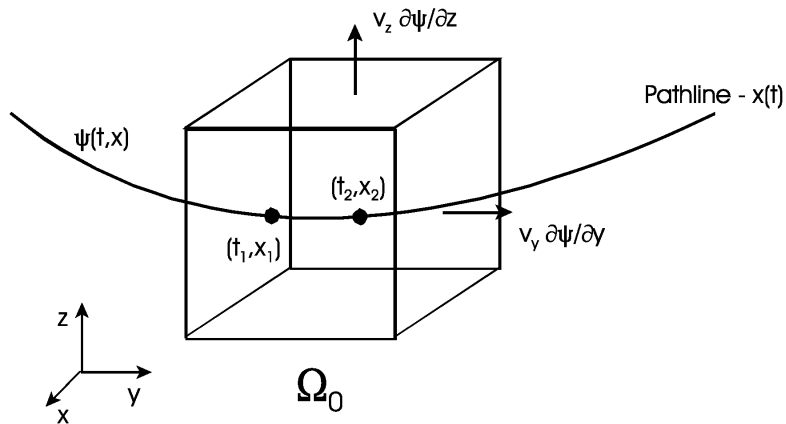


Quellen: Kobus et al. (1995), Kolditz (2002)

# Lagrange Konzept (1.1.1)



# Euler Konzept (1.1.1)



- ▶ Volumenintegral (Zeichnung)

$$\int_{\Omega} d\Omega \quad (1)$$

$$\int_a^b f(x) dx = \lim_{(x_{k+1}-x_k) \rightarrow 0} \sum_{k=1}^{\infty} (f(x_{k+1}) - f(x_k)) (x_{k+1} - x_k) \quad (2)$$

- ▶ Oberflächen-(Ring)-Integral (Zeichnung)

$$\oint_{\partial\Omega} \mathbf{n} \cdot d\mathbf{S} \quad (3)$$

- ▶ Materielle Ableitung

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v} \cdot \nabla\psi \quad (4)$$

- ▶ Gradient (Vektor)

$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \quad (5)$$

- ▶ Divergenz (Skalar)

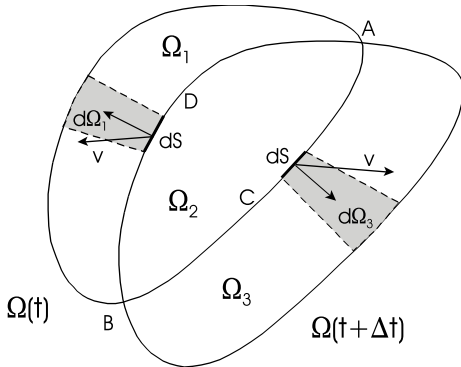
$$\nabla \cdot \mathbf{v} = \partial v_x / \partial x + \partial v_y / \partial y + \partial v_z / \partial z \quad (6)$$

Aufgabe: Was ist  $\mathbf{v} \cdot \nabla\psi$ ?



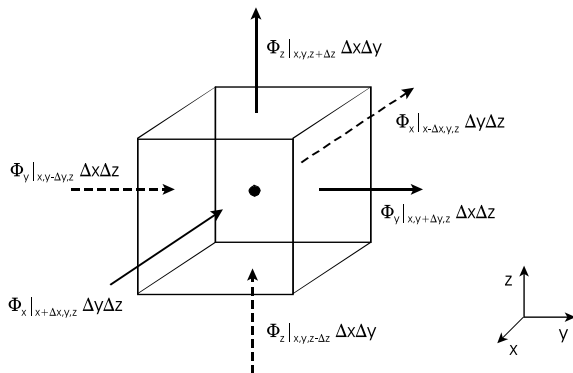
# Reynolds Transport Theorem (Lagrange) (1.1.3)

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \oint_{\partial \Omega} \psi(t) \mathbf{v} \cdot d\mathbf{S} = \int_{\Omega} q^{\psi} d\Omega \quad (7)$$



Beweisführung:  
Siehe Skript  
Abschn. 1.1.3

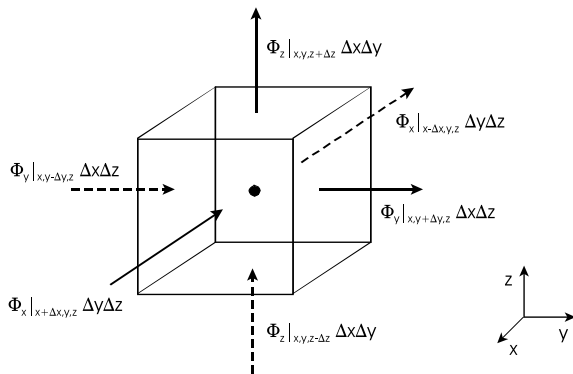
# Reynolds Transport Theorem (Euler) (1.1.3)



$$\Phi_x^\psi = \partial\psi/\partial x \quad , \quad \Phi^\psi = \nabla\psi$$

Frage: Ist  $\Phi^\psi$  eine skalare oder vektorielle Größe ?

# Reynolds Transport Theorem (Euler) (1.1.3)



$$\oint_{\partial\Omega} \Phi^\psi \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^\psi d\Omega \quad (8)$$

# Reynolds Transport Theorem (Euler) (1.1.3)

## Divergence

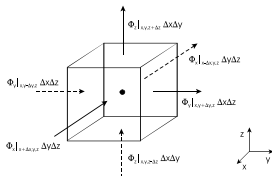
$$\oint_{\partial\Omega} \Phi^\psi \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^\psi d\Omega \quad (9)$$

$$\lim_{\Omega \rightarrow 0} \frac{1}{\Omega} \oint_{\partial\Omega} \Phi \cdot d\mathbf{S} = \nabla \cdot \Phi \quad (10)$$

## Point-Flux

# Reynolds Transport Theorem (Euler) (1.1.3)

## General Balance Equation



$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \underbrace{\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega}_1 + \underbrace{\oint_{\partial\Omega} \Phi \psi \cdot d\mathbf{S}}_2 = \underbrace{\int_{\Omega} q \psi d\Omega}_3 \quad (11)$$

with:

- 1 Rate of change of total amount of quantity  $\psi$  in the control volume,
- 2 Net rate of increase / decrease of  $\psi$  due to fluxes,
- 3 Rate of increase / decrease of  $\psi$  due to sources.

# Reynolds Transport Theorem (Euler) (1.1.3)

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \underbrace{\frac{\partial}{\partial t} \int_{\Omega} \psi d\Omega}_1 + \underbrace{\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S}}_2 = \underbrace{\int_{\Omega} q^{\psi} d\Omega}_3 \quad (12)$$

using

$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \quad (13)$$

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega = \int_{\Omega} q^{\psi} d\Omega \quad (14)$$

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega = \int_{\Omega} q^{\psi} d\Omega \quad (15)$$

$$\forall \Omega : \frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \nabla \cdot \Phi^{\psi} = q^{\psi} \quad (16)$$

# Reynolds Transport Theorem

$$\frac{d}{dt} \int_{\Omega} \psi d\Omega = \int_{\Omega} \left( \frac{\partial \psi}{\partial t} + \nabla \cdot \Phi^{\psi} \right) d\Omega = \int_{\Omega} q^{\psi} d\Omega$$

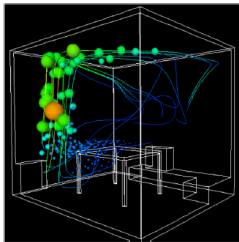
$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^E \nabla \psi$$



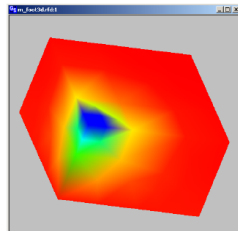
Lagrange



Euler



<http://www.cscs.ch/~mvalle/Libro/>

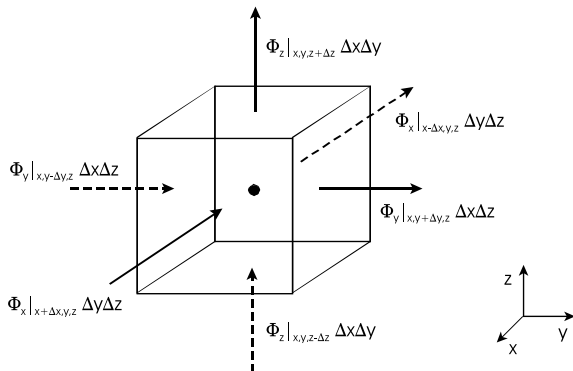




# Fluxes (1.1.6)

The total flux  $\Phi^\psi$  of a quantity  $\psi$  is defined as

$$\Phi^\psi = \mathbf{v}^E \psi \quad (17)$$



$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot \Phi^\psi = q^\psi \quad (18)$$

$$\Phi^\psi = \mathbf{v}^E \psi \quad (19)$$

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}^E \psi) = q^\psi \quad (20)$$

$$\Phi^\psi = \mathbf{v}^E \psi = \underbrace{\mathbf{v} \psi}_{\Phi_A^\psi} + \underbrace{(\mathbf{v}^E - \mathbf{v}) \psi}_{\Phi_D^\psi} \quad (21)$$

and, therefore, decomposed into two parts: an advective flux  $\Phi_A^\psi$  and a diffusive flux  $\Phi_D^\psi$  relative to the mass-weighted velocity:

- ▶ advective flux of quantity  $\psi$

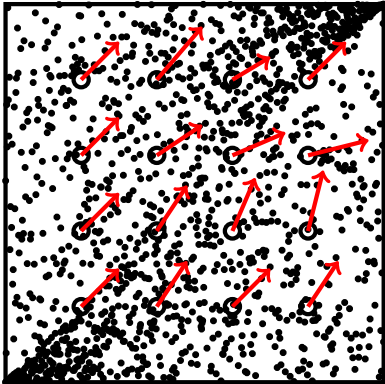
$$\Phi_A^\psi = \mathbf{v} \psi \quad (22)$$

- ▶ diffusive flux of quantity  $\psi$  (Fick's law)

$$\Phi_D^\psi = -\mathbf{D}^\psi \nabla \psi \quad (23)$$

# Fluxes (1.1.6)

## Velocities



$$\mathbf{v}^E = \mathbf{U} \mathbf{v}_i$$

$$\mathbf{v}^E = \mathbf{v} + \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \mathbf{v}^E - \mathbf{v}$$

# General Balance Equation (1.1.7)

► Integral form

$$\int_{\Omega} \frac{d\psi}{dt} = \int_{\Omega} \frac{\partial\psi}{\partial t} + \int_{\Omega} \nabla \cdot (\mathbf{v}\psi) - \int_{\Omega} \nabla \cdot (\mathbf{D}^{\psi} \nabla\psi) = \int_{\Omega} Q^{\psi} \quad (24)$$

► Differential form

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^{\psi} \nabla\psi) = Q^{\psi} \quad (25)$$

# Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume  $\Omega$  is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \quad (26)$$

where  $\Psi$  is an extensive conservation quantity (i.e. mass, momentum, energy) and  $\psi$  is the corresponding intensive conservation quantity such as mass density  $\rho$ , momentum density  $\rho\mathbf{v}$  or energy density  $e$ .

Extensive quantity	Symbol	Intensive quantity	Symbol
Mass	$M$	Mass density	$\rho$
Linear momentum	$\mathbf{m}$	Linear momentum density	$\rho\mathbf{v}$
Energy	$E$	Energy density	$e = \rho i + \frac{1}{2}\rho v^2$