From dynamic groundwater head measurements to regional aquifer parameters
Assessing the power of spectral analysis

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Projecting the response of regional aquifer systems to strongly changing conditions is paramount.

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We need regional GW models!

- Regional groundwater models are difficult to parameterize!
- Usually aquifers are investigated through pumping tests.

How to parameterize regional groundwater models?
Methodology
Spectral Analysis (SA)

Answer
Take GW level time series and derive the aquifer parameters from the spectral response!

Example: disentangling a sinusoidal signal

![Diagram showing time series and power spectrum with components 1, 2, and 3.]
Temporal and spatial variation and scaling of groundwater levels in a bounded unconfined aquifer

Xiuyu Liang, You-Kuan Zhang

Evoking the Dupuit-Assumptions:

\[ S_{hh}(x', \omega) = \frac{16}{\pi^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n} B_mB_n S_{WW}}{(2m^2 + 2n^2 + 2m + 2n + 1)} \]

\[ \times \frac{(2m + 1)^2}{(2m + 1)^4 / t_c^2 + \omega^2} \]

\[ S_{storativity} \]

\[ T_{transmissivity} \]

\[ t_c = \frac{4L^2S}{\pi^2T} \]
Our Approach
Schematized

1. Num. Modelling

1. homogeneous
2. heterogeneous
3. block domain

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Our Approach
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1. Num. Modelling
2. Time Series
3. Fit with anal. Solution

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\( S_{hh} (T, S, L, x) \)
- \( T \): Transmissivity
- \( S \): Storativity
- \( L \): Aquifer Length
- \( x \): Distance from Water Table

Input parameter space

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Our Approach
Schematized

1. Num. Modelling
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3. Fit with anal. Solution
4. Comparison of in- and output

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$S_{hh}(T, S, L, x)$
- $T$: Transmissivity
- $S$: Storativity
- $L$: Aquifer Length
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Input parameter space
1 - Homogeneous Domain Setup

Motivation

- Analytical solution of Dupuit-Eq. also valid for the 2D groundwater equation?
- Does the location influence the derived parameter?
- What is the required time step and length of the time series?
1 - Homogeneous Domain
Setup

**Motivation**

- Analytical solution of Dupuit-Eq. also **valid** for the **2D groundwater equation**?
- Does the **location** influence the **derived parameter**?
- What is the required **time step** and **length of the time series**?

**Setup**

**Input Parameter Space:**

$t_c = 10^{-1} \ldots 10^4 \text{ days}$

\[
t_c = \frac{4 \cdot L^2 \cdot S}{\pi^2 \cdot T}
\]

$t_c$: characteristic time

- **Recharge:** white noise
- **Time Step Size:** 1 day
- **Time Steps:** 10,950 (30 years)
1 - Homogeneous Domain

Results

The output transmissivity **matches** the input transmissivity almost exactly!

The error in transmissivity increases if the characteristic time is **larger than 1/10** of the length of the time series.

Transmissivity

![Graph showing error in transmissivity over time for different locations](image)

**Error [%]** = \( \frac{|\text{Input} - \text{Output}|}{\text{Input}} \times 100 \)
2 - Heterogeneous Domain Setup

Motivation

- Aquifers are often considered as being homogeneous - but they aren’t!
- What does the SA tell us when applying it to heterogeneous media?
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- What does the SA tell us when applying it to heterogeneous media?

Setup

Variance: 1
Len Scale: 5 and 15 m
Ari-Mean: 7.50e-05 ms\(^{-1}\)
Geo-Mean: 4.54e-05 ms\(^{-1}\)
Har-Mean: 2.75e-05 ms\(^{-1}\)
Realizations: 200

Recharge: white noise
Time Step Size: 1 day
Time Steps: 10,950
Spec. Storage: 0.001
2 - Heterogeneous Domain

Results
Motivation

- We have investigated homogeneous and log-normal domains > works well!
- Introduce an aquifer with different **hydraulic zones**.
Motivation

- We have investigated homogeneous and log-normal domains > works well!
- Introduce an aquifer with different hydraulic zones.

A) Flow from T1 to T2

B) Flow from T2 to T1

spec. storage: 0.0001
forcing & geometry: like previous examples

low conductive zone, $T_1 = 3 \times 10^{-4} \text{ m}^2/\text{s}$
high conductive zone, $T_2 = 3 \times 10^{-2} \text{ m}^2/\text{s}$
3 - Block Domain

Results

A) T1 → T2

<>

High conductivities at the outlet of the aquifer slightly affect lower transmissivities located upgradient.

> High conductivities at the outlet of the aquifer slightly affect lower transmissivities located upgradient.
3 - Block Domain

Results

A) T1 → T2

- Low (left) to high (right)

B) T2 → T1

- High (left) to low (right)

> **High conductivities** at the outlet of the aquifer **slightly** affect lower transmissivities located upgradient.

> **Low conductivities** at the outlet dominate the entire flow regimes resulting in lower effective transmissivities upgradient.
To sum it up...

Conclusion

- The analytical solution for the head spectrum based on the linearized Boussinesq equation and evoking the Dupuit assumptions is valid for the 2D GW equation with mentioned constrains.

- Measure at least ~10 times as long as your expected $t_c$!

- Spectral analysis reveals the effective parameter!

  > log-normal distributed conductivity domains: geometric mean

  > block domains: low conductivities at the outlet dominate the hydraulic regime
Thank you for your attention!
To sum it up...

Conclusion

- The analytical solution for the head spectrum based on the linearized Boussinesq equation and evoking the Dupuit assumptions is valid for the 2D GW equation with mentioned constrains.

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Our Approach

Schematized

1. **Num. Modelling**
   - 1: homogeneous
   - 2: heterogeneous
   - 3: block domain

2. **Time Series**
   - h(t)

3. **Fit with anal. Solution**
   - $S_{hh}(T, S, L, x)$
   - T: Transmissivity
   - S: Storativity
   - L: Aquifer Length
   - x: Distance from river

4. **Comparison of in- and output**
   - T & S input
   - T & S output
   - different locations x

OpenGeoSys

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The output transmissivity matches the input transmissivity almost exactly!

The error in transmissivity increases if the characteristic time is larger than 1/10 of the length of the time series.

\[
\text{Error} \% = \left( \frac{|\text{Input} - \text{Output}|}{\text{Input}} \right) \times 100
\]
2 - Heterogeneous Domain

Results

Location of observation point [m]

Transmissivity [m²s⁻¹]

Storativity

Transmissivity

5 m

15 m

geomean

harmean

arimean

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3 - Block Domain

Results

> **High conductivities** at the outlet of the aquifer *slightly* affect lower transmissivities located upgradient.

> **Low conductivities** at the outlet dominate the entire flow regimes resulting in lower effective transmissivities upgradient.

A) $T_1 \rightarrow T_2$

- low
- high

B) $T_2 \rightarrow T_1$

- high
- low

Transm. [m$^2$ s$^{-1}$]

>>> direction of flow >>>
Homogeneous Domain Setup

characteristic time $t_c$ [d] 
for $L = 1000$ m

$$t_c = \frac{4 \cdot L^2 \cdot S}{\pi^2 \cdot T}$$

Example 1 
$L = 1000$ m 
$S = 0.1$ 
$T = 1e-3$ m$^2$s$^{-1}$ 
$t_c = 469$ d

Example 2 
$L = 1000$ m 
$S = 0.001$ 
$T = 1e-4$ m$^2$s$^{-1}$ 
$t_c = 46$ d

Storativity [-] 
Transmissivity [m$^2$s$^{-1}$]
References


