



# From dynamic groundwater head measurements to regional aquifer parameters

## Assessing the power of spectral analysis

M.Sc. Timo Houben

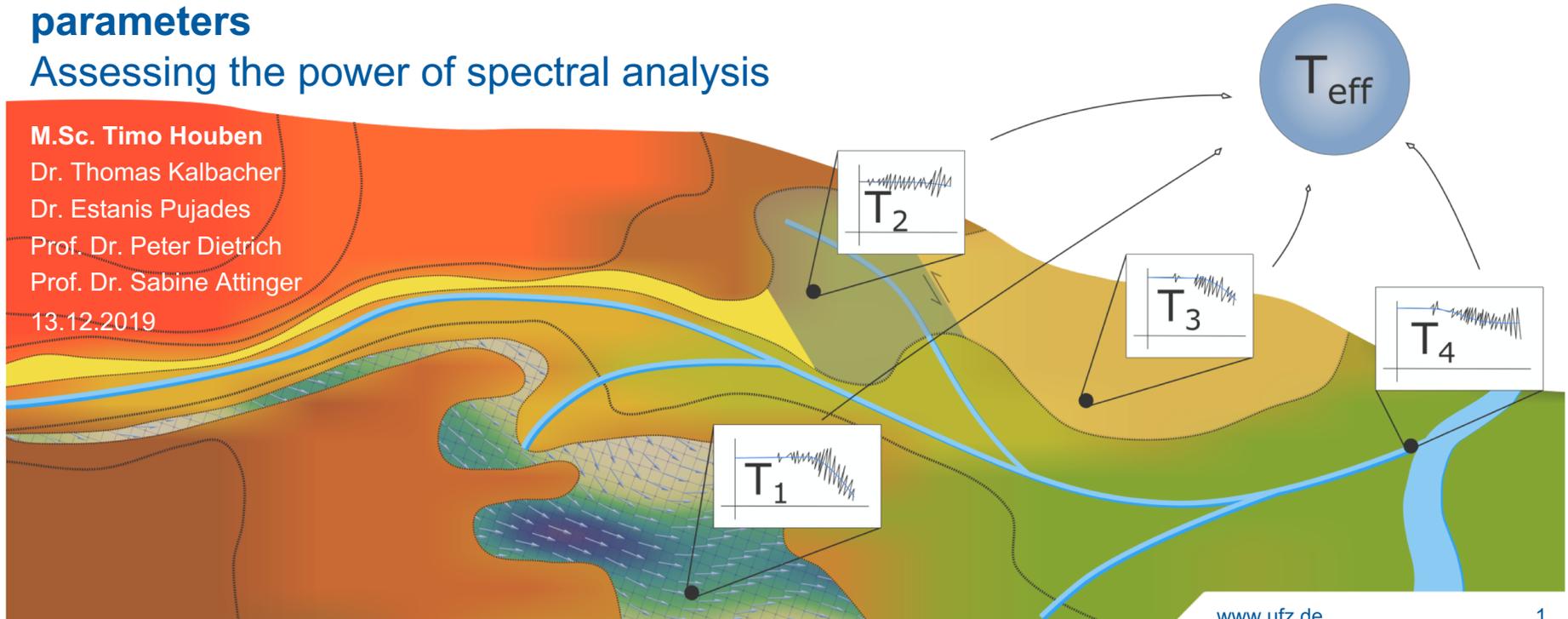
Dr. Thomas Kalbacher

Dr. Estanis Pujades

Prof. Dr. Peter Dietrich

Prof. Dr. Sabine Attinger

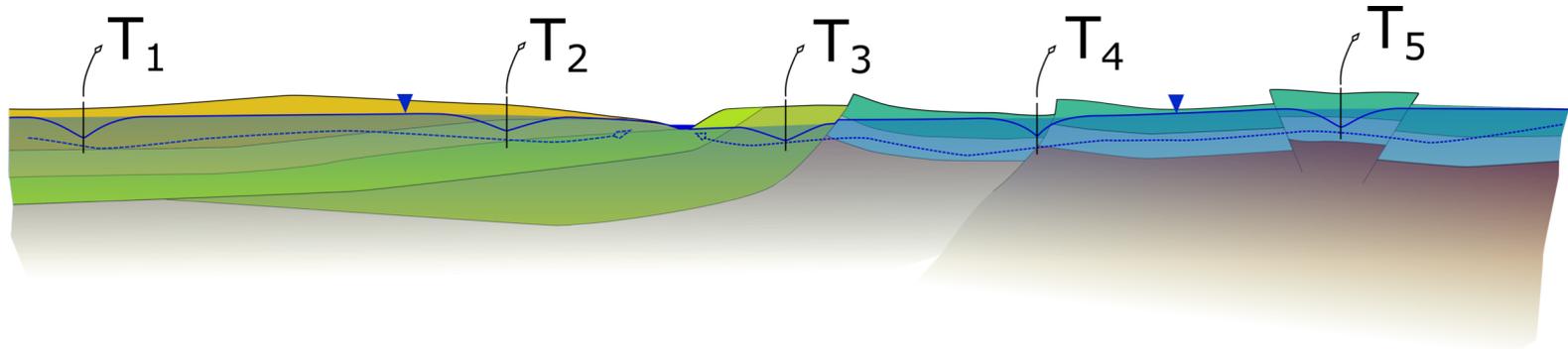
13.12.2019



# Motivation

- Projecting the response of regional aquifer systems to strongly changing conditions is paramount.

**We need regional GW models!**

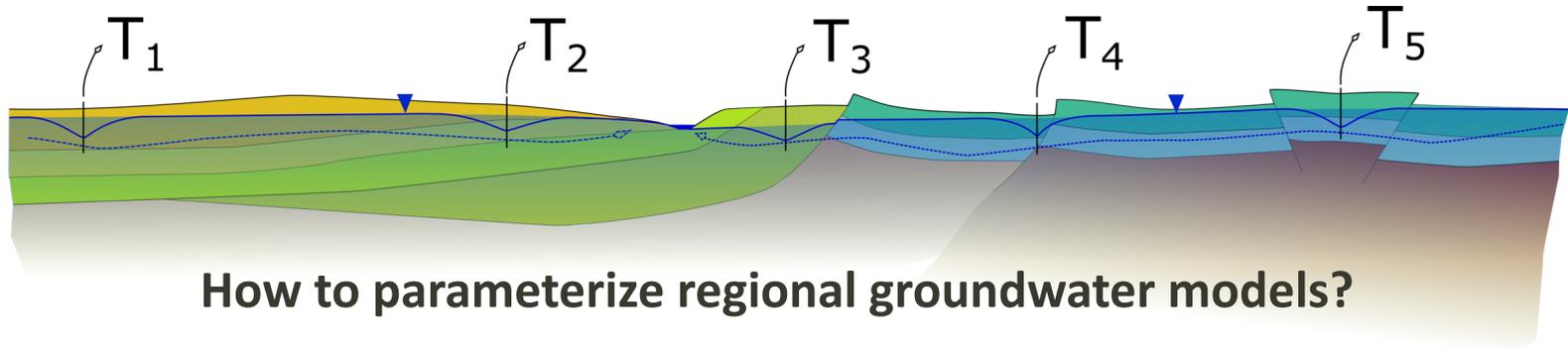


# Motivation

- Projecting the response of regional aquifer systems to strongly changing conditions is paramount.

## We need regional GW models!

- **Regional** groundwater models are **difficult to parameterize!**
- Usually aquifers are investigated through **pumping tests**.



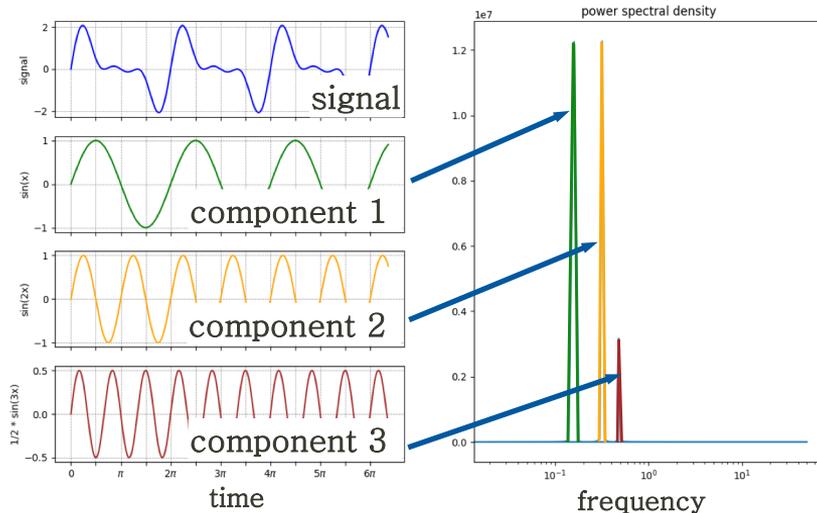
# Methodology

## Spectral Analysis (SA)

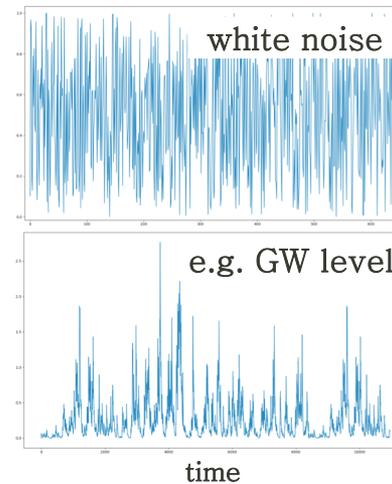
### Answer

Take GW level time series and derive the aquifer parameters from the spectral response!

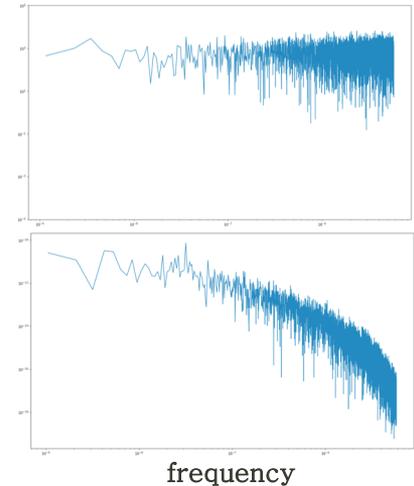
Example: disentangling a sinusoidal signal



time series



power spectrum





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Temporal and spatial variation and scaling of groundwater levels  
in a bounded unconfined aquifer

Xiuyu Liang<sup>a,b</sup>, You-Kuan Zhang<sup>a,b,c,\*</sup>

Evoking the Dupuit-Assumptions:

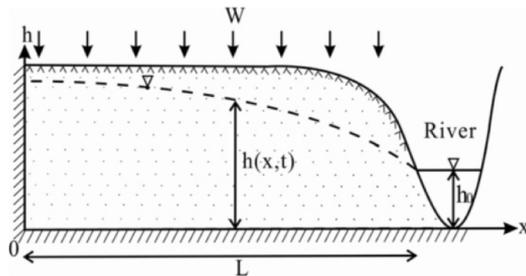
$$S_{hh}(x', \omega) = \frac{16}{\pi^2 S_Y} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n} B_m B_n S_{WW}}{(2m^2 + 2n^2 + 2m + 2n + 1)}$$

$$\times \frac{(2m + 1)^2}{(2m + 1)^4 / t_c^2 + \omega^2}$$

*Storativity*

*T* *transmissivity*

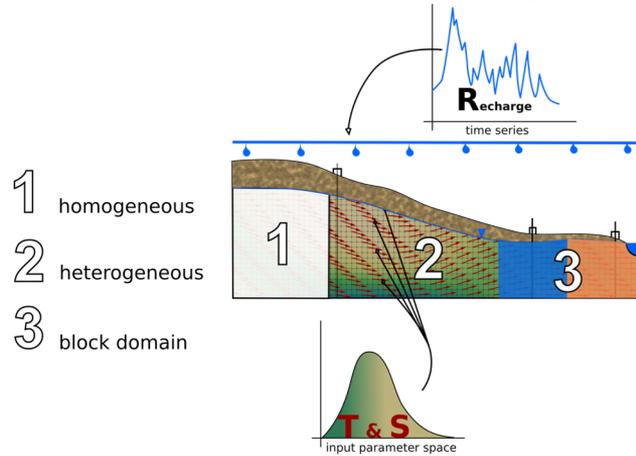
$$t_c = \frac{4 \cdot L^2 \cdot S}{\pi^2 \cdot T}$$



# Our Approach

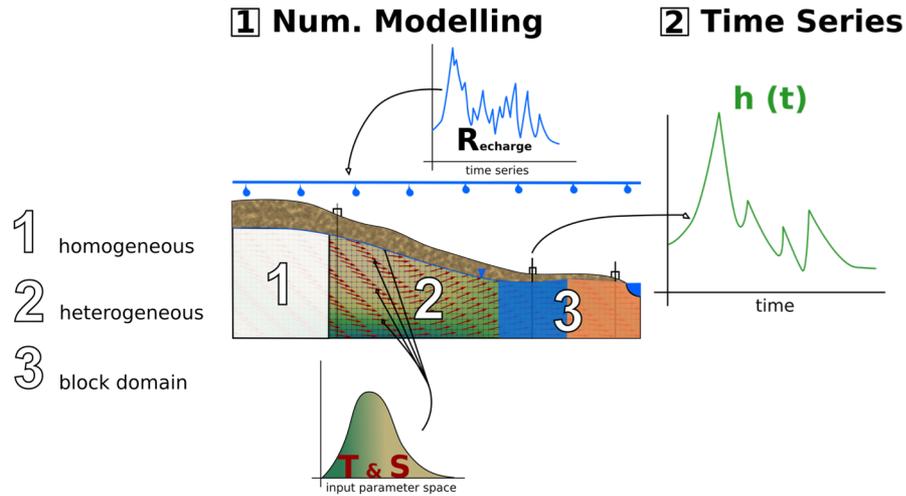
## Schematized

### 1 Num. Modelling



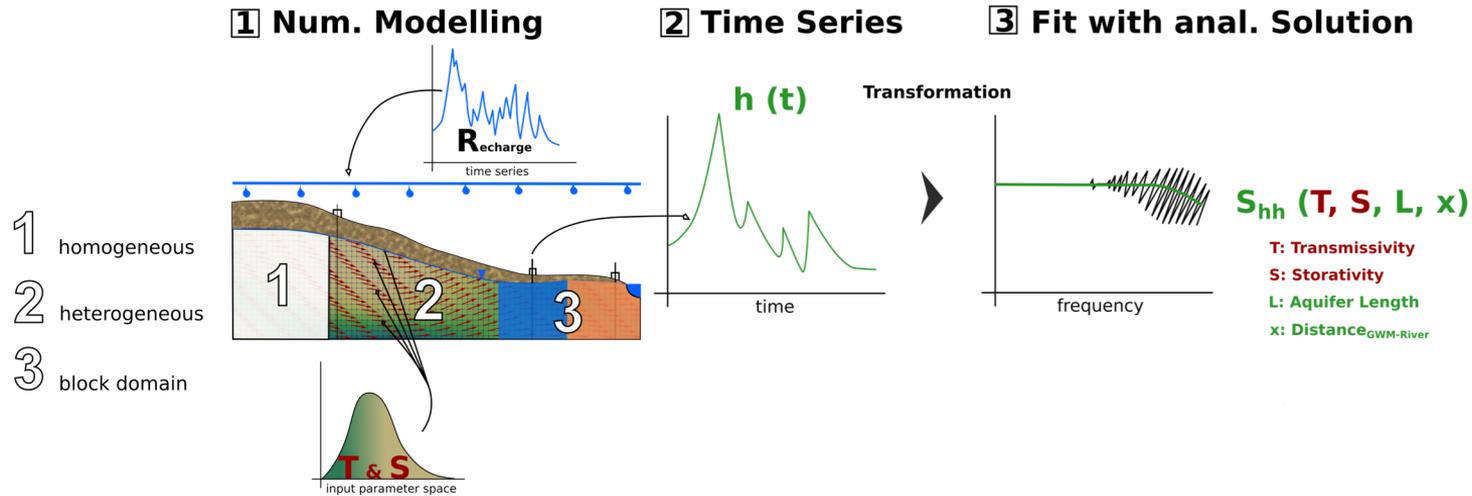
# Our Approach

## Schematized

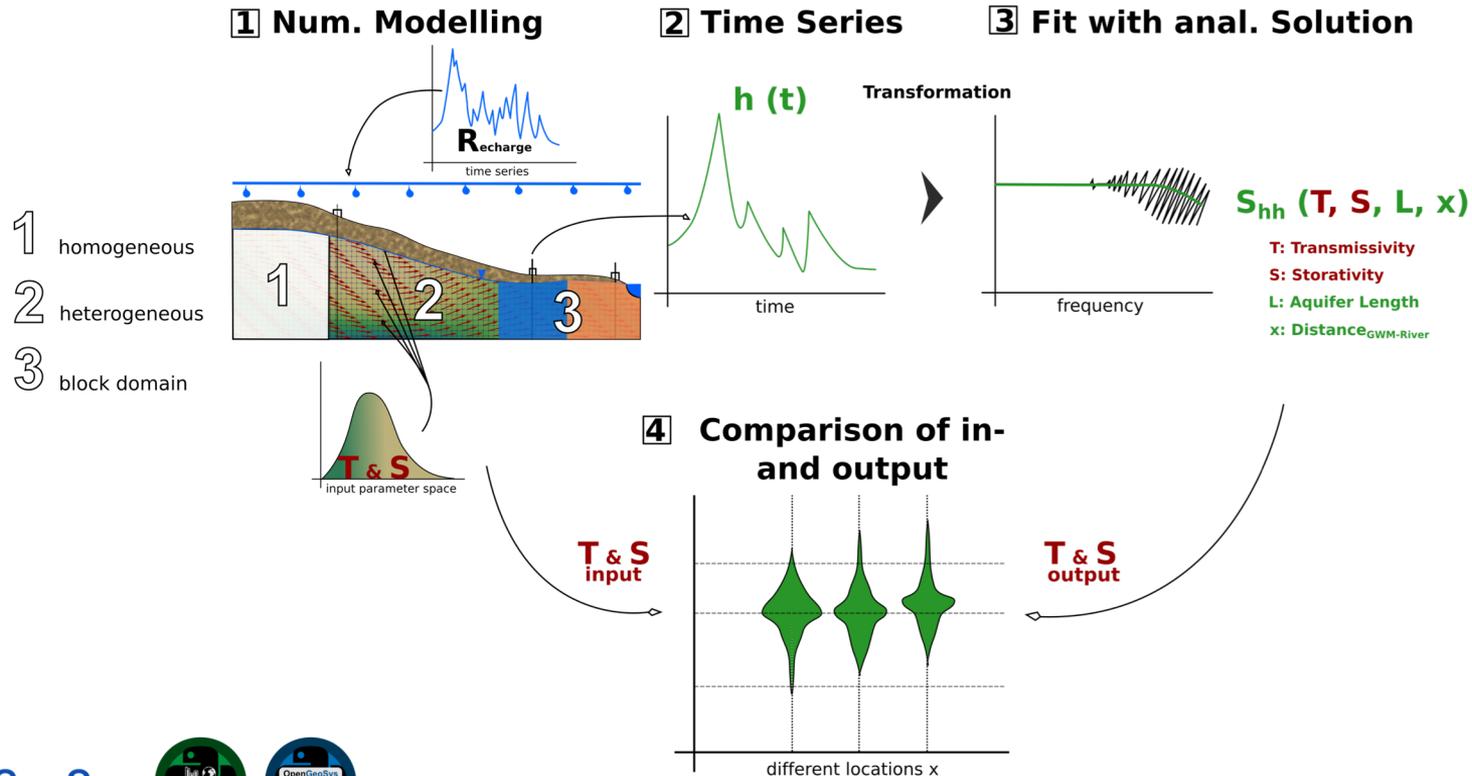


# Our Approach

## Schematized



# Our Approach Schematized



# 1 - Homogeneous Domain

## Setup

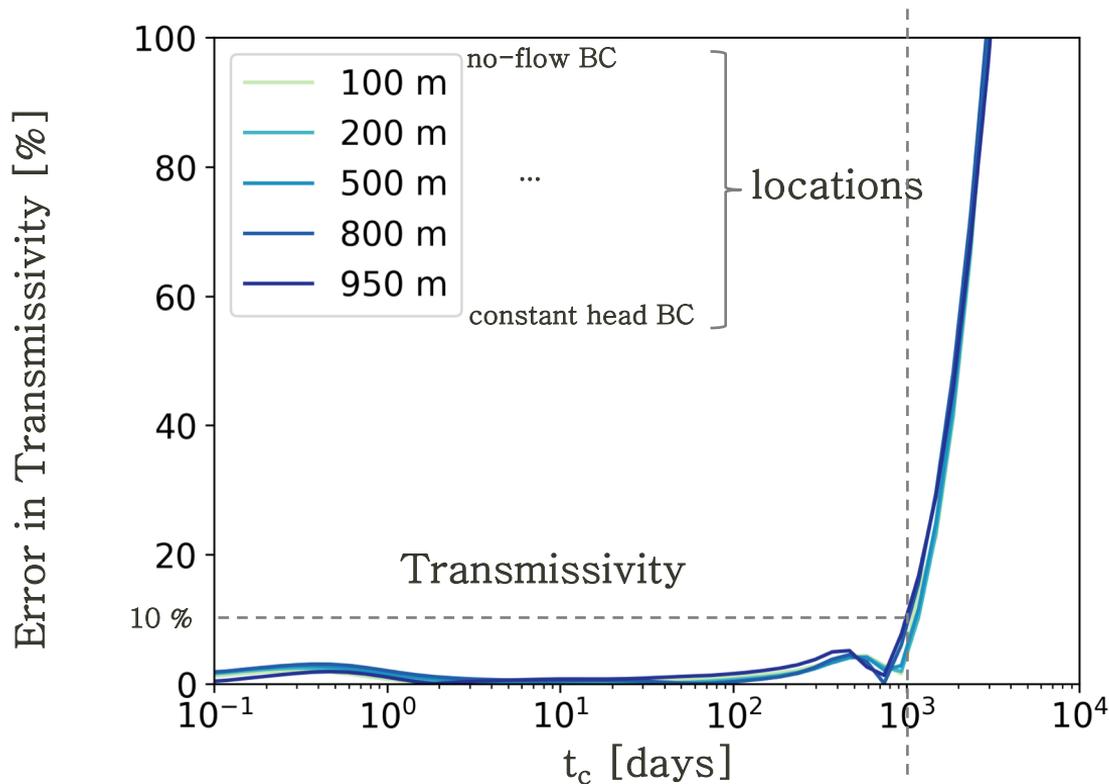
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### Motivation

- Analytical solution of Dupuit-Eq. also **valid** for the **2D groundwater equation**?
- Does the **location** influence the **derived parameter**?
- What is the required **time step** and **length of the time series**?



# 1 - Homogeneous Domain Results



The output transmissivity **matches** the input transmissivity almost **exactly!**

The error in transmissivity increases if the characteristic time is **larger than 1/10** of the length of the time series.

$$Error [\%] = \frac{|Input - Output|}{Input} \times 100$$

## 2 - Heterogeneous Domain

### Setup

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#### Motivation

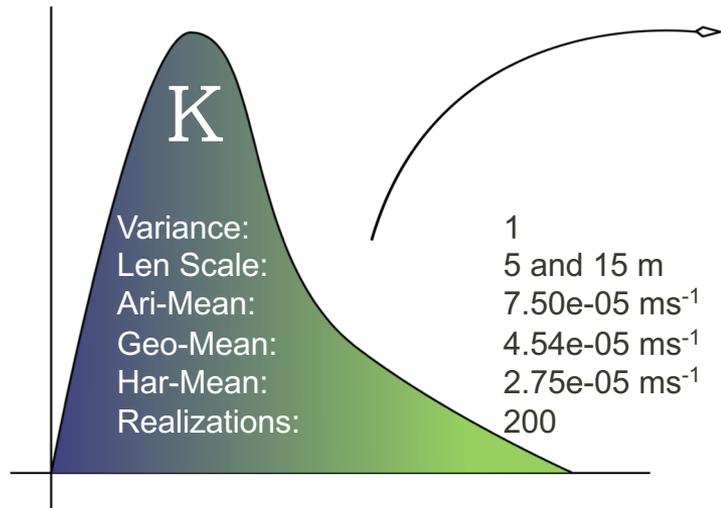
- Aquifers are often considered as being homogeneous - **but they aren't!**
- What does the SA tell us when applying it to **heterogeneous media?**

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### Setup



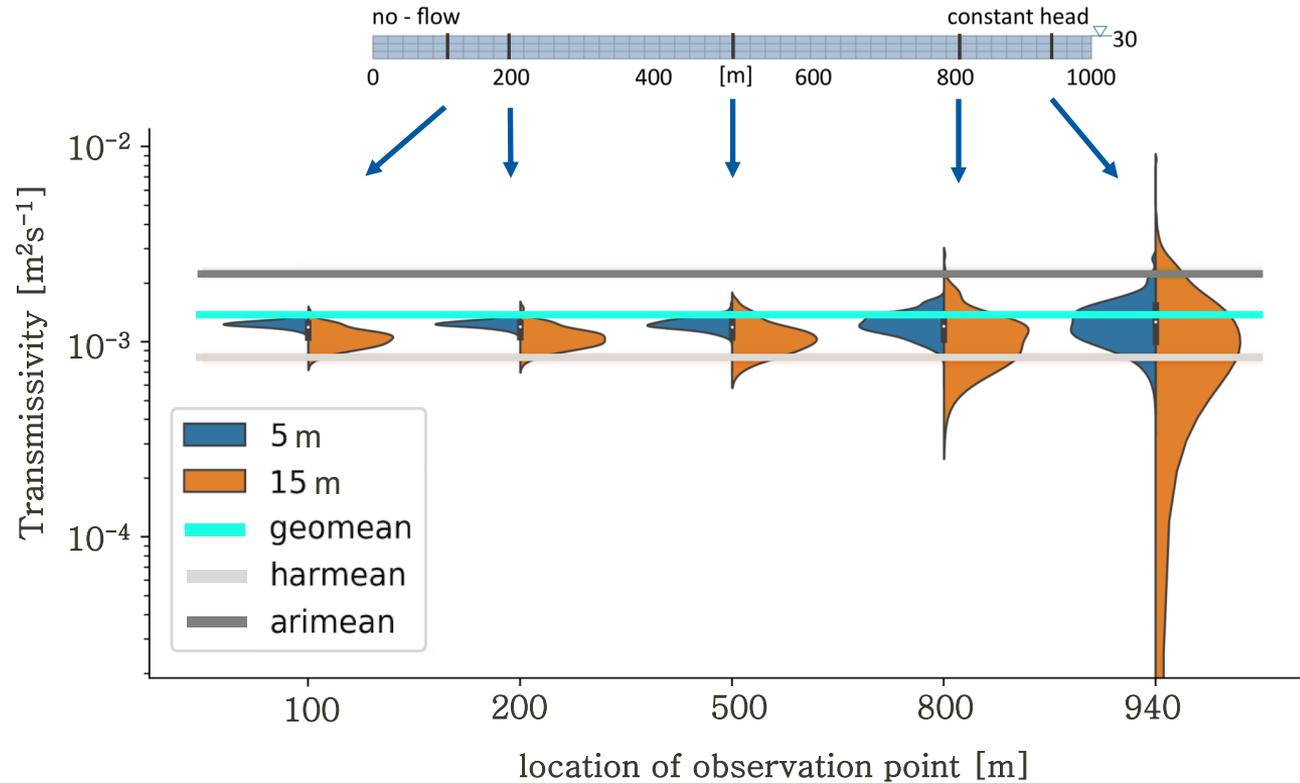
5 m

15 m

Recharge: white noise  
Time Step Size: 1 day  
Time Steps: 10,950  
Spec. Storage: 0.001



## 2 - Heterogeneous Domain Results



## 3 - Block Domain Setup

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### Motivation

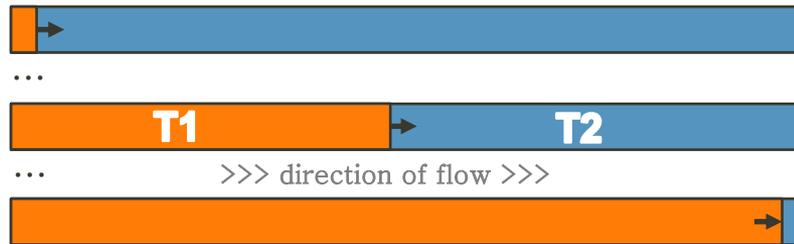
- We have investigated homogeneous and log-normal domains > **works well!**
- Introduce an aquifer with different **hydraulic zones**.

# 3 - Block Domain Setup

## Motivation

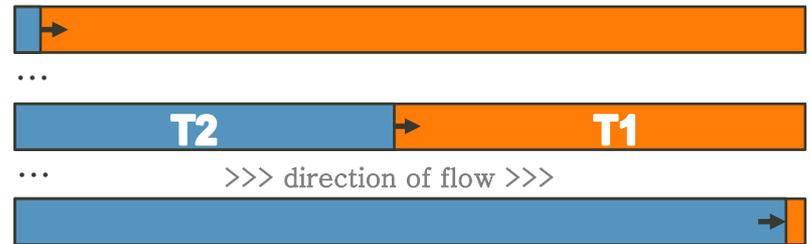
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- Introduce an aquifer with different **hydraulic zones**.

A) Flow from **T1** to **T2**



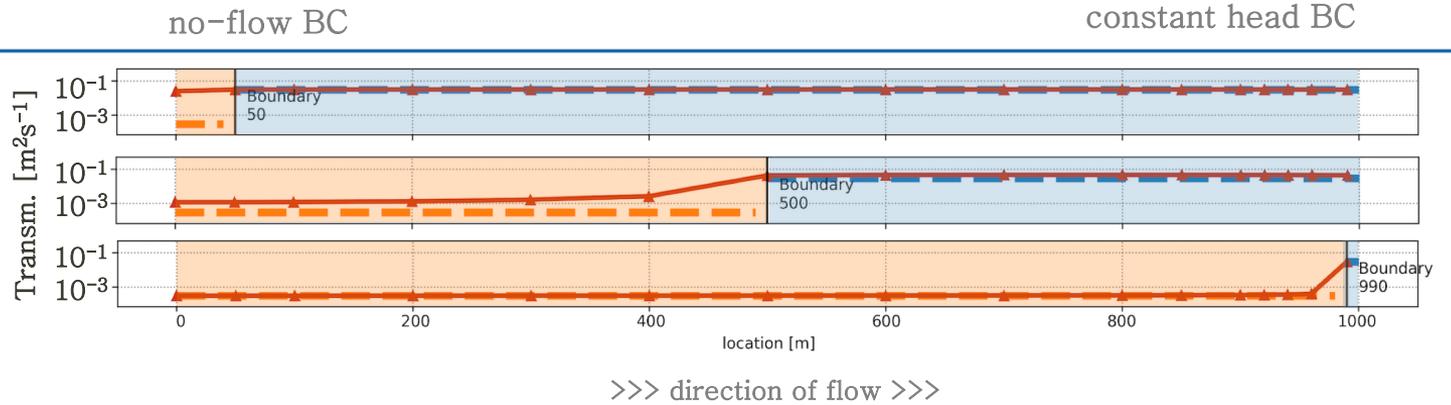
spec. storage: 0.0001  
forcing & geometry: like previous examples

B) Flow from **T2** to **T1**



 low conductive zone,  $T1 = 3e^{-4} \text{ m}^2/\text{s}$   
 high conductive zone,  $T2 = 3e^{-2} \text{ m}^2/\text{s}$

# 3 - Block Domain Results



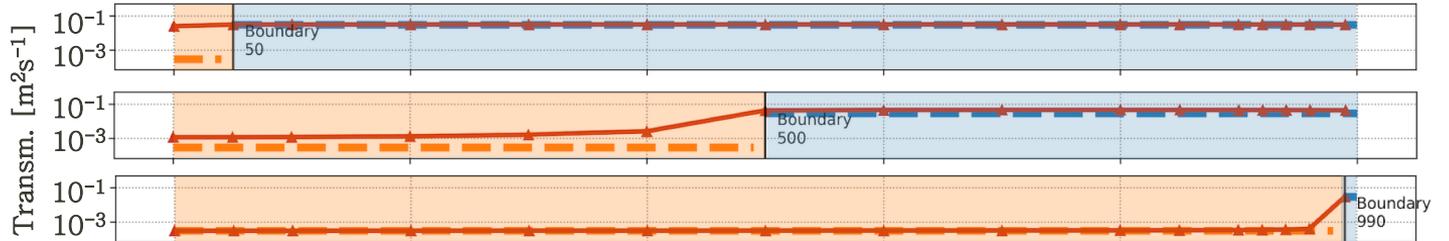
> **High conductivities** at the outlet of the aquifer **slightly** affect lower transmissivities located upgradient.

# 3 - Block Domain Results

no-flow BC

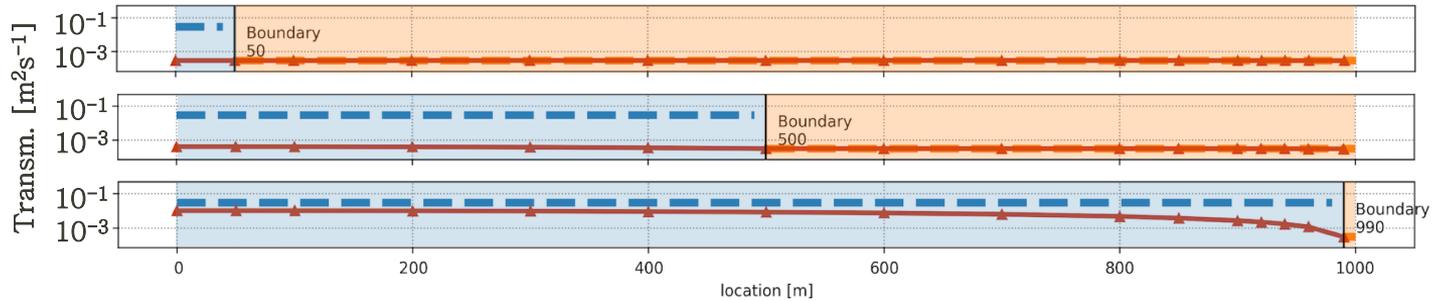
constant head BC

A) T1 → T2  
low high



>>> direction of flow >>>

B) T2 → T1  
high low



- > **High conductivities** at the outlet of the aquifer **slightly** affect lower transmissivities located upgradient.
- > **Low conductivities** at the outlet dominate the entire flow regimes resulting in lower effective transmissivities upgradient.

## To sum it up...

### Conclusion

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- The analytical solution for the head spectrum based on the linearized Boussinesq equation and evoking the Dupuit assumptions **is valid for the 2D GW equation** with mentioned constraints.
- Measure at least **~10 times** as long as your **expected  $t_c$** !
- Spectral analysis reveals the **effective parameter!**
  - > log-normal distributed conductivity domains: **geometric mean**
  - > block domains: **low conductivities at the outlet dominate** the hydraulic regime



# Thank you for your attention!



[ufz.de/index.php?en=43660](https://ufz.de/index.php?en=43660)  
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@HoubenTimo



[geostat-framework.github.io](https://geostat-framework.github.io)

# OpenGeoSys

OPEN-SOURCE MULTI-PHYSICS

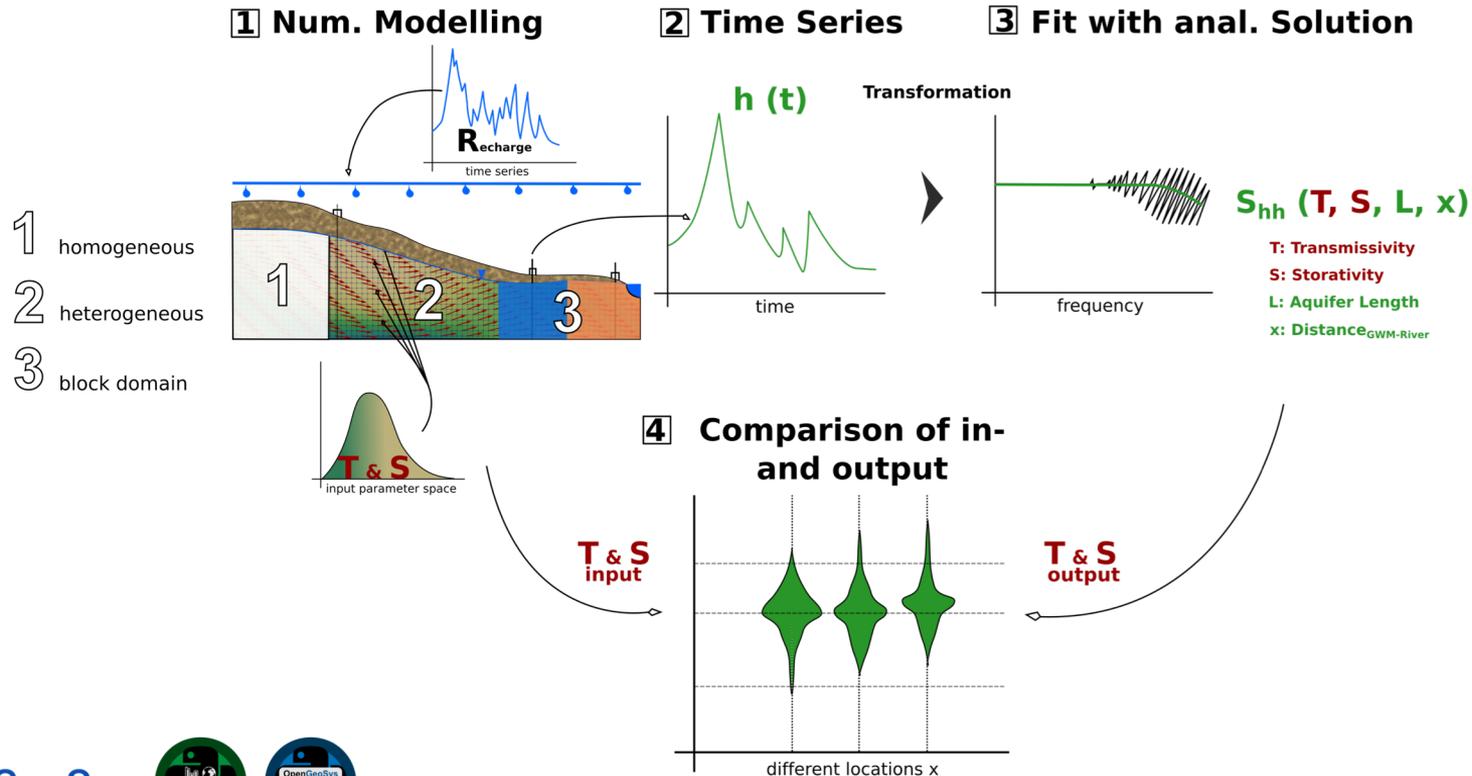
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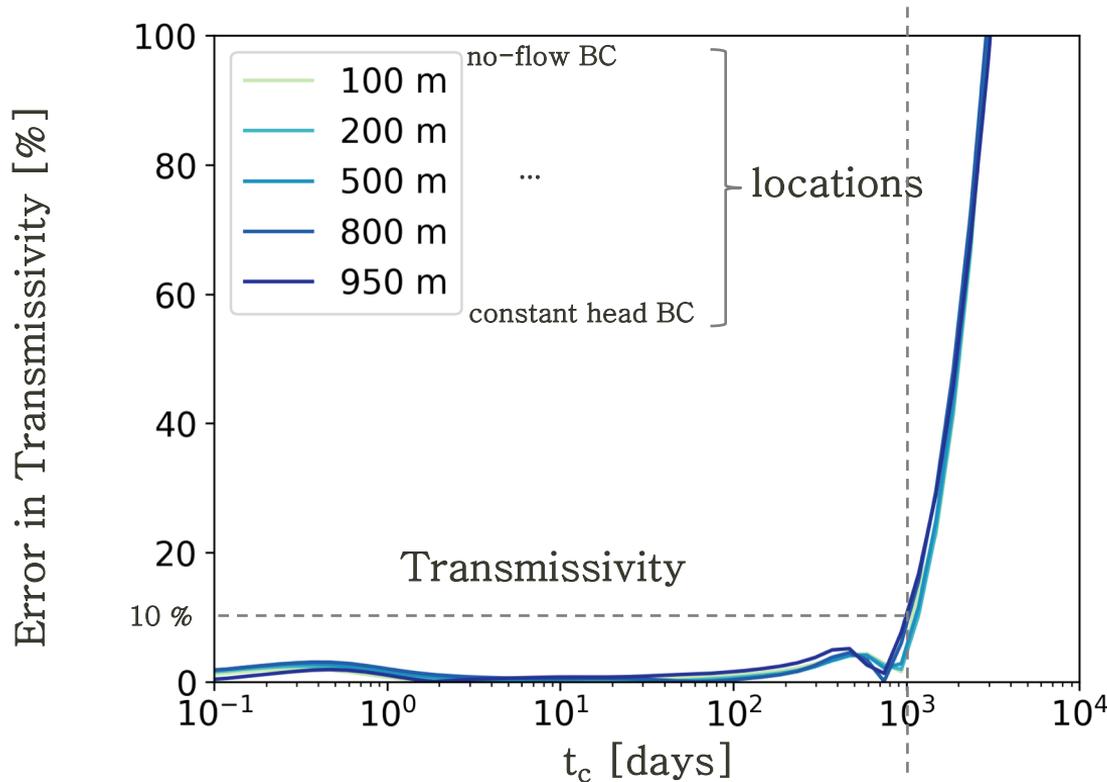
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# Our Approach Schematized



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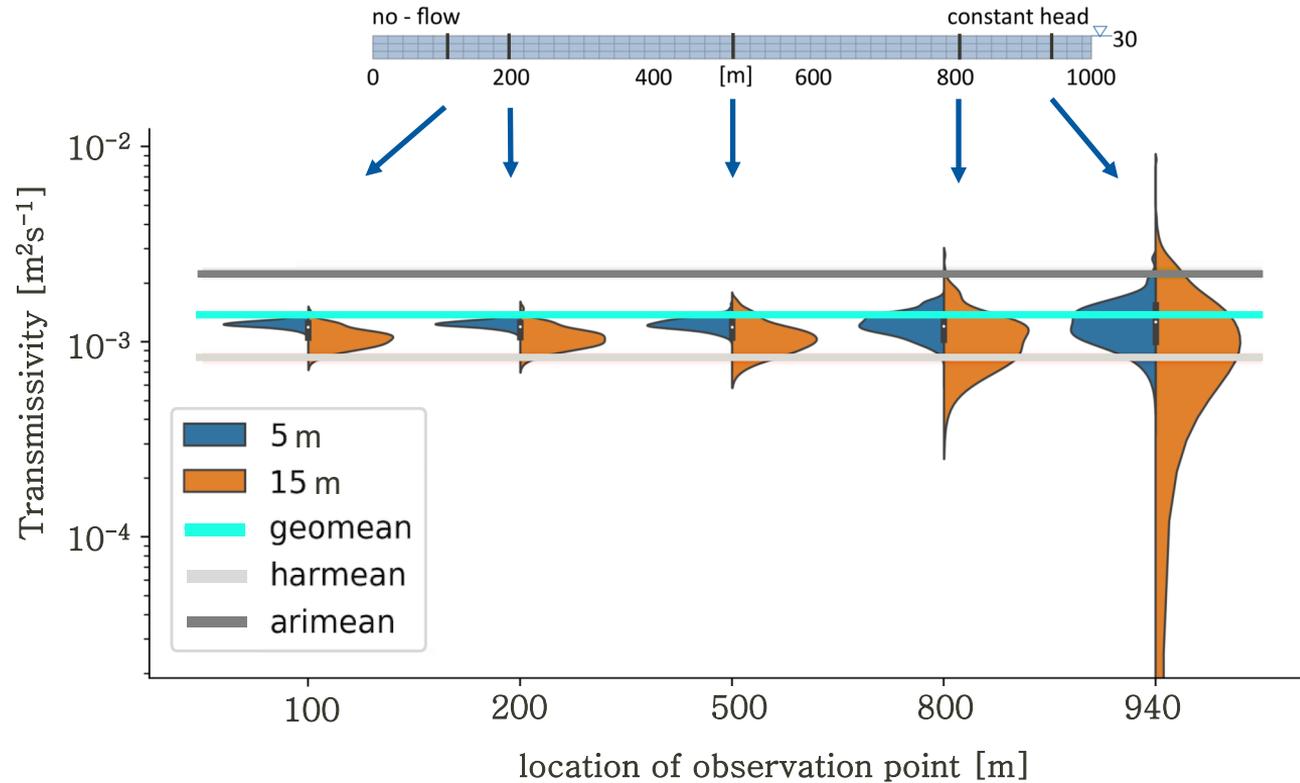


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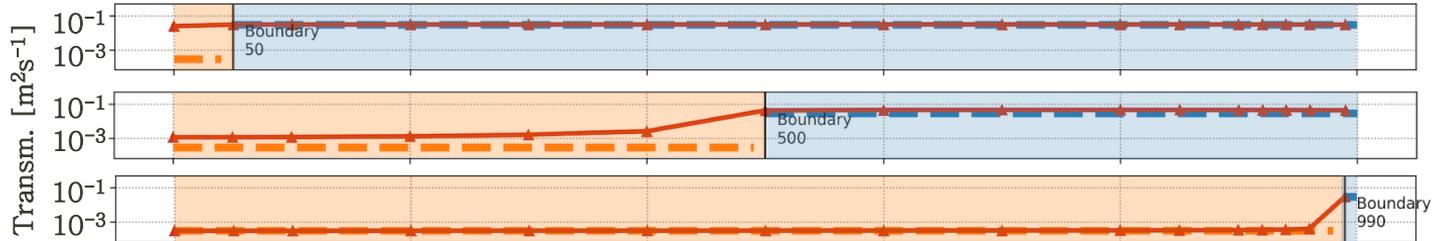


# 3 - Block Domain Results

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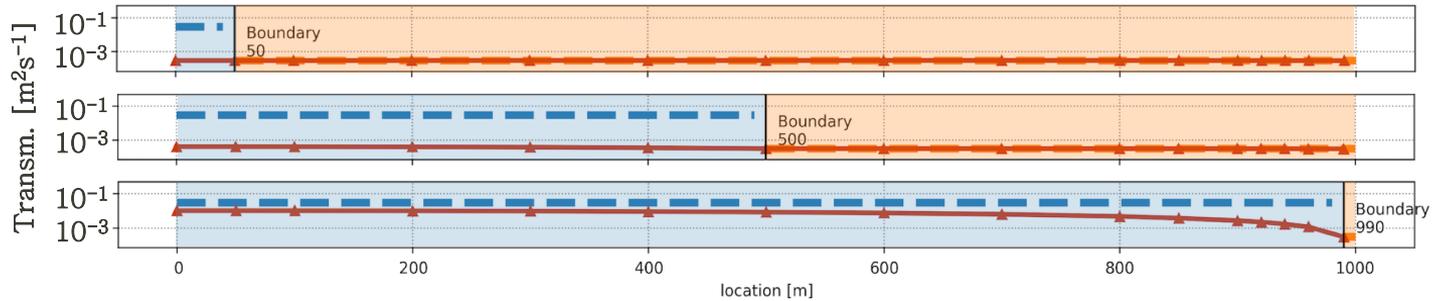
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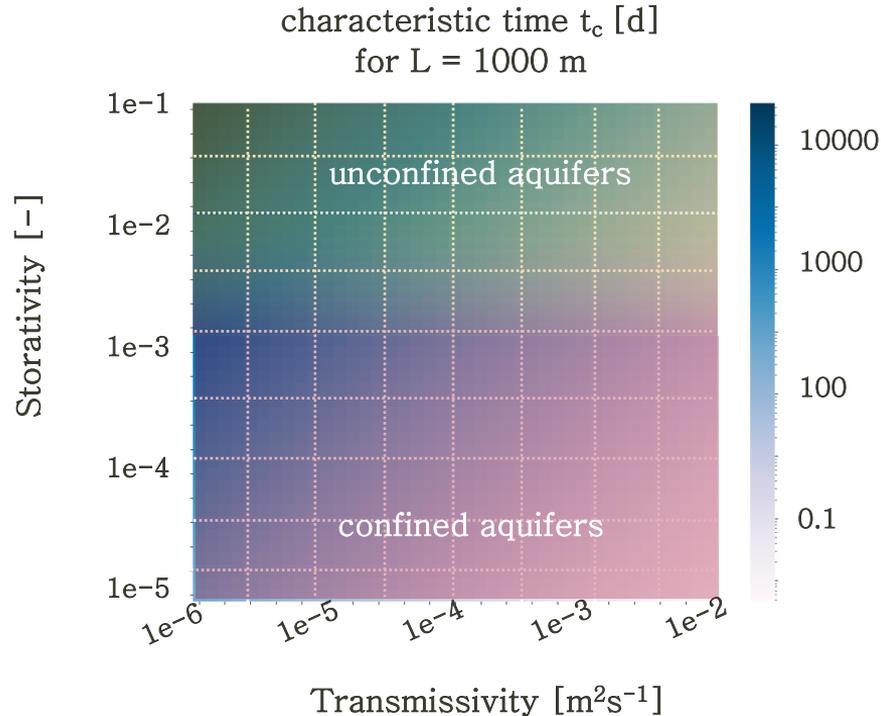
>>> direction of flow >>>

B) T2 → T1  
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- > **High conductivities** at the outlet of the aquifer **slightly** affect lower transmissivities located upgradient.
- > **Low conductivities** at the outlet dominate the entire flow regimes resulting in lower effective transmissivities upgradient.

# Homogeneous Domain Setup



$$t_c = \frac{4 \cdot L^2 \cdot S}{\pi^2 \cdot T}$$

Example 1

$L = 1000$  m

$S = 0.1$

$T = 1\text{e-}3 \text{ m}^2\text{s}^{-1}$

**$t_c = 469$  d**

Example 2

$L = 1000$  m

$S = 0.001$

$T = 1\text{e-}4 \text{ m}^2\text{s}^{-1}$

**$t_c = 46$  d**

## References

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