BHYWI-08: Semester-Fahrplan 2019

Vorlesungen

Datum	V	Thema
05.04.2019	01	IT: GitHub / Qt Installation
12.04.2019	02	Grundlagen: Kontinuumsmechanik
19.04.2019	--	Ostern
26.04.2019	03	Grundlagen: Hydromechanik
03.05.2019	04	Grundlagen: Partielle Differentialgleichungen
10.05.2019	05	Grundlagen: Numerik, Qt Übung: Funktionsrechner
17.05.2019	06	Numerik: Finite Differenzen Methode I (explizit)
24.05.2019	07	Numerik: Finite Differenzen Methode II (implizit)
31.05.2019	08	Gerinnehydraulik: Theorie – Grundlagen
07.06.2019	09	Gerinnehydraulik: Programmierung, Übung
14.06.2019		Pfingsten
21.06.2019	10	Grundwassermodellierung: Catchment Übung
28.06.2019	11	Grundwassermodellierung: Datenbasierte Methoden I
05.07.2019	12	Grundwassermodellierung: Datenbasierte Methoden II
12.07.2019	13	Beleg

TECHNISCHE UNIVERSITÄT DRESDEN

Hydroinformatik II

"Prozesssimulation und Systemanalyse"

BHYWI-08-06 @ 2019

Finite-Differenzen-Methode

Olaf Kolditz

*Helmholtz Centre for Environmental Research – UFZ

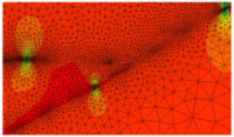
[1]Technische Universität Dresden – TUDD

[2]Centre for Advanced Water Research – CAWR

17.05.2019 - Dresden

Konzept

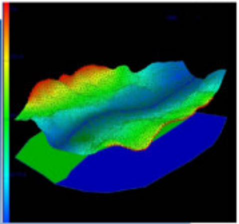
$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v}^E \nabla\psi$$



Basics

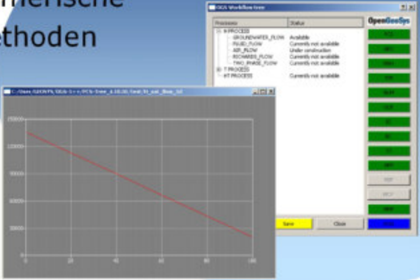
Mechanik

Anwendung



Numerische

Methoden



Programmierung

Visual C++

Prozessverständnis

Fahrplan

Übung

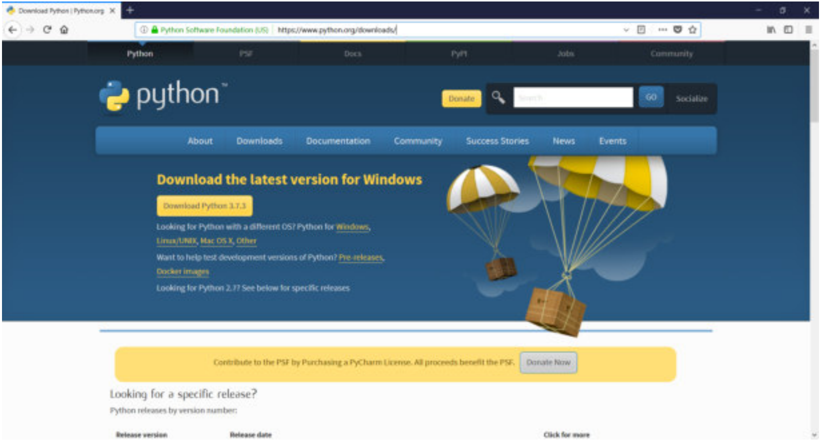
- Python ...

Vorlesung

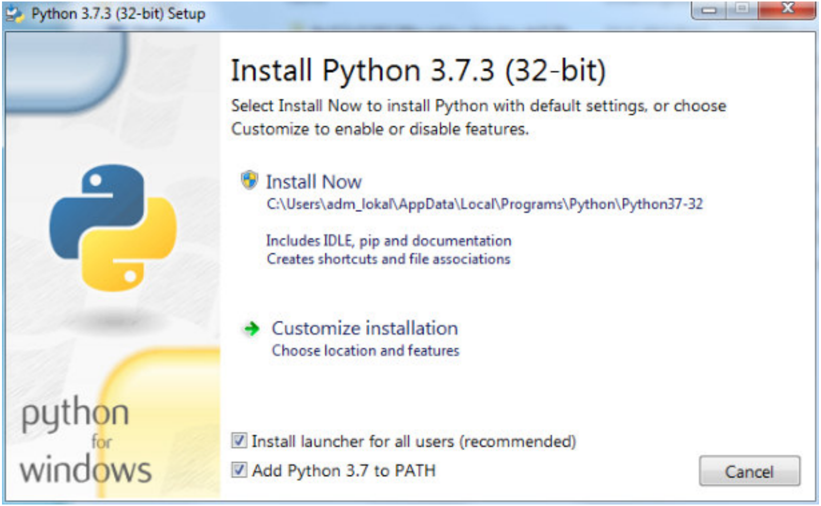
- Grundlagen der Finite Differenzen Methode
- Approximation methods
- Finite difference method – FDM (Ch. 3)
- Taylor series expansion
- Derivatives
- Diffusion equation
- (Finite element method – FEM $\Rightarrow$ Hydrosystemanalyse)

Python, die Zweite (Path)

`https://www.python.org/downloads/`

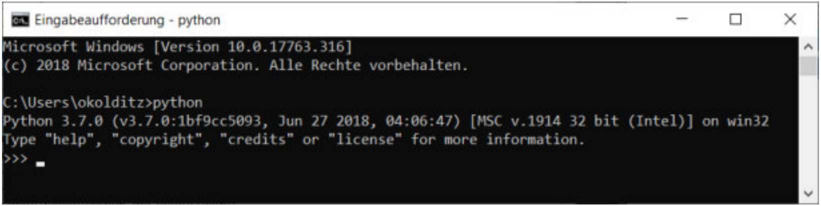


Python, die Zweite (Path)



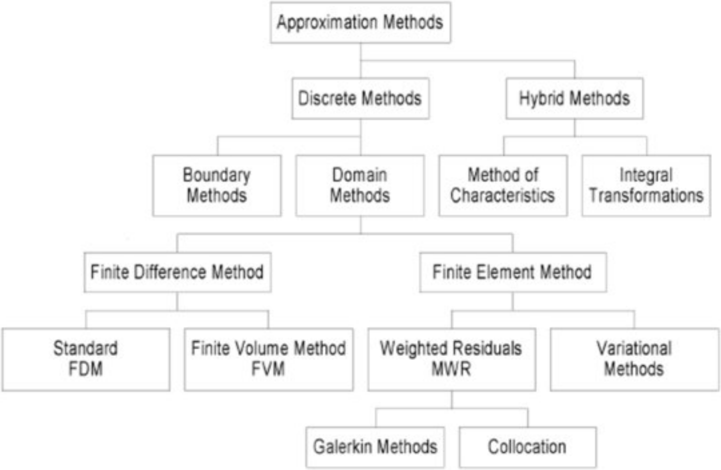
Zwei Optionen:

1. Python deinstallieren, neu installieren und "Add Python to PATH"
2. "PATH" nachträglich ergänzen (unterschiedlich für verschiedene Windows-Versionen), am besten googeln



TECHNISCHE UNIVERSITÄT DRESDEN

Näherungsverfahren



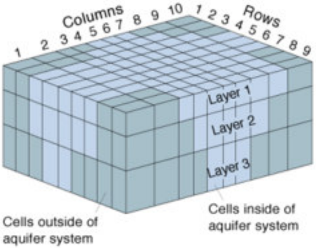


Figure 2. Example of model grid for simulating three-dimensional ground-water flow.

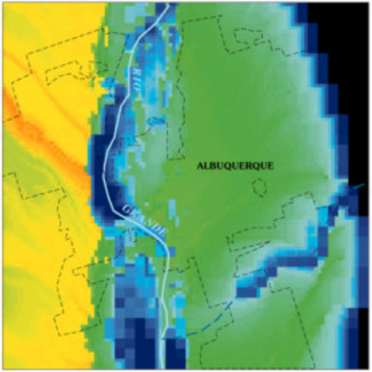
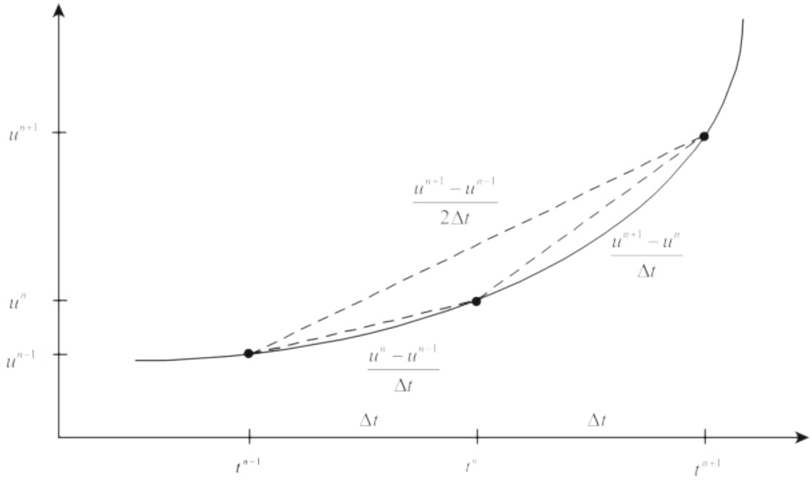


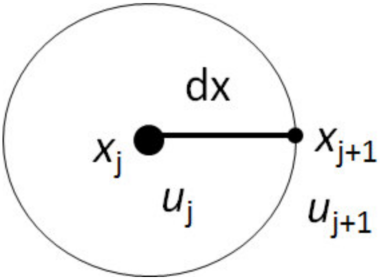
Figure 7. Application of particle tracking to estimate ground-water travel time.

http://water.usgs.gov/pubs/FS/FS-121-97/images/fig7.gif

TECHNISCHE UNIVERSITÄT DRESDEN


u^{n+1}
u^{n}
u^{n-1}
$\frac{u^{n+1}-u^{n-1}}{2\Delta t}$
$\frac{u^{n+1}-u^{n}}{\Delta t}$
$\frac{u^{n}-u^{n-1}}{\Delta t}$
Δt
Δt
t^{n-1}
t^{n}
t^{n+1}

Taylor-Reihe



in time

$$u_j^{n+1} = \sum_{m=0}^{\infty} \frac{\Delta t^m}{m!} \left[\frac{\partial^m u}{\partial t^m} \right]_j^n \quad (1)$$

$$\Delta t = t^{n+1} - t^n$$

in space

$$u_{j+1}^{n} = \sum_{m=0}^{\infty} \frac{\Delta x^m}{m!} \left[\frac{\partial^m u}{\partial x^m} \right]_j^n \quad (2)$$

$$\Delta x = x_{j+1} - x_j$$

Trunkation

$$u_j^{n+1} = u_j^n + \Delta t \left[\frac{\partial u}{\partial t}\right]_j^n + \frac{\Delta t^2}{2}\left[\frac{\partial^2 u}{\partial t^2}\right]_j^n + 0(\Delta t^3) \qquad (3)$$

$$u_{j+1}^{n} = u_j^n + \Delta x \left[\frac{\partial u}{\partial x}\right]_j^n + \frac{\Delta x^2}{2}\left[\frac{\partial^2 u}{\partial x^2}\right]_j^n + 0(\Delta x^3) \qquad (4)$$

1. Ableitung

$$\left[\frac{\partial u}{\partial t}\right]_j^n = \frac{u_j^{n+1} - u_j^n}{\Delta t} - \frac{\Delta t}{2}\left[\frac{\partial^2 u}{\partial t^2}\right]_j^n + 0(\Delta t^2) \qquad (5)$$

$$\left[\frac{\partial u}{\partial x}\right]_j^n = \frac{u_{j+1}^{n} - u_j^n}{\Delta x} - \frac{\Delta x}{2}\left[\frac{\partial^2 u}{\partial x^2}\right]_j^n + 0(\Delta x^2) \qquad (6)$$

Differenzen-Schemata

Forward difference approximation

$$\left[\frac{\partial u}{\partial x}\right]_j^n = \frac{u_{j+1}^n - u_j^n}{\Delta x} + 0(\Delta x) \tag{7}$$

Backward difference approximation

$$\left[\frac{\partial u}{\partial x}\right]_j^n = \frac{u_j^n - u_{j-1}^n}{\Delta x} + 0(\Delta x) \tag{8}$$

Central difference approximation

$$\left[\frac{\partial u}{\partial x}\right]_j^n = \frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} + 0(\Delta x^2) \tag{9}$$

TECHNISCHE UNIVERSITÄT DRESDEN

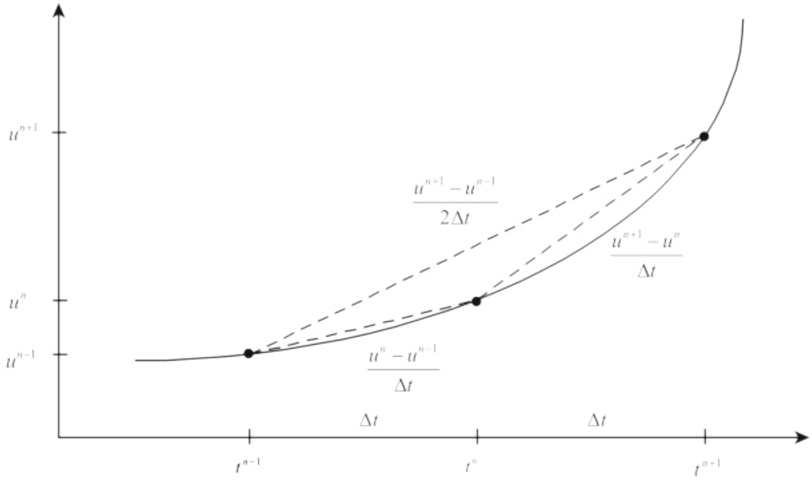
Zentrale Differenzen

$$
\begin{aligned}
u_{j+1}^n &= u_j^n + \Delta x \left[\frac{\partial u}{\partial x}\right]_j^n + \frac{\Delta x^2}{2}\left[\frac{\partial^2 u}{\partial x^2}\right]_j^n + 0(\Delta x^3) \\
u_{j-1}^n &= u_j^n - \Delta x \left[\frac{\partial u}{\partial x}\right]_j^n + \frac{\Delta x^2}{2}\left[\frac{\partial^2 u}{\partial x^2}\right]_j^n - 0(\Delta x^3)
\end{aligned}
\qquad (10)
$$

Central difference approximation

$$
\left[\frac{\partial u}{\partial x}\right]_j^n = \frac{u_{j+1}^n - u_{j-1}^n}{2\Delta x} + 0(\Delta x^2) \qquad (11)
$$

Ableitungen



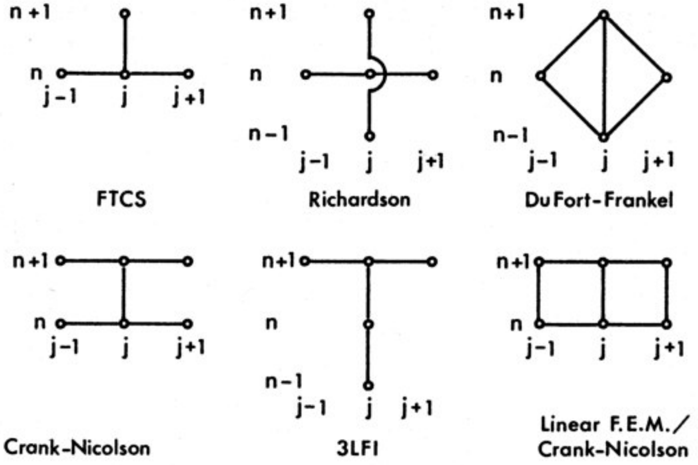
2. Ableitung

$$\left[\frac{\partial^2 u}{\partial x^2}\right]_j^n \approx \frac{1}{\Delta x}\left(\left[\frac{\partial u}{\partial x}\right]_{j+1}^n - \left[\frac{\partial u}{\partial x}\right]_j^n\right)$$

$$\approx \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2} \qquad (12)$$

$$\left[\frac{\partial^2 u}{\partial x^2}\right]_j^n = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2} + \frac{\Delta x^2}{12}\left[\frac{\partial^4 u}{\partial x^4}\right]_j^n + \ldots \qquad (13)$$

Übersicht Differenzenverfahren



TECHNISCHE UNIVERSITÄT DRESDEN

Diffusionsgleichung

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2} = 0 \qquad (14)$$

TECHNISCHE UNIVERSITÄT DRESDEN

Analytical solution for diffusion equation (Skript 5.2.2)

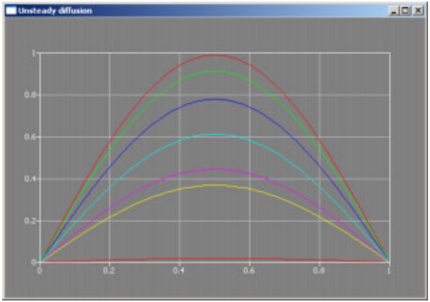
- Diffusion equation

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2} = 0 \qquad (15)$$

- Analytical solution

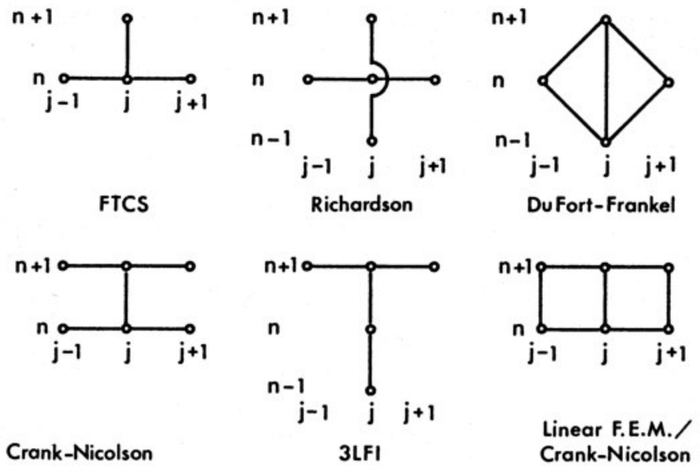
$$u = sin(\pi x) e^{-\alpha t^2} \qquad (16)$$

- K: validity



$\Rightarrow$ Übung

Übersicht Differenzenverfahren



Explizite FDM - FTCS Verfahren (Skript 3.2.2/4.1)

- PDE for diffusion processes

$$\frac{\partial u}{\partial t} - \alpha \frac{\partial^2 u}{\partial x^2} = 0 \tag{17}$$

- forward time / centered space

$$\left[\frac{\partial u}{\partial t}\right]_j^n \approx \frac{u_j^{n+1} - u_j^n}{\Delta t} \quad \left[\frac{\partial^2 u}{\partial x^2}\right]_j^n \approx \frac{u_{j-1}^n - 2u_j^n + u_{j+1}^n}{\Delta x^2} \tag{18}$$

- substitute

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} - \alpha \frac{u_{j-1}^n - 2u_j^n + u_{j+1}^n}{\Delta x^2} = 0 \tag{19}$$

- FTCS scheme for diffusion equations

$$u_j^{n+1} = u_j^n + \frac{\alpha \Delta t}{\Delta x^2}\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right), \; Ne = \frac{\alpha \Delta t}{\Delta x^2} \tag{20}$$

TECHNISCHE UNIVERSITÄT DRESDEN

Analysis of approximation schemes consists of three steps:

- Develop the **algebraic scheme**,
- Check **consistency** of the algebraic approximate equation,
- Investigate **stability** behavior of the scheme.

08.06.2018

Analysis of approximation schemes consists of three steps:

- Develop the **algebraic scheme**,

$$u_j^{n+1} = u_j^n + \frac{\alpha \Delta t}{\Delta x^2}(u_{j-1}^n - 2u_j^n + u_{j+1}^n) \qquad (21)$$

- Check **consistency** of the algebraic approximate equation,

$$\lim_{\Delta t, \Delta x \to 0} |\hat{L}(u_j^n) - L(u[t_n, x_j])| = 0 \qquad (22)$$

- Investigate **stability** behavior of the scheme.

$$Ne = \frac{\alpha \Delta t}{\Delta x^2} \leq 1/2 \qquad (23)$$

TECHNISCHE UNIVERSITÄT DRESDEN

Lösung des FTCS Schemas

Algebraische Schema

$$u_j^{n+1} = u_j^n + \frac{\alpha \Delta t}{\Delta x^2}(u_{j-1}^n - 2u_j^n + u_{j+1}^n) \tag{24}$$

Resultierendes Gleichungssystem

$$\mathbf{u}^{n+1} = \mathbf{A}\mathbf{u}^n \qquad , \qquad n = 0, 1, 2, ... \tag{25}$$

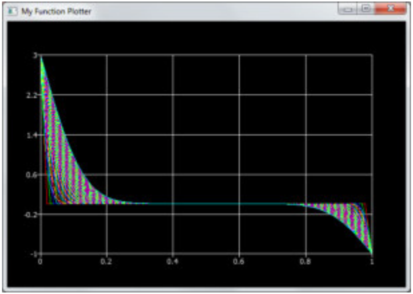
$$\mathbf{A} = \begin{bmatrix} 1 - 2\frac{\alpha \Delta t}{\Delta x^2} & \frac{\alpha \Delta t}{\Delta x^2} & & & \\ \frac{\alpha \Delta t}{\Delta x^2} & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \frac{\alpha \Delta t}{\Delta x^2} \\ & & & \frac{\alpha \Delta t}{\Delta x^2} & 1 - 2\frac{\alpha \Delta t}{\Delta x^2} \end{bmatrix}, \mathbf{u}^n = \begin{bmatrix} u_2^n \\ u_3^n \\ ... \\ u_{np-2}^n \\ u_{np-1}^n \end{bmatrix}$$

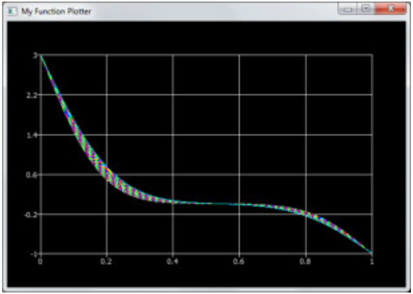
TECHNISCHE UNIVERSITÄT DRESDEN

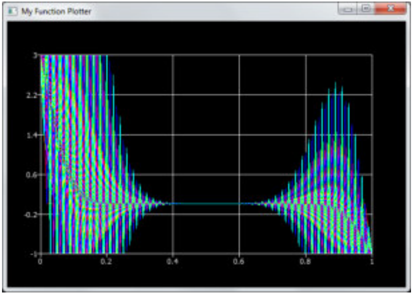
Übung BHYWI-08-03-E

Dialog-Klasse: Konstruktor
`Dialog::Dialog`

1. Elemente
2. Connects
3. Layout
4. Datenstrukturen (Speicherreservierung)





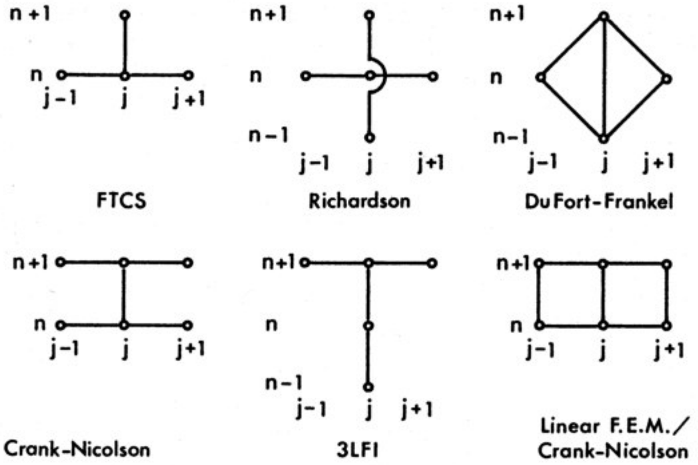


$$Ne = \frac{\alpha \Delta t}{\Delta x^2} \leq 0.5 \qquad (26)$$

How sensitive ?

TECHNISCHE UNIVERSITÄT DRESDEN

Explizite und implizite Differenzenverfahren



Algebraische Schema:

$$\left[\frac{\partial^2 u}{\partial x^2}\right]_j^{n+1} \approx \frac{u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}}{\Delta x^2} \tag{27}$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} - \alpha \frac{u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}}{\Delta x^2} = 0 \tag{28}$$

$$\frac{\alpha \Delta t}{\Delta x^2} \left(-u_{j-1}^{n+1} + 2u_j^{n+1} - u_{j+1}^{n+1}\right) + u_j^{n+1} = u_j^n \tag{29}$$

TECHNISCHE UNIVERSITÄT DRESDEN

Datum	E	Übungen
05.04.2019	00	Git und QT (Lars Bilke)
03.05.2019	01	Qt: Hallo World
10.05.2019 17.05.2019	02	Qt: Funktionsrechner
	03	Qt: Explizite Finite-Differenzen-Methode
	04	Qt: Implizite Finite-Differenzen-Methode
	05	Qt: Gerinnehydraulik I (QAD)
	06	Qt: Gerinnehydraulik II (OOP)
	08	Qt: Gerinnehydraulik IV (interaktiv)
		...

https://github.com/envinf/Hydroinformatik-II

TECHNISCHE UNIVERSITÄT DRESDEN