Hydroinformatik II: Grundlagen der Kontinuumsmechanik

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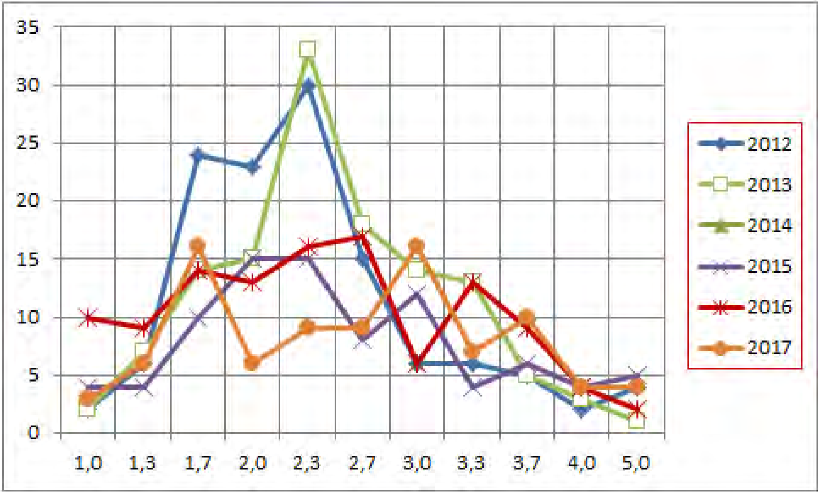
Vorlesungsplan Hydroinformatik II SoSe 2017

#	Datum	Thema
01	07.04.2017	Einführung
02	07.04.2017	Grundlagen: Kontinuumsmechanik
–	14.04.2017	Ostern
03	21.04.2017	Grundlagen: Hydromechanik
04	28.04.2017	HW: Qt Installation (2017)
05	05.05.2017	Grundlagen: Partielle Differentialgleichungen / TEX
06	12.05.2017	Qt Einführung
07	19.05.2017	Qt Übung: Funktionsrechner; Grundlagen Numerik
08	26.05.2017	Numerik: (exp) Finite Differenzen Methode
09	02.06.2017	Numerik: (imp) Finite Differenzen Methode
–	09.06.2017	Pfingsten
10	16.06.2017	Gerinnehydraulik: Theorie - Grundlagen
11	23.06.2017	HW: Gerinnehydraulik: Programmierung, Übung 1
12	30.06.2017	Gerinnehydraulik: Programmierung, Übung 2
13	07.07.2017	Einführung - Grundwassermodellierung (MW)
14	14.07.2017	Kurs-Zusammenfassung, Ausblick und Beleg

Klausur Hydroinformatik I: 2012 > 2017

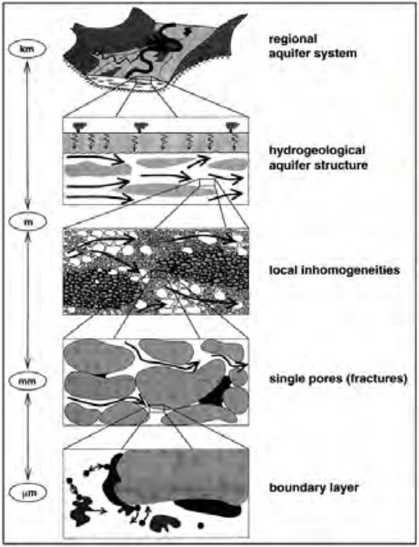


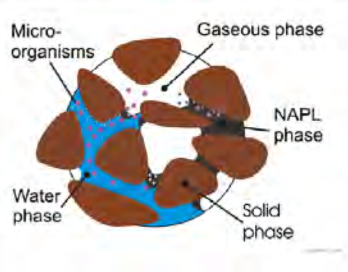
Fahrplan für heute ...

- Motivation
- Lagrange Konzept
- Euler Konzept
- Reynolds Transport Theorem
- Fluxes
- Bilanzgleichungen
- Erhaltungsgrößen

$\Longrightarrow$ viel Theorie - vor allem die mathematische Schreibweise verstehen "zu lesen"

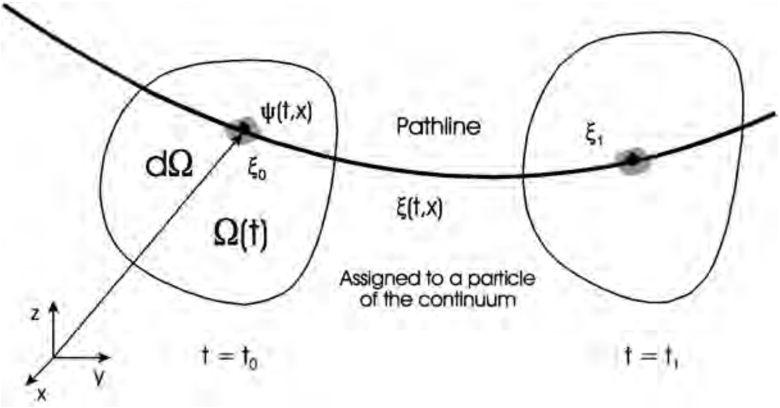
Skalen



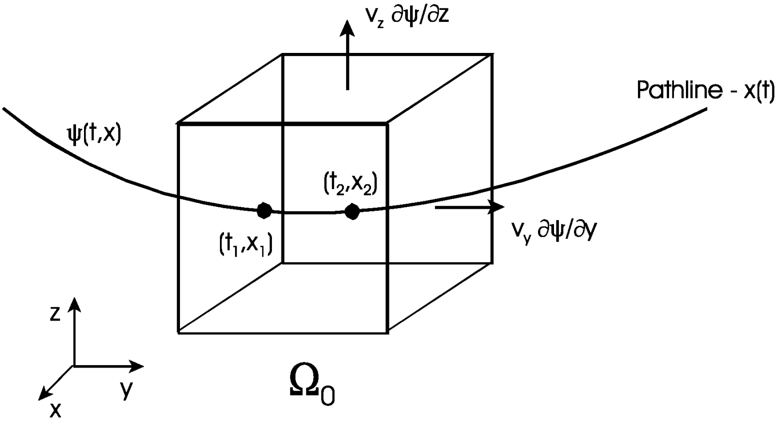


Quellen: Kobus et al. (1995), Kolditz (2002)

Lagrange Konzept (1.1.1)

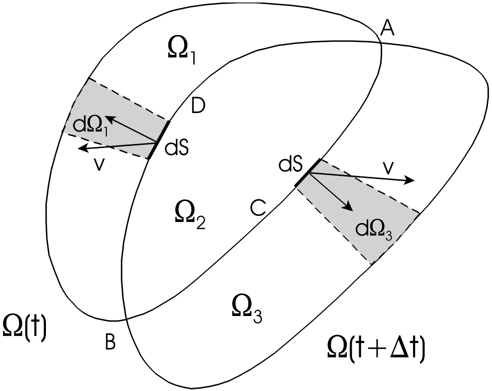


Euler Konzept (1.1.1)

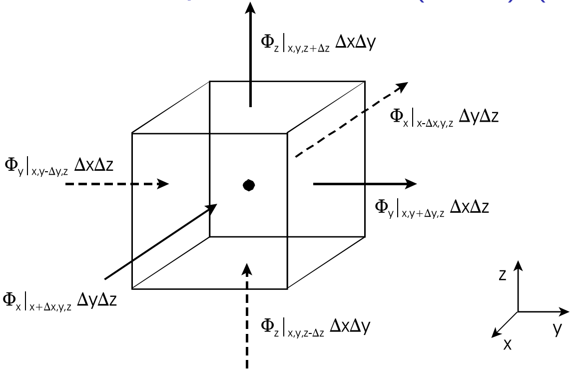


Reynolds Transport Theorem (Lagrange) (1.1.3)

$$\frac{d}{dt}\int\limits_{\Omega} \psi d\Omega = \int\limits_{\Omega} \frac{\partial \psi}{\partial t} d\Omega + \oint\limits_{\partial\Omega} \psi(t)\mathbf{v} \cdot d\mathbf{S} = \int\limits_{\Omega} q^{\psi} d\Omega \qquad (1)$$



Reynolds Transport Theorem (Euler) (1.1.3)



$$\oint_{\partial\Omega} \mathbf{\Phi}^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \mathbf{\Phi}^{\psi} d\Omega \quad (2)$$

$$\nabla \cdot \mathbf{\Phi} = \lim_{\Omega \to 0} \frac{1}{\Omega} \oint_{\partial\Omega} \mathbf{\Phi} \cdot d\mathbf{S} \quad (3)$$

Reynolds Transport Theorem (Euler) (1.1.3)

$$\underbrace{\frac{\partial}{\partial t}\int\limits_{\Omega}\psi d\Omega}_{1} = \underbrace{-\oint\limits_{\partial\Omega}\mathbf{\Phi}^{\psi}\cdot d\mathbf{S}}_{2} + \underbrace{\int\limits_{\Omega} q^{\psi} d\Omega}_{3} \tag{4}$$

with:

1. Rate of change of total amount of quantity ψ in the control volume,
2. Net rate of increase / decrease of ψ due to fluxes,
3. Rate of increase / decrease of ψ due to sources.

Reynolds Transport Theorem (Euler) (1.1.3)

$$\underbrace{\frac{\partial}{\partial t}\int\limits_{\Omega} \psi d\Omega}_{1} = \underbrace{-\oint\limits_{\partial\Omega} \mathbf{\Phi}^{\psi} \cdot d\mathbf{S}}_{2} + \underbrace{\int\limits_{\Omega} q^{\psi} d\Omega}_{3} \tag{5}$$

using

$$\oint\limits_{\partial\Omega} \mathbf{\Phi}^{\psi} \cdot d\mathbf{S} = \int\limits_{\Omega} \nabla \cdot \mathbf{\Phi}^{\psi} d\Omega \tag{6}$$

$$\int\limits_{\Omega} \frac{\partial \psi}{\partial t} d\Omega = -\int\limits_{\Omega} \nabla \cdot \mathbf{\Phi}^{\psi} d\Omega + \int\limits_{\Omega} q^{\psi} d\Omega \tag{7}$$

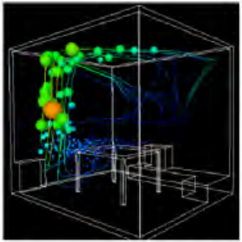
Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega} \psi \, d\Omega = \int\limits_{\Omega} \left(\frac{\partial \psi}{\partial t} + \nabla \cdot \mathbf{\Phi}^{\psi} \right) d\Omega = \int\limits_{\Omega} q^{\psi} \, d\Omega$$

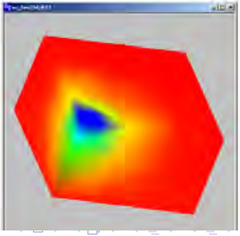
$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^{E} \nabla \psi$$

Lagrange

Euler



http://www.cscs.ch/~mvalle/Libro/



Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \frac{\partial}{\partial t}\int\limits_{\Omega}\psi d\Omega + \oint\limits_{\partial\Omega}\nabla\cdot\mathbf{\Phi}\partial\Omega = \int\limits_{\Omega} q^{\psi} d\Omega \qquad (8)$$

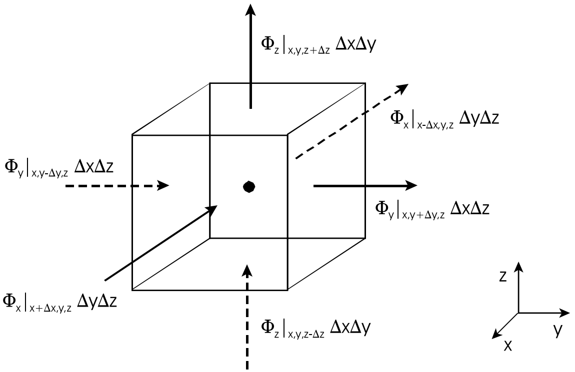
$$\frac{\partial}{\partial t}\int\limits_{\Omega}\psi d\Omega = \int\limits_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \nabla\cdot\oint\limits_{\partial\Omega}\mathbf{\Phi}\partial\Omega = \int\limits_{\Omega} q^{\psi} d\Omega \qquad (9)$$

$$\forall\Omega : \frac{d\psi}{dt} = \frac{\partial\psi}{dt} + \nabla\cdot\mathbf{\Phi} = q^{\psi} \qquad (10)$$

Fluxes (1.1.6)

The total flux $\mathbf{\Phi}^\psi$ of a quantity ψ is defined as

$$\mathbf{\Phi}^\psi = \mathbf{v}^E \psi \qquad (11)$$



Fluxes (1.1.6)

$$\mathbf{\Phi}^{\psi} = \mathbf{v}^E \psi = \underbrace{\mathbf{v}\psi}_{\mathbf{\Phi}_A^{\psi}} + \underbrace{(\mathbf{v}^E - \mathbf{v})\psi}_{\mathbf{\Phi}_D^{\psi}} \quad (12)$$

and, therefore, decomposed into two parts: an advective flux $\mathbf{\Phi}_A^{\psi}$ and a diffusive flux $\mathbf{\Phi}_D^{\psi}$ relative to the mass-weighted velocity:

- advective flux of quantity ψ

$$\mathbf{\Phi}_A^{\psi} = \mathbf{v}\psi \quad (13)$$

- diffusive flux of quantity ψ (Fick's law)

$$\mathbf{\Phi}_D^{\psi} = -\alpha \nabla \psi \quad (14)$$

General Balance Equation (1.1.7)

- Integral form

$$\int_\Omega \frac{d\psi}{\partial t} = \int_\Omega \frac{\partial\psi}{\partial t} + \int_\Omega \nabla \cdot (\mathbf{v}\psi) - \int_\Omega \nabla \cdot (\mathbf{D}^\psi \nabla\psi) = \int_{\mathbf{\Omega}} \mathbf{Q}^\psi \quad (15)$$

- Differential form

$$\frac{d\psi}{\partial t} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}\psi) - \nabla \cdot (\mathbf{D}^\psi \nabla\psi) = \mathbf{Q}^\psi \quad (16)$$

Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume Ω is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{17}$$

where Ψ is an extensive conservation quantity (i.e. mass, momentum, energy) and ψ is the corresponding intensive conservation quantity such as mass density ρ, momentum density $\rho\mathbf{v}$ or energy density e.

Extensive quantity	Symbol	Intensive quantity	Symbol
Mass	M	Mass density	ρ
Linear momentum	$\mathbf{m}$	Linear momentum density	$\rho\mathbf{v}$
Energy	E	Energy density	$e = \rho i + \frac{1}{2}\rho v^2$