Deformation Processes in Porous Media

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The mechanical part of coupled THM processes in porous media is closely associated with the deformation of the solid phase, and the interaction of deformation and flow processes.

Motion of a solid body

- Rigid body motion (translation or rotation of the body without changing its volume or shape)
- Deformation (local relative change of lengths and/or angles referred to neighboring particles)

Interaction of mechanical and hydraulic processes

- Effects on the stress state within the solid phase due to pore pressure evolution
- Variations of the pore size distribution due to the deformation of the solid skeleton

Constitutive models

- Material-independent balance relations do not represent a mathematically closed system of equations to solve initial-boundary value problems of mechanics
- Observation:
  Different mechanical response of individual materials to the impact of external forces and temperature
- Mechanical constitutive models:
  Material-dependent relations between measures of deformation (*strains*) and internal force densities (*stresses*)

Theoretical framework for the analysis of deformation processes

- Micromechanics
- Continuum mechanics

Modeling of deformation processes in porous media

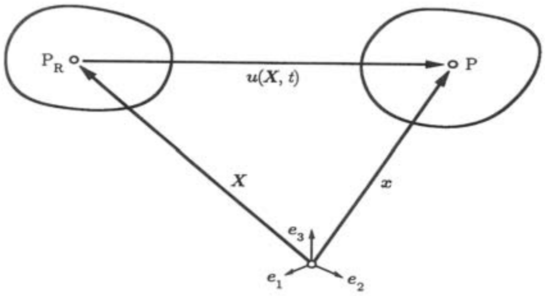
- Porous media distinguish themselves by a sophisticated complex microstructure
- Realistic modeling and numerical simulation not efficient
- Continuum mechanics provides the preferred constitutive models for porous media
  - Models are not based on a physical characterization of the real microstructure
  - Effects of the microstructure on the physical behavior is considered in a phenomenological, macroscopic manner
  - Constitutive relations are not laws of nature
  - Mathematical models based on physically reasonable assumptions
  - Characterize the typical material behavior

Kinematics analyzes the state and the evolution of geometry of motion in general, and of deformation processes in particular.

Assumptions for kinematic models

- Material body $\mathcal{B}$ (set of material points $\mathcal{P}$) at each time t can be uniquely defined with certain parts of space
- Considering the image of the material body in the Euclidean space of physical observations, the location of material points can be identified with the position vector $\mathbf{x}(t)$
- Unique modeling of motion requires a reference state
- Domain in space occupied by the material body at time t_0 as reference domain (position vector $\mathbf{X}$)
- Reference domain in porous media:
  Appropriately chosen initial state of solid skeleton

Definition of the displacement vector



Euclid of Alexandria
(323 BC-283 BC)

Deformation vector

$$\mathbf{u} = \mathbf{x} - \mathbf{X}$$

Deformation field

$$\mathbf{u}(\mathbf{X}, t) = \bar{\mathbf{u}}(\mathbf{x}, t)$$

- Displacement vector connects the current position $\mathbf{x}$ of a material point, which has been moved under the impact of external forces, with its location $\mathbf{X}$ at time t_0
- Displacements of the solid phase of porous media as primary variable in THM processes

For the sake of comparability of deformation processes it is reasonable to introduce relative physical variables.

Strain measure in case of small deformations

- Displacement gradient

$$\nabla \bar{\mathbf{u}}(\mathbf{x}, t) = \frac{\partial u_i}{\partial x_j} \mathbf{e}_i \otimes \mathbf{e}_j$$

- Strain tensor – symbolic representation

$$\varepsilon(\mathbf{x}, t) = \frac{1}{2}\left(\nabla \bar{\mathbf{u}}(\mathbf{x}, t) + (\nabla \bar{\mathbf{u}}(\mathbf{x}, t))^{\mathrm{T}}\right)$$

- Matrix of the coefficients of the strain tensor

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

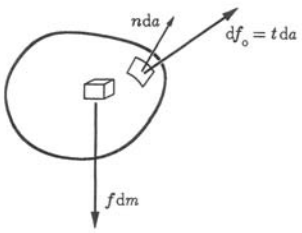
External forces, acting on a material body, represent the mechanical effect of the surroundings.

Resultant force, acting on a material body

$$\mathcal{F} = \int_{\partial \mathcal{B}} \mathbf{t} \, da + \int_{\mathcal{B}} \mathbf{f} \, dm = \int_{\Gamma} \mathbf{t}(\mathbf{x}, t, \mathbf{n}) \, d\Gamma + \int_{\Omega} \mathbf{f}(\mathbf{x}, t) \, \varrho(\mathbf{x}, t) \, d\Omega$$

- Surface forces with the surface force density $\mathbf{t}$ (traction)
- Volume forces with the volume force density $\mathbf{f}$
- Assumption for porous media:
  Only gravity $\varrho \mathbf{g}$ should be considered as specific volume force

Definition of the Cauchy stress tensor



Differential surface force: $d\mathbf{f}_0 = \mathbf{t} \, d\Gamma$

Differential surface element: $d\mathbf{\Gamma} = \mathbf{n} \, d\Gamma$

A. L.Cauchy (1789-1857)

Cauchy's theorem

$$\mathbf{t}(\mathbf{x}, t, \mathbf{n}) = \boldsymbol{\sigma}(\mathbf{x}, t) \, \mathbf{n}$$

Cauchy stress tensor

$$\boldsymbol{\sigma} = \sigma_{ij} \, \mathbf{e}_i \otimes \mathbf{e}_j$$

- Traction vector is considered to be a function of the location, time and of the normal vector (orientation of surface element): $\mathbf{t}(\mathbf{x}, t, \mathbf{n})$
- Assumption (Cauchy's theorem):
  Linear relation between the traction and the normal vector arranged by the stress measure $\boldsymbol{\sigma}(\mathbf{x}, t)$

The total Cauchy stress tensor in porous media is decomposed in partial stresses referring to the participating phases.

$$\boldsymbol{\sigma} = (1-n)\,\boldsymbol{\sigma}^s - n\left(\sum_\gamma S^\gamma\, p^\gamma\right)\mathbf{I}$$

Attention:
Note the sign convention of positive fluid phase pressure p^γ, but negative compressive normal stress for the solid phase!

Effective solid stress

Modification of the stress representation

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_E^s - \left(\sum_\gamma S^\gamma\, p^\gamma\right)\mathbf{I} \quad \text{with} \quad \boldsymbol{\sigma}_E^s = (1-n)\left[\boldsymbol{\sigma}^s + \left(\sum_\gamma S^\gamma\, p^\gamma\right)\mathbf{I}\right]$$

Effective solid stress:
Total solid stress reduced by the excess pore liquid pressure, but referred to the domain of the overall porous medium

Constitutive models for solid skeleton

$$\boldsymbol{\sigma}_E^s = (1-n)\left[\boldsymbol{\sigma}^s + \left(\sum_\gamma S^\gamma\, p^\gamma\right)\mathbf{I}\right]$$

- Constitutive relations for the solid phase of porous media combine the solid skeleton strain with the effective solid stress
- Equilibrium condition for the porous medium

$$\varrho\,\mathbf{g} + \nabla\cdot\boldsymbol{\sigma}_E^s - \nabla\left(\sum_\gamma S^\gamma\, p^\gamma\right) = \mathbf{0}$$

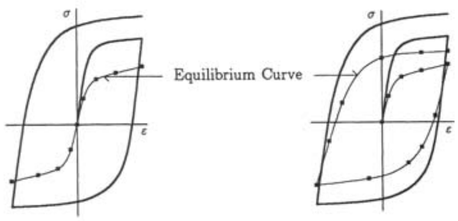
- Constitutive relations represent idealized and simplified models according to the most dominating phenomena observed in practical applications under consideration

Experimental observations

- Performing lab tests to investigate the mechanical behavior of various materials diverse experimental observations can be made regarding stress-strain curves
- Similar stress-strain curves can be caused by different physical effects, e.g. a nonlinear stress-strain curve does not necessarily suggest inelastic material behavior
- For the sake of clarity, it is useful to introduce a classification of materials based on some essential distinctly identifiable material phenomena
- Comparatively simple experiments can be performed to investigate if the stress-strain curves are rate-dependent, and if hysteresis phenomena occur

Generalized material classes

Experimentally observed rate-independent solid material behavior. Cyclic uniaxial stress-strain curves: without hysteresis (left), with hysteresis (right)



Experimentally observed rate-dependent solid material behavior. Cyclic uniaxial stress-strain curves: without hysteresis (left), with hysteresis (right)

Material classes – Mathematical models

According to the experimental observations, there are four classes of mathematical models matching the material classes defined above:

- Theory of elasticity describes rate-independent material behavior without hysteresis
- Theory of (elasto)plasticity describes rate-independent material behavior with hysteresis
- Theory of viscoelasticity describes rate-dependent material behavior without hysteresis
- Theory of viscoplasticity describes rate-dependent material behavior with hysteresis

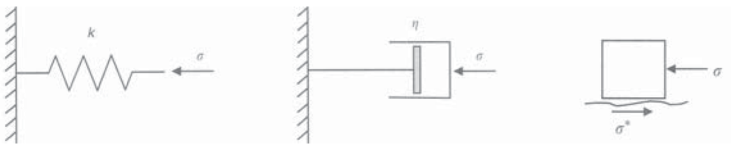
Classification of material behavior depends on real loading regime (e.g. small or large strains), environmental conditions (e.g. temperature), and the time scale of the physical processes under consideration

Deformation Processes
Mechanical Properties
Material Classes
Elasticity
Elastoplasticity
Viscoelasticity
Viscoplasticity

Material classes – Physico-mathematical substitute models

Physically significant constitutive relations in the uniaxial case can be defined for material classes based on so-called rheological models (simple networks of individual rheological elements)

Individual rheological elements



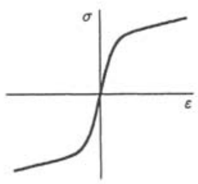
Elastic spring — Viscous dashpot — Coulomb friction element

$$\sigma = k\,\varepsilon$$

$$\sigma = \eta\,\dot{\varepsilon}$$

$$\varepsilon = \begin{cases} 0, & \text{if} \quad \sigma < \sigma^\star \\ \varepsilon(t), & \text{if} \quad \sigma \geq \sigma^\star \end{cases}$$

Theory of elasticity



Generalized uniaxial stress-strain curve

Rheological substitute model
(Spring element)

Material Class	Technical/Natural Material	Geomaterial
elasticity	metals at small strains, ceramics, bone, most other materials at small strains	igneous rocks (e.g. granite), hard sedimentary rocks (e.g. sandstone)

General remarks

- Micromechanically, elasticity is predominantly caused by the evolution of interatomic forces in response to the impact of external forces
- Observed for crystalline substances (atoms are established in regular structures) as well as for amorphous materials (atoms compose irregular structures)
- Characterized by reversibility of the deformation processes
- Absence of any hysteresis
- Current stress state does not depend on the strain history
  - ⇒ Stress tensor represents a function of the strain tensor, and does not depend on the strain rate

Specific constitutive relations

Robert Hooke (1635-1703)

Isothermal isotropic linear elastic material model (Hooke's law)

$$\boldsymbol{\sigma} \; = \; 2\mu\, \boldsymbol{\varepsilon} \; + \; \lambda\, \mathrm{tr}(\boldsymbol{\varepsilon})\, \mathbf{I}$$

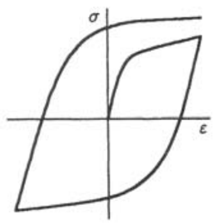
Material parameters: Lamé constants μ and λ, alternatively Young's modulus E and Poisson's ratio ν

$$E = \mu\,\frac{2\mu + 3\lambda}{\mu + \lambda}, \qquad \nu = \frac{\lambda}{2(\mu + \lambda)}$$

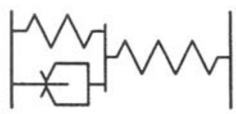
Coefficients of the consistent material matrix (numerics)

$$\mathbb{C}_{ijkl} \; = \; \frac{d\sigma_{ij}}{d\varepsilon_{kl}} \; = \; 2\mu\, \delta_{ik}\, \delta_{jl} \; + \; \lambda\, \delta_{ij}\, \delta_{kl}$$

Theory of (elasto)plasticity



Generalized uniaxial stress-strain curve



Rheological substitute model
(Spring and frictional elements)

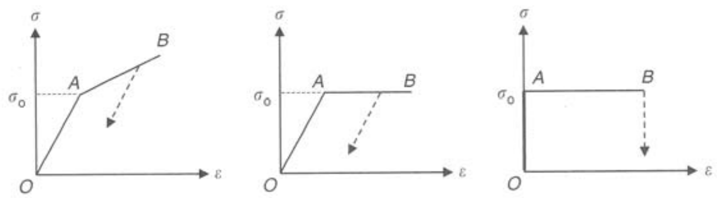
Material Class	Technical/Natural Material	Geomaterial
elastoplasticity	metals at large strains	most soils, soft sedimentary rocks (e.g. tuff)

General remarks

- Elastic material response until a critical stress (yield stress) is reached, and plastic flow occurs
- Initial elastic behavior at each change of load direction
- Micromechanically, plastic yielding is associated with motion of defects in atomic structure (dislocations, discontinuities)
- Typical for crystalline substances
- Irreversible deformation processes
- Strain energy dissipation during plastic flow (hysteresis)
- No explicit stress strain relation due to hysteresis effects
  - $\Rightarrow$ Stress rate tensor represents a function of the elastic strain rate tensor

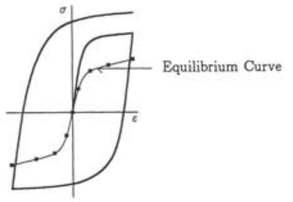
Special cases of (elasto)plasticity

- Elastoplastic material behavior with hardening effects
- Elastic-perfectly plastic material behavior
- Rigid-perfectly plastic response

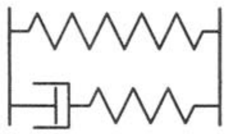


Schematic representation of material behavior exhibiting plastic yielding: elastic-plastic with strain hardening (left), elastic-perfectly plastic (middle), and rigid-perfectly plastic (right)

Theory of viscoelasticity



Generalized uniaxial stress-strain curve



Rheological substitute model
(Spring and dashpot elements)

Material Class	Technical/Natural Material	Geomaterial
viscoelasticity	rubber, glass, soft biological tissues	rock salt (halite)

General remarks

- Micromechanically, viscoelasticity is characterized by elastic behavior of individual macromolecular chains, and internal friction of macromolecular networks
- Typical for amorphous substances, particularly polymers
- Distinctive rate-dependent mechanical properties
- Strain energy dissipation due to internal friction (hysteresis)
- Equilibrium states without hystereses at sufficiently small loading rates (asymptotic elastic properties)
  - Stress relaxation at constant strain
  - Strain retardation (creep) at constant stress
- Possible to reestablish shape after mechanical loading due to heat supply and/or recovery period

Viscoelastic Maxwell model

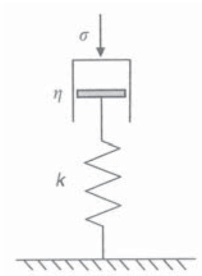
- Series connection of the spring and the dashpot
- Equal stress values in both individual elements
- Total strain as sum of the partial strains in the elements
- Constitutive model (differential equation)

$$\dot{\varepsilon} = \frac{1}{\eta}\sigma + \frac{1}{k}\dot{\sigma}$$

- Response to an instantaneous stress jump:
  Instantaneous elastic and long-term viscous response
- Response to an instantaneous strain jump ε_0:
  Stress decrease (relaxation) to zero value
  (similar to viscoelastic fluids)

$$\sigma = k\,\varepsilon_0\,\mathrm{e}^{-kt/\eta}$$

J.C. Maxwell
(1831-1879)



Viscoelastic Kelvin-Voigt model

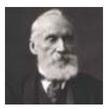
- Parallel connection of the spring and the dashpot
- Total stress value as sum of the stresses in spring and dashpot
- Equal strain in both individual elements
- Constitutive model (differential equation)

$$\sigma = k\,\varepsilon + \eta\,\dot{\varepsilon}$$

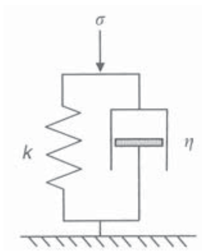
- Response to an instantaneous stress jump σ_0:
  Strain increases asymptotically to elastic state

$$\varepsilon = \frac{\sigma_0}{k}\left[1 - \mathrm{e}^{-kt/\eta}\right]$$

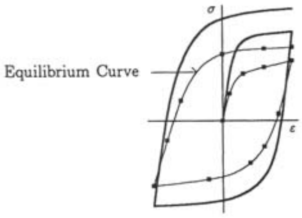
$\Rightarrow$ Typical strain retardation (viscoelastic creep) neglecting any instantaneous strain



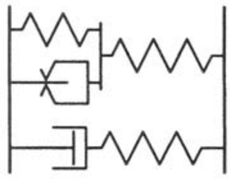
Lord Kelvin
(1824-1907)



Theory of viscoplasticity



Generalized uniaxial stress-strain curve



Rheological substitute model
(Spring, dashpot, and
frictional elements)

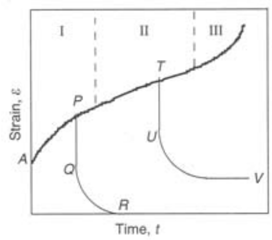
Material Class	Technical/Natural Material	Geomaterial
viscoplasticity	polymers (plastics), wood, bitumen, metals at high temperature	clay soils, clay stone

Deformation Processes
Mechanical Properties
Material Classes
Elasticity
Elastoplasticity
Viscoelasticity
Viscoplasticity

General remarks

- Most general material class
- Combines elements of all other classes
- Micromechanical phenomena are exceptionally complex
- Typical effect: Viscoplastic creep (three typical periods without reaching any asymptotical value)

- Primary (transient) creep:
  decreasing creep rate
- Secondary (stationary) creep:
  constant strain rate
- Tertiary creep causing structural failure:
  ever-increasing strain rate



Typical viscoplastic creep curve
Point A: Instantaneous elastic strain

Further aspects of the mechanics of geomaterials

- Separation of rate-dependent deformation processes (pore pressure evolution, intrinsic solid viscous effects)
- Anisotropy (dependency on direction) due to layered structure (e.g. shale, sandstone)
- Damage and failure of rocks
- Analysis of wave propagation (dynamic phenomena)
- Design of lab tests for fundamental characterization of material behavior, and calibration of constitutive models