UFZ-Discussion Papers

Department of
Ecological Economics and
Environmental Sociology (OEKUS)

2/1999

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Bernd Klauer

1999

Bernd Klauer
UFZ Centre for Environmental Research
PO–Box 500136
D–04301 Leipzig
Germany

e-mail: klauer@alok.ufz.de
fon: +49 341 235–2204
fax: +49 341 235–2825

Pricing in Ecosystems:
A Generalized Linear Production Model

by Bernd Klauer[*]

Abstract

In this paper a method is developed to derive prices for natural goods from information about material and energy flows within ecosystems. The derivation is based on an analogy between ecological and economic systems: both systems are characterized by flows of material and energy. To derive ecosystem prices the mathematical structure of Koopmans' economic linear production model—his activity analysis—is applied to a material flow model of ecosystems. The ecological interpretation of these prices is discussed and the uniqueness of the price system is investigated. An algorithm for price calculation is derived and demonstrated with a numerical example. Finally, it is discussed whether ecosystem prices may be suitable as surrogates for economic valuations of natural goods.

JEL–classification: Q2

Keywords: Price theory, evaluation of natural goods, activity analysis, general equilibrium theory, ecosystems, material flows, energy flows

1 Introduction

Valuing natural goods is one of the major problems of ecological economics. According to economic theory, values of goods are determined by individuals' preferences, and these preferences are in turn revealed by their economic decisions on markets. However, people cannot be expected to analyze the behavior of ecosystems when making economic decisions. Consequently, the preferences of individuals doubtless do not reflect everything scientists have found out about the functioning of ecosystems. Nevertheless, it would be desirable for scientific knowledge to be integrated into the economic valuation of natural goods. In this paper I discuss a way of doing this. The idea is to derive prices for natural goods from information about the material and energy flows within ecosystems. This produces surrogates for economic prices which I will call "ecosystem prices".

[*] UFZ Centre for Environmental Research Leipzig-Halle, Permoserstr. 15, D–04318 Leipzig, Germany. The author gratefully acknowledges critical and helpful comments by Malte Faber, Manuel Frondel, and Frank Jöst.

The procedure for deriving ecosystem prices described below is based on an analogy between ecological and economic systems. In contrast to economies, prices cannot be perceived in ecosystems. We will develop an ecosystem model that takes the mathematical structure of economic price theory and applies it to ecosystems. It is crucial that the mathematical structure of the economic model can be ecologically interpreted in a plausible manner.

Ecological and economic systems have several structural similarities: Both systems are characterized by the relations and interactions of living beings. These relations and interactions are expressed in flows of material and energy. Ecologists use studies of the material and energy flows as important building blocks to understand ecosystems. In economics the French physiocrat QUESNAY (1694–1774) coined the still popular picture of the economic process as two dual circles of commodities and money (see e.g. SAMUELSON 1964). Hence both economic ecological systems may be characterized by flows of material and energy. It makes sense to use this structural similarity for the derivation of prices in ecosystems.

However, ecosystems are also distinguished from economies in many respects. In contrast to economies, for instance, we typically observe in ecosystems not voluntary exchange but material and energy flows caused by forced giving, eating-and-being-eaten, as well as physical laws of nature. This difference between economic and ecological systems impedes the search for an economic price model suitable for a plausible ecological interpretation since the concept of exchange is frequently central to economic price theories. Nevertheless, there are also economic price theories which are not based on exchange but on the duality of quantities and values. So far there have been two approaches in the literature for the derivation of prices in ecosystems founded on this duality: HANNON (1985) used LEONTIEF's input–output analysis and AMIR (1975, 1987, 1989, 1994, 1995) based his ecosystem model on a generalized linear production model (as e.g. VON NEUMAN 1945; KOOPMANS 1951; MALINVAUD 1953).

Motivated by the Non-substitution Theorem (SAMUELSON 1951; cf. HANNON 1995: 332), HANNON (1985) showed that prices can be derived in an ecosystem if an equilibrium of the sectoral balance is presupposed, i.e. the value of outputs in each sector equals the value of inputs. He also developed a dynamic version of the model and calculated ecosystem prices from empirical ecosystem data. However, the model is unsatisfactory because it is assumed that there is only one non-produced input[1], whereas many ecosystems depend not only on the import of sunlight but also on the import of rainwater or certain nutrients not produced by the ecosystem. Moreover, in HANNON's model it is assumed that each component of the ecosystem produces only one single output. This means for instance that if "plants" are taken as an ecosystem component in a model, it would not be possible to differentiate between the outputs "wood", "dead plant material", "fruits" etc.

Compared to an input-output approach, a generalized linear production model as is used both by AMIR and in this paper has the advantages that each component may have several outputs and that the system may have several non-produced inputs. However, AMIR's studies (1975, 1987, 1989, 1994, 1995) are not satisfactory, either, since they give no hints of:

1. What objective appropriately describes the behavior of an ecosystem or how whether a certain objective function appropriately describes the behavior can be verified;
2. What data are needed for the price calculation and how calculation should be performed;
3. How the prices can be numerically calculated from empirical data.

In this paper we will develop a third model for ecosystem prices which does not have the disadvantages mentioned of HANNON's and AMIR's appoaches. Our aim is to critically assess the suitability of ecosystem prices as surrogates for economic valuations of natural goods. The paper is structured as follows: In the next section we will first explain the basic structure of the model and in particular how the mathematical structure can be calculated ecologically. Then we will use a result of KOOPMANS (1951) to derive prices in our ecosystem model. We will discuss the significance of the derived prices and the uniqueness of the price system. As the practical application of the ecosystem prices entails their numerical calculation, in section 3 we will develop a algorithm for price calculation and demonstrate the calculation with an numerical example. Finally, in section 4 we will discuss our ecosystem model and compare it to traditional economic evaluation methods. We will show the limitations and prospects for ecosystem prices for evaluating natural goods.

[1] HANNON (1985) assumes also that there is no joint production. Later HANNON, COSTANZA, and HEREDEEN (1986) developed a model where this assumption is weakened.

2 The derivation of prices in ecosystems

Our ecosystem model is based on the generalized linear production model by KOOPMANS (1951). KOOPMANS postulates a relationship between efficiency and prices in a manner similar to that used in general equilibrium theory (cf. ARROW/DEBREU 1954; DEBREU 1959; ARROW/HAHN 1971). However, in contrast to general equilibrium theory, the consumption side of KOOPMANS' model has a very simple structure. He uses efficiency as the sole criterion for allocation. This enables prices to be derive prices only using the structure of production, i.e. the network of material and energy flows between the sectors of an economy.

Similarly, we perceive an ecosystem as a network in which the knots are components of the ecosystem and the linkages are the flows of services. Several possibilities exist concerning what can be considered as components and as services. Let us first turn to the latter. Services can be more or less aggregated. For instance, individual chemical compounds such as oxygen, carbon dioxide, phosphorus compounds, water, etc. can be perceived as services, or aggregates like plant biomass and animal biomass can be perceived as services. The economic counterpart to services flowing between the components of an ecosystem are goods.

The components of an ecosystem are the locations where the incoming services are used and services for other components are produced. Above all, living beings or groups of living beings (such as a population of animals, a plant species or all the herbivores) are perceived as components of an ecosystem. The transformation processes within the living beings mainly take place for the purpose of the life-preserving metabolism. However, parts of abiotic nature in which chemical processes like the decomposition of biotic material take place can be perceived as ecosystem components, too. The economic counterparts of components of ecosystems are economic actors or groups of economic actors, e.g. an economic sector. To remain compatible with KOOPMANS' notation we will also call the components in the context of the model activities.[2]

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[2] How one dertermines the components as well as the services best for a certain empirical investigation depents on the purpose of the investigation as well as on the available data. Examples of ecological studies which collected data of flows in ecosystems are ODUM (1957), TILLY (1968), STEEL (1974), DAME and PATTEN (1981). They all determine the components depending on the trophical levels (plants, herbivores, carnivores, detritivores etc.). Mostly only one kind of service (e.g. energy) is considered. An exception is the study by FASHAM (1985) who comprehends flows of carbon as well as of nitrogen.

Next we will explain how production is modeled, i.e. how flows are transformed within the activities.

The services are divided into:

- Primary factors (e.g. sunlight, water or certain nutrients), characterized by the fact more is imported than exported
- Final products (e.g. biomass), whose production is the objective of the transformation processes.

Deriving prices for services in the ecosystem model requires (as we will see below) the ecosystem to behave in such a way as to maximize its net-output of final products.[3] In this sense, final products are wanted services. There could be two other kinds of services: unwanted services, and neutral services.

In our model we neglect the existence of unwanted services. However, this can be done without a loss of generality since instead of an unwanted service one can consider the service "avoiding this unwanted service". This avoiding service then becomes wanted.

Furthermore, we assume that all primary factors are neutral. This can be done without a loss of generality, too: If a certain primary factor is desired, an additional activity can be introduced which converts one unit of the (by assumption) unwanted primary factor to one unit of a (new) wanted final product. To sum up: In our model services are either primary factors and neutral or final products and wanted.

We assume homogeneity and separability of the services. As our ecosystem model is static, we do not have to indicate the time period. We denote by y_i the entire net-output of service i of the ecosystem in one period. If y_i is negative than the (net-)imports of service i exceed the amount produced within the ecosystem. Altogether there are n services; those with the subscript $i = 1, \ldots, r$ are final products and those with subscript $i = r+1, \ldots, n$ are primary factors. $\mathbf{y} \in \mathrm{R}^n$ denotes the net-output vector of the ecosystem, where we also write

$$\mathbf{y} = (y_1, \ldots, y_r, y_{r+1}, \ldots, y_n)^T = (y^{\text{fin}}, \mathbf{y}^{\text{pri}})^T .$$

[3] There are several studies in the ecological literature which work with similar hypothesis of maximization (cf. e.g. ODUM 1969; REICHELE ET AL. 1975; WHITTAKER 1975; HANNON 1976).

The formulation of the transformation processes from primary factors to final products in biotic and abiotic nature is the kernel of our ecosystem model. The activities are the basic units of transformation: A certain combination of services flows per period into the activity and is converted into outputs. For instance, the component "plants" takes water and nutrients from the soil as well as sunlight, and develops biomass by means of photosynthesis. The biomass is then eventually distributed, e.g. among herbivores.

We suppose a linear production structure: the net-output of an activity is proportional to its so-called level of production. a_{ij} denotes the net amount of service, which is produced per period by the activity j (where $j = 1, \ldots, m$) per unit of production level. A negative sign of the coefficient indicates that the service is in the sum used.

The level of production of the j-th activity is denoted by x_j, where $x_j \geq 0$. Then the net-output y_i^j of the activity j of service i is expressed as $y_i^j = a_{ij} x_j$. As the very same output can be produced by different activities, the net-output y_i of the entire ecosystem is $y_i = \sum_{j=1}^{m} a_{ij} x_j$. If the coefficients a_{ij} are arranged in an $n \times m$-matrix $\mathbf{A} := (a_{ij})$,

$\mathbf{y} = \mathbf{A}\mathbf{x}$ can also be written,

where $\mathbf{x} = (x_1, \ldots, x_m)^T \in \mathrm{R}_+^m$. The net-output of the ecosystem is hence a linear function of the level of production. This equation describes all possible transformation processes within the ecosystem. To determine which net-outputs are actually feasible, the restrictions of the primary factors also need to be considered.

We assume in our model that the import of primary factors is absolutely restricted. This assumption is plausible: For instance, the amount of sunlight which can be used by plants cannot be determined by the ecosystem itself but only by exogenous factors like the sun's intensity of radiation, and the area and angle of incoming radiation. The restrictions of the primary factors reads $h_i \leq y_i$ for $i = r+1, \ldots, n$, where h_i is negative.[4]

[4] A primary factor need not necessarily be exclusively imported (as e.g. sunlight). It may also be produced within the ecosystem (as e.g. certain nutrients). However, the definition requires that in the sum the imports exceed the production of the ecosystem.

As the primary factors are identical with the net-imports of the ecosystems, for $i = r+1,\dots,n$ holds $y_i \le 0$, whereas for all other services which are final products $i = 1,\dots,r$ holds $y_i \ge 0$. To simplify the notation we define

$$h \quad := (0,...,0,h_{r+1},...,h_n)^T = (0^{\text{fin}}, h^{\text{pri}})^T \text{, where } h \leqq 0 \text{,}^5$$

such that the restrictions of the primary factors can be summarized as

$$h \leqq \mathbf{y}.$$

Now we are able to define the set of feasible net-outputs, i.e. the set of net-outputs that can in principle be produced by the given linear "technology" and the restrictions of the primary factors.

Definition 1 *A net-output* $\mathbf{y} \in \mathrm{R}^n$ *is said to be feasible if*

1. *There is a level of production* $\mathbf{x} \geqq 0$*, such that the net-output* $\mathbf{y}$ *can be produced with the given technology, i.e.* $\mathbf{y} = \mathbf{Ax}$ *;*
2. *The restrictions of the primary factors* $\mathbf{y}^{\text{pri}} \geqq h^{\text{pri}}$ *are obeyed;*
3. *No final products are used as net-inputs, i.e.* $\mathbf{y}^{\text{fin}} \geqq 0$.

The set $\left\{\mathbf{y} \in \mathrm{R}^n \middle| \mathbf{y} = \mathbf{Ax}, \mathbf{x} \geqq 0, \mathbf{y} \geqq h = (0^{\text{fin}}, h^{\text{pri}})^T \right\}$ *of feasible net-products is called Y.*

We note that the set of feasible net-products is convex. This can be shown by straightforward verification of the definition of convexity.

The central notion of our ecosystem model is efficiency.

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[5] We use the following notation: Let $\mathbf{a} = (a_1,\dots,a_n)^T$ and $\mathbf{b} = (b_1,\dots,b_n)^T$ be two vectors of the R^n. Then we denote

$\mathbf{a} > \mathbf{b}$, if $a_i > b_i$ for all $i = 1,\dots,n$,

$\mathbf{a} \geqq \mathbf{b}$, if $a_i \ge b_i$ for all $i = 1,\dots,n$, and

$\mathbf{a} \ge \mathbf{b}$, if $a_i \ge b_i$ for all $i = 1,\dots,n$ and $a_i > b_i$ for at least one $j \in \{1,\dots,n\}$.

The relations <, $\leqq$ and $\le$ are defined analogously.

Definition 2 *A feasible point* $\mathbf{y} \in Y$ *is called efficient if there is no feasible point* $\mathbf{y}' \in Y$*, which contains in all components of final products an amount at least as big and in at least one component of final products a properly greater amount. That is* $\mathbf{y}'^{\text{fin}} - \mathbf{y}^{\text{fin}} \geq 0$.

The presupposition for deriving ecosystem prices in our model is that — roughly speaking — the objective of the ecosystem is to produce as much of the final products as possible. One way of specifying this objective is the notion of efficiency. Instead of the notion of efficiency, the ecosystem's objective of allocation can be characterized by an objective function, which assigns each net-output a certain value representing its desirability. We will use the weighed sum of net-outputs of the final products as an objective function.

The two concepts "efficiency" and "maximizing an objective function" are closely related in our ecosystem model: Efficiency is a necessary condition for a feasible net-output to maximize the objective function. Efficiency is even a sufficient condition if a certain objective function is presupposed, i.e. if the weights of the final products of the objective function are properly defined. This is the essence of the following theorem. The theorem is of key importance for us since it allows conditions to be formulated for interpreting the weights of the final products of the objective function as ecosystem prices.

Before stating the theorem, we must introduce the distinction between scarce and free primary factors. In contrast to the restriction of free primary factors, the restriction of scarce primary factors is exhausted by the system. Let $\mathbf{y} = (\mathbf{y}^{\text{fin}}, \mathbf{y}^{\text{pri}})^T$ be a feasible net-output. After suitably renumbering the subscripts we can write $\mathbf{y} = (\mathbf{y}^{\text{fin}}, \mathbf{y}^{\text{pri}}_{=}, \mathbf{y}^{\text{pri}}_{>})^T$, where $\mathbf{y}^{\text{pri}}_{=}$ summarizes the scarce factors and $\mathbf{y}^{\text{pri}}_{>}$ summarizes the free factors. The partitioning may be different for different net-outputs, since then different primary factors may be scarce or free. We will also transfer the partitioning to price vectors related to a certain net-output. In economics the price for a free primary factor is zero and the price for a scarce primary factor is positive (or zero in extreme cases). Fortunately, our ecosystem prices will exhibit the same properties. We call a price vector which has these properties admissible.

Definition 3 *A price vector which is assigned to a feasible net-output is said to be admissible if the following condition holds:*

$$\mathbf{p}^{\text{fin}} > 0,\ \mathbf{p}^{\text{pri}}_{=} \geqq 0 \ \textit{and}\ \mathbf{p}^{\text{pri}}_{>} = 0\,.$$

Theorem 1 (KOOPMANS 1951: 86) *A net-output* $\mathbf{y}^* \in Y$ *is efficient if and only if there is an admissible price vector* $\mathbf{p} \in \mathrm{R}^n$ *such that* $\mathbf{y}^*$ *is a solution of the optimization problem*

$$\max \mathbf{p}^{\mathrm{fin}\,T}\mathbf{y}^{\mathrm{fin}}$$

subject to

$$\mathbf{y}^* \in Y\,.$$

It is enough for an understanding of our ecosystem model to explain the main idea of the proof of the implication "$\Rightarrow$".[6] Let $\mathbf{y}^*$ be an efficient net-output. The main task is to find an admissible vector **p** such that $\mathbf{y}^*$ is a solution to the optimization problem.

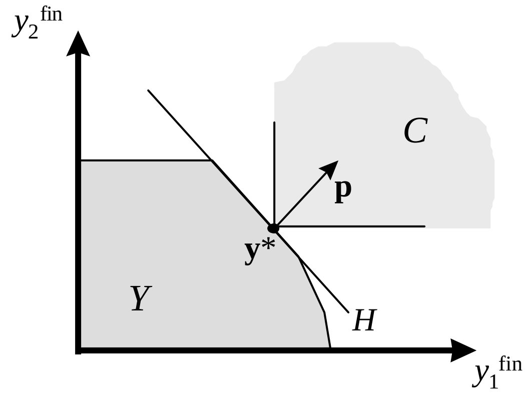

Figure 1. Separating hyperplane between the set Y of feasible net-outputs and the set C at the efficient point **y***.

For this purpose, consider the set C of all points $\mathbf{y} \in \mathrm{R}^n$ which have a properly bigger net-output of final products than $\mathbf{y}^*$ and also satisfy the factor restrictions (but need not be produced by the given technique):

$$C = \left\{\mathbf{y} \in \mathrm{R}^n \middle| \mathbf{y}^{\mathrm{fin}} - \mathbf{y}^{*\mathrm{fin}} \geq 0 \text{ and } \mathbf{y} \geqq h\right\}.$$

The set C is not closed (note the definition of $\geq$, cf. Footnote 5) and $\mathbf{y}^* \notin C$ since it does not hold $\mathbf{y}^{*\mathrm{fin}} \geq \mathbf{y}^{*\mathrm{fin}}$. The closure $\overline{C}$ of the set C is a convex cone whose vertex is the point **y***. The edges

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[6] A rather formal proof can be looked up at KOOPMANS (1951: 82–88) and a more intuitive proof is given in KLAUER (1998: 147-148, 171-172).

of the cone $\overline{C}$ lie parallel to the axis of the graph in the positive direction (see Figure 1). The set C as well as the set of feasible net-outputs Y are convex. Since $\mathbf{y}^*$ is efficient, C and Y are disjunct. Therefore, there is a separating hyperplane H between C and Y (cf. HADLEY 1961: 6-6). This hyperplane is uniquely determined (up to multiplication by a positive scalar) by a normal vector **p** at the net-output $\mathbf{y}^*$ pointing at C: $H = \left\{ \mathbf{y} \in \mathrm{R}^n \middle| \mathbf{p}^T \mathbf{y} = \mathbf{p}^T \mathbf{y}^* \right\}$ (see Figure 1).

Let us assume for the moment that **p** is admissible according to Definition 3. All $\mathbf{y} \in Y$ are contained in the halfspace $H_Y := \left\{ \mathbf{y} \in \mathrm{R}^n \middle| \mathbf{p}^T(\mathbf{y} - \mathbf{y}^*) \leqq 0 \right\}$. Since $\mathbf{p}_>^{\mathrm{pri}} = 0$ and by definition $\mathbf{y}^{*\mathrm{pri}}_= = h^{\mathrm{pri}}_=$, for all $\mathbf{y} \in Y$ holds

$$\begin{aligned} 0 &\geqq \mathbf{p}^T(\mathbf{y} - \mathbf{y}^*) \\ &= \mathbf{p}^{\mathrm{fin}\,T}(\mathbf{y}^{\mathrm{fin}} - \mathbf{y}^{*\mathrm{fin}}) + \mathbf{p}_=^{\mathrm{pri}\,T}(\mathbf{y}_=^{\mathrm{pri}} - \mathbf{y}_=^{*\mathrm{pri}}) + \mathbf{p}_>^{\mathrm{pri}\,T}(\mathbf{y}_>^{\mathrm{pri}} - \mathbf{y}_>^{*\mathrm{pri}}) \\ &= \mathbf{p}^{\mathrm{fin}\,T}(\mathbf{y}^{\mathrm{fin}} - \mathbf{y}^{*\mathrm{fin}}) + \mathbf{p}_=^{\mathrm{pri}\,T}(\mathbf{y}_=^{\mathrm{pri}} - h_=^{\mathrm{pri}}). \end{aligned}$$

But neither $\mathbf{p}_=^{\mathrm{pri}}$ nor $\mathbf{y}_=^{\mathrm{pri}} - h_=^{\mathrm{pri}}$ is non-negative, such that

$$0 \geqq \mathbf{p}^{\mathrm{fin}\,T}(\mathbf{y}^{\mathrm{fin}} - \mathbf{y}^{*\mathrm{fin}})$$

for all $\mathbf{y} \in Y$. Therefore, $\mathbf{y}^*$ is indeed a solution of the optimization problem.

It remains to show that the separating hyperplane can be chosen such that the normal vector p is admissible. This part of the proof which is not important for an understanding of the ecosystem prices can be found in (KLAUER 1998: 171-172). ■

The theorem allows not only the statement of prices within an economy as was intended by KOOPMANS, but can also be applied to ecosystems to derive prices there under certain circumstances: Whether an the empirically observed net-output $\mathbf{y}^*$ of an ecosystem is efficient (in accordance with the assumed technology and primary restrictions) can be verified, as we will show in the next section. If $\mathbf{y}^*$ is efficient, then according to the theorem this is equivalent to the statement "the ecosystem realizes the net-output with the highest total value subject to the linear technology and the restrictions of the primary factors." The total value of net-output is calculated by first weighting the flows of services by p_i and then adding them. These weights of the objective function can be interpreted as prices of the service flows. If the service i has the price p_i, this means that an additional (marginal) unit of the service i would lead to an increase of the objective function of the magnitude p_i.

Note that the ecosystem prices are solely determined by the structure of the transformation processes within the ecosystem and by the restriction of the primary factors. Observe furthermore that our model provides a method to verify whether the behavior of the ecosystem can be described by the stated objective function: It can be confirmed whether the observed net-output is efficient; but then the theorem yields that the objective function correctly describes the behavior of the ecosystem and ecosystem prices can be derived.

The existence of a feasible net-output is of course necessary for the derivation of prices. The existence can be shown (using the Theorem) under the assumption which KOOPMANS (1951: 50) called the *"impossibility of the Land of Cockaigne"* (KOOPMANS 1951: 88). The assumption excludes the unrealistic case of production without inputs.[7]

One problem that may arise is the ambiguity of the price vector, which we will now investigate. The uniqueness of a price vector (up to a scalar multiple) would be a desirable property if we wanted to use the prices for e.g. the aggregation of the varying natural capital goods to a one-dimensional stock of capital. Unfortunately, it may happen that the ecosystem prices derived from our model are not unique. Figure 2 shows a situation where there are many (even infinitely many) possibilities to place a separating hyperplane between *Y* and the set *C* and, therefore, many different price vectors.

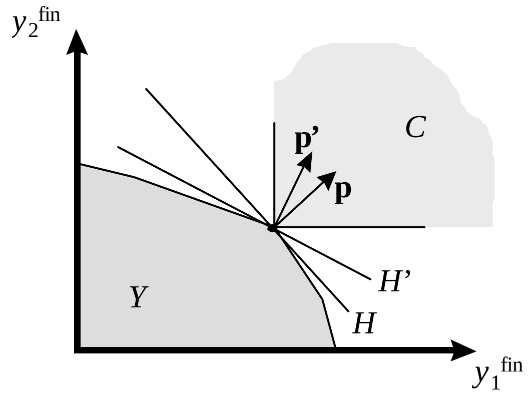

Figure 2. Situation in which there are several separating hyperplanes and price systems.

However, we can check for a given situation whether a unique price system exists. If the flows of services between the components of an ecosystem are observed, the vertexes and facets of the

[7] The assumption reads formally: There is no level of production $\mathbf{x} \geqq 0$, such that $\mathbf{y} = \mathbf{Ax} \geq 0$.

polyhedron of feasible points can (as we will show in the next section) be calculated. If the observed net-output $\mathbf{y}^*$ lies not on a vertex or edge of the polyhedron but inside a facet, the price system is unique; in this case the normal vector of the facet (pointing towards C) is the (up to a scalar) unique price vector (see Figure 1).

3 Calculating ecosystem prices

The applicability of the ecosystem model we developed in the last section closely depends on whether prices can be derived from the data observed. We assume that it is possible to observe the net-outputs of the individual activities. We will now develop an algorithm to determine the prices. We will first describe the principle of price calculation using a simple example with hypothetical numbers. Then we will explain a general approach for calculating prices and discus the difficulties which may arise.

Example. We consider an ecosystem with three activities and three kinds of services. All three activities are populations of plants which produce biomass by means of sunlight and nutrients. In this model biomass is the final product, whereas sunlight and nutrients are the primary inputs. We observe the following net-outputs:

	plant 1	plant 2	plant 3	total
sunlight	-2	-1	-1	-4
nutrients	-4	-3	-1	-8
biomass	4	3	1	8

The columns in the center of the table correspond to the net-outputs $\mathbf{y}^1$, $\mathbf{y}^2$ and $\mathbf{y}^3$ of the activities *plant 1, plant 2* and *plant 3*. The column on the right side is identical with the net-output $\mathbf{y}^*$ of the whole ecosystem. $\mathbf{y}^*$ is given as the sum of $\mathbf{y}^1$, $\mathbf{y}^2$ and $\mathbf{y}^3$.

The first step to calculate prices is to derive the transformation matrix $\mathbf{A}$ of the ecosystem from these observations. The coefficients a_{ij} of the matrix $\mathbf{A}$ are determined from the equation $y_i^j = a_{ij}x_j$. If we assume that the level of production equals 1 for all activities, then $y_i^j = a_{ij}$ holds for all i and j. Hence the observed data can be summarized by the following equation:

$$\mathbf{Ax}^* = \begin{pmatrix} -2 & -1 & -1 \\ -4 & -3 & -1 \\ 4 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \\ 8 \end{pmatrix} = \mathbf{y}^*$$

The matrix $\mathbf{A} = (a_{ij})$ describes the transformation processes which are in principle possible with respect to the assumption of linearity. The set $\overline{Y} = \left\{\mathbf{y} \in \mathrm{R}^n \,\middle|\, \mathbf{y} = \mathbf{Ax} \text{ for an } \mathbf{x} \geqq 0\right\}$ of all net-outputs that can be produced by this techninology (neglecting the restrictions of the primary factors) forms a convex cone which is spanned by the column vectors $\mathbf{a}^1,\ldots,\mathbf{a}^m$ of the matrix **A**. The vertex of the cone lies in the origin of the graph.

Not all net-outputs that are in principle possible, i.e. which lie within the cone $\overline{Y}$, are indeed feasible since the availability of the primary factors is limited. We assume in our example that the observed net-output **y*** totally uses up both primary factors, sunlight and nutrients. Hence, the factor restrictions can be described by the vector $h = (-4, -8, 0)$. The set of feasible outputs Y then consists of those points **y** of the convex cone $\overline{Y}$ which satisfy the inequation $\mathbf{y} \geqq h$:

$$Y = \left\{\mathbf{y} \in \mathrm{R}^n \,\middle|\, \mathbf{y} = \mathbf{Ax} \text{ for an } \mathbf{x} \geqq 0 \text{ and } \mathbf{y} \geqq h\right\}.$$

The next step of the price calculation is to determine the vertexes of the polyhedron *Y*, as we will now explain. The wanted price vector is, as we explained above, the normal vector of a separating hyperplane between the set $C = \left\{\mathbf{y} \in \mathrm{R}^n \,\middle|\, \mathbf{y}^{\text{end}} - \mathbf{y}^{*\text{end}} \geq 0,\, \mathbf{y} \geqq h\right\}$ and the convex polyhedron Y at the observed net-output **y***. If the point **y*** is efficient and lies on the facet *F* of the polyhedron *Y*, then the facet determines the separating hyperplane (see Figure 1). The normal vector of the facet *F* is thus the normal vector of a separating hyperplane and, therefore, a price vector of the ecosystem. In order to calculate the price vector, the vertexes of the polyhedron *Y* and thus of the facets must first be determined. The facet to which **y*** belongs must then be examined. The corresponding normal vectors represent a respective price system.

In our example the vertexes of the polyhedron are characterized by the fact that an activity produces at the highest possible level.[8] The maximal level of production of activity *j* solves

[8] This characterization is not generally correct. We will later give a characterization which is always valid.

$$\max_{\mathbf{x}} x_j \quad \textit{subject to} \quad \mathbf{Ax} \geqq h\ .$$

We obtain for the first, second and third activity the maximum levels of production $\mathbf{x}^1 = (2,0,0)^T$, $\mathbf{x}^2 = (0,8/3,0)^T$ and $\mathbf{x}^3 = (0,0,4)^T$. Thus the vertexes $\mathbf{e}^1$, $\mathbf{e}^2$ and $\mathbf{e}^3$ of the polyhedron Y are $\mathbf{e}^1 = \mathbf{Ax}^1 = (-4,-8,8)^T$, $\mathbf{e}^2 = \mathbf{Ax}^2 = (-8/3,-8,8)^T$ and $\mathbf{e}^3 = \mathbf{Ax}^3 = (-4,-4,4)^T$. The fourth vertex is the origin of the graph $\mathbf{0} = (0,0,0)^T$.

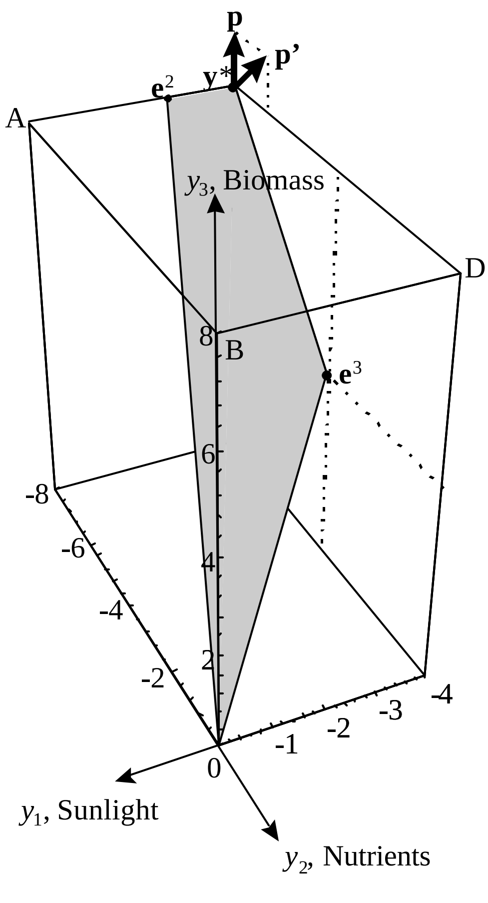

Figure 3. Set of feasible net-outputs *Y* and the observed net-output **y*** in the numerical example 1. The 3-dimensional polyhedron *Y* is degenerated to a 2-dimensional quadrilateral.

The vertex $\mathbf{e}^1$ is identical with the observed net-output **y***. Figure 3 shows the polyhedron Y as well as the point **y***. Note that the four vertexes $\mathbf{0}$, $\mathbf{e}^1$, $\mathbf{e}^2$ and $\mathbf{e}^3$ are positioned in a plane, i.e. the 3-dimensional polyhedron is degenerated to a 2-dimensional quadrilateral. To understand this, it must be realized that $\mathbf{y}^* = \mathbf{e}^1$ can be generated by activity 1 at production level 2 (i.e. $\mathbf{x} = (2,0,0)^T$), and also by all three activities at the unit production level (i.e. $\mathbf{x} = (1,1,1)^T$). Hence, the activities $\mathbf{a}^1$, $\mathbf{a}^2$ and $\mathbf{a}^3$, which span the polyhedron *Y*, are linearly dependent, i.e. they belong to the same plant.[9]

All efficient net-outputs lie, as can be seen in Figure 3, on the junction-line from $\mathbf{e}^1$ to $\mathbf{e}^2$, because only these net-outputs reach a biomass output of 8 units. Thus we have verified that $\mathbf{y}^* = \mathbf{e}^1$ is efficient. Now we have all the information needed to determine the admissible price vectors. In Figure 3 it can also be perceived that the vector $\mathbf{p} = (0,0,1)^T$ is an admissible price vector since the plane $H_\mathbf{p}$ defined by $\mathbf{p}$ separates at the point $\mathbf{y}^*$ the polyhedron *Y* from the set *C*.[10] The separating (hyper)-plane $H_\mathbf{p}$ lies parallel to the axis "sunlight" and "nutrients" in the plane of the quadrilateral ABD$\mathbf{y}^*$. In our numerical example the price vector is not unique because the efficient net-output $\mathbf{y}^*$ observed lies at the vertex $\mathbf{e}^1$. The plane spanned by the points $\mathbf{0}$, $\mathbf{e}^1$, $\mathbf{e}^2$, and $\mathbf{e}^3$ also separates *Y* from *C*. The corresponding price vector $\mathbf{p}'$ stands perpendicular to the vectors $\mathbf{e}^1$ and $\mathbf{e}^2$, i.e. $\mathbf{p}'$ is identical with the solution $(0,1,1)^T$ of the system of linear equations

$$\begin{pmatrix} \mathbf{y}^1 \\ \mathbf{y}^2 \end{pmatrix} \begin{pmatrix} p_1' \\ p_2' \\ p_3' \end{pmatrix} = \begin{pmatrix} -4, -8, 8 \\ -\tfrac{8}{3}, -8, 8 \end{pmatrix} \begin{pmatrix} p_1' \\ p_2' \\ p_3' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$\mathbf{p}'$ is an admissible price vector, too. The price of the primary nutrients is at $\mathbf{p}'$ (in contrast to **p)** not zero.

All admissible price vectors are represented by the convex combination of $\mathbf{p}$ and $\mathbf{p}'$, i.e. they can be written in the form $(0,p,1)^T$ with $0 \le p \le 1$. The price of the primary factor sunlight is zero, since it is a free factor: A (marginal) reduction of the sunlight used would not lead to a reduction of the produced biomass. ■

We will now present a general algorithm for the calculation of prices from given data.

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[9] We have chosen an example with only three services, because it is then possible to graphically display the polyhedron *Y*. To avoid another problem (which in general does not occur in models with several final products) we have determined the activities to be linearly dependent. If the numbers of an example with two primary factors and one final product such that the activities are linearly independent, it can be shown that the observed net–output cannot be efficient and, therefore, no price system can be derived.

[10] One can easily imagine the location of the set *C* in Figure 3: Let *O* be the positive orthant of the graph, one gets the set *C* by parallel translation of *O* by vector $\mathbf{y}^*$.

1st step: The net-output $\mathbf{y}^*$ of the whole system is calculated from the observed net-outputs of the individual activities.

2nd step: The restriction of the primary factors h must be determined on the basis of ecological knowledge of the ecosystem. For instance, the restriction of the primary factor "rainwater" may be determined by the amount of rainfall observed. As it may be more difficult to ascertain other restrictions (e.g. the amount of imported nutrients), it may be useful to assume that the observed net-output $\mathbf{y}^*$ fully exhausts these restrictions.

3rd step: The vertexes and facets of the polyhedron Y must then be calculated and the normal vectors determined for each facet. A method for doing this is described in the Appendix. Now the vectors which are admissible according to Definition 3 must be filtered out from the final set of normal vectors $\{\mathbf{p}^1,\ldots,\mathbf{p}^r\}$.

4th step: We must check whether the observed net-output $\mathbf{y}^*$ is efficient, because only in this case will there be a separating hyperplane between Y and the efficiency cone C at the point $\mathbf{y}^*$. According to Theorem 1, $\mathbf{y}^*$ is efficient if and only if there is an admissible price vector such that $\mathbf{y}^*$

$$\max \mathbf{p}^{\text{fin},T}\mathbf{y}^{\text{fin}} \quad \textit{subject to} \quad \mathbf{y}^* \in Y$$

In order to check whether $\mathbf{y}^*$ is efficient, it is sufficient to check the final set of admissible normal vectors $\{\mathbf{p}^1,\ldots,\mathbf{p}^r\}$. Three cases are possible:

1. There is no vector in $\{\mathbf{p}^1,\ldots,\mathbf{p}^r\}$ such that $\mathbf{y}^*$ solves the optimization problem. Then $\mathbf{y}^*$ is not efficient and no price system can be derived.
2. There is exactly one vector $\mathbf{p}^k$ in $\{\mathbf{p}^1,\ldots,\mathbf{p}^r\}$ such that $\mathbf{y}^*$ solves the optimization problem. Then $\mathbf{p}^k$ is the (up to a scalar) unique admissible price vector.
3. $\mathbf{y}^*$ solves the optimization problem for the vectors $\mathbf{p}^{k_1},\ldots,\mathbf{p}^{k_l}$ in $\{\mathbf{p}^1,\ldots,\mathbf{p}^r\}$. Then all vectors within the convex envelope of $\mathbf{p}^{k_1},\ldots,\mathbf{p}^{k_l}$ are admissible price systems to $\mathbf{y}^*$.

The first case in which $\mathbf{y}^*$ is not efficient is particularly precarious. This case can be ruled out by an additional assumption: If every activity produces only one final product, which is moreover not produced by any other activity (i.e. it is specific), and if all primary factor restrictions are fully exhausted, then **y*** is always efficient:[11] If the production level of any activity is increased, then (because of the factor restrictions) the production level of some other activity must inevitably be reduced and thus the output of the respective (specific) final product decreases. Nevertheless, whether the ecosystem prices make sense depends on the (ecological) plausibility of this (and the other) assumptions.

4 Discussion: Limitations and prospects for the evaluation of natural goods by ecosystem prices

The main result of our model is that prices for the services of an ecosystem can be derived if the net-output observed is efficient. The ecosystem prices are characterized by the fact that the statements "the observed net-output is efficient" and "the total value of the net-output" (calculated with these prices) are equivalent. The ecosystem prices are solely determined by the structure of the ecosystem's transformation processes and by the restriction of the primary factors. If it is empirically verified that the observed net-output is efficient (by the method described in section 3) and if the assumptions of our model prove to be ecologically plausible, the mass of empirical data can be condensed to aggregated information about the overall state of the ecosystem. This process is comparable to national accounting for economies. Such aggregated information about ecosystems can support decision-making in environmental policy.

Our model has several advantages over the existing approaches to ecosystem prices of HANNON (1985) and AMIR (1975, 1987, 1989, 1994, 1995) as described in Section 1. However, our model currently contains a number of restrictions and shortcomings (most of which also occur in HANNON's and AMIR's models), which ought to be eliminated by further research:

1. It may happen that the observed net-output of the system is not efficient and no price system can be derived. Under certain assumptions this precarious case can be avoided. However, it is crucial for the application of the model that these additional assumptions prove to be ecologically plausible, too.

[11] This assumption is even weaker than the premise of the Non–substitution Theorem supposed by HANNON (1985), since it is not claimed that there is only one primary factor.

2. It may happen that the derived price system is not unique. If the ecosystem prices are used to assess practical environmental problems, the different price systems may produce contradictory advice.

3. The model is static. Indeed, the model can be interpreted as quasi-static: If we suppose a myopic objective function which depends only on flows of the actual period, and if we assume the ecosystem is in a stationary state, all the results of the model remain valid. One consequence of this is that stocks of natural capital cannot be reflected in the model. However, the build up and exhaustion of natural capital stocks are important for describing natural processes. One starting-point for the dynamization of the model and the consideration of stocks in our model could be the study by MALINVAUD (1953), which augments the model of KOOPMANS (1951) with intertemporal aspects. An extension of my model in this direction would seem desirable.

4. Another problem of our model is that structural changes to the ecosystem, i.e. the occurrence and disappearance of components such as the migration of new species or the local extinction of others, cannot be reflected. If the structure of the ecosystem changes, the respective price systems cannot be compared. The probability of a structure becoming apparent in the model decreases with the level of aggregation of the components. However, a high level of aggregation means that energy and material flows can only be coarsely comprehended by the model and the information contained in the ecosystem prices is comparatively low.

Since neoclassical methods of natural goods also face severe practical difficulties as well as theoretical weaknesses, we want to discuss the following questions:

- To what extent are our ecosystem prices suitable for evaluating natural goods?
- What are the advantages and disadvantages of the ecosystem prices compared to the evaluation methods of the neoclassical theory?

To answer these questions, it must be realized that prices which are derived in ecosystem models are not based on the same evaluation criteria as neoclassical theory. The latter is founded on the principle of methodological individualism. According to this principle, only human individuals, but not a state, community, or nature, determine values and make decisions. By contrast, ecosystem prices cannot be directly traced back to evaluations of individuals of society. If decisions were made using only ecosystem prices, this would violate the principle of methodological individualism. Hence, ecosystem prices are not valuations in the traditional economic sense.

Values which are derived independently of human beings, e.g. those which were already in the world before humans occurred, are called ecocentric values (KREBS 1997). Are the prices we have derived in our ecosystem model ecocentric prices? Does the use of ecosystem prices to evaluate natural goods mark a shift away from anthropocentrism towards ecocentrism? In our opinion this is not the case. Using ecosystem prices means that information on the relationships in nature flows into the decision process. However, the resulting evaluations or decisions need not necessarily be adopted by society, which can always reject these evaluations.

The German ACADEMY OF SCIENCE doubts that a departure from anthropocentrism is at all possible when evaluating nature (1992: 27, our translation):

> *"The current discussion between an anthropocentric and a cosmocentric (physiocentric, ecocentric) approach proves to be insincere when the structure of the human relationship with the environment is considered in the crossfire between the threat to life and the condition of life. Since Man has to prevail and liberate himself vis-à-vis his environment, he is unable to position himself as an apparently neutral observer and judge over the environments of all living beings, granting each living being with a patronizing attitude the same right to life."*

It is therefore justified to speak of an *"absence of a way out of anthropocentrism"* when dealing with the evaluation of nature (ACADEMY OF SCIENCE 1992: 32 with reference to HOFMANN 1988: 277 cont.).

Nevertheless, there is no *direct* relation between ecosystem prices and evaluations by the individuals of society. This means that ecosystem prices cannot be directly compared to economic prices. Moreover, recommendations cannot be directly concluded for actions for society from ecosystem prices since they reflect the functional interrelations in an ecosystem but not directly the social desirability. However, the aggregate information about functional interrelations can of course support the decision-making process.

The prices which can be derived from these approaches can be used as surrogates for economic prices. This should in particular be taken into consideration if for natural goods no market exists and the respective neoclassical methods of evaluation (contingent evaluation method, hedonic pricing, travel cost method) are too expensive or the results are unsatisfactory (cf. HAUSMAN 1993; HANLEY/SPASH 1993).

To sum up, we believe that the derivation of prices in ecosystems is a promising aid to decision-making if traditional economic evaluation methods cannot be successfully applied. However, more research is needed before ecosystem prices can be put to practical use. In particular, ecosystem studies should be undertaken in order to empirically confirm that the model (and in particular the objective function) positively describes the behavior of the system, and that the prices positively reflect the functioning of the ecosystem.

5 Appendix

We will now describe a method to calculate the normal vectors of the facets of the polyhedron $Y = \{\mathbf{y} \in \mathrm{R}^n | \mathbf{y} = \mathbf{Ax} \text{ for an } \mathbf{x} \geqq 0 \text{ und } \mathbf{y} \geqq h\}$. The method is based on mathematical results which are also used in the simplex method of linear programming developed by DANTZIG (1951).

We consider the set $X = \{\mathbf{x} \in \mathrm{R}^m | \mathbf{Ax} \geqq h, \mathbf{x} \geqq 0\}$ of all production levels $\mathbf{x}$ which satisfy the primary restrictions. Both X and Y are convex polyhedrons. The set Y is a linear transformation of the set X, i.e. Y can be written as $Y = \{\mathbf{y} \in \mathrm{R}^n | \mathbf{y} = \mathbf{Ax} \text{ for an } \mathbf{x} \in X\}$. We will explain how the set N_X of normal vectors of the facets of the polyhedron X can be determined. The set N_Y of normal vectors of the facets of the polyhedron Y is then yielded as the linear transformation of N_X:

$$\mathbf{A}(N_X) := \{\mathbf{p} \in \mathrm{R}^n | \mathbf{p} = \mathbf{Aq} \text{ for an } \mathbf{q} \in N_X\} = N_Y .$$

The polyhedron $X \subseteq \mathrm{R}^m$ is identical with the set of solutions of the system of linear inequations

$$\begin{array}{ccccccccc} a_1(\mathbf{x}) & - & b_1 & := & a_{11}x_1 + & \cdots & + a_{1m}x_m & -h_1 & \geqq & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_n(\mathbf{x}) & - & b_n & := & a_{n1}x_1 + & \cdots & + a_{nm}x_m & -h_n & \geqq & 0 \\ a_{n+1}(\mathbf{x}) & - & b_{n+1} & := & x_1 & & & & \geqq & 0 \\ & \vdots & \vdots & & & \ddots & & & \vdots & \vdots \\ a_{n+m}(\mathbf{x}) & - & b_{n+m} & := & & & x_m & & \geqq & 0 \end{array} .$$

The vertexes of X can be calculated using the following theorem:

Theorem 2 (e.g. FISCHER 1985: 101) *Let $X \in \mathrm{R}^m$ be the set of solutions of the system of linear inequations*

$$\begin{array}{ccccc} a_1(\mathbf{x}) & - & b_1 & \geqq & 0 \\ \vdots & & \vdots & \vdots & \vdots \\ a_{n+m}(\mathbf{x}) & - & b_{n+m} & \geqq & 0 \end{array} .$$

Then $\mathbf{e}$ *is a vertex of the polyhedron X if and only if there are indices* $i_1,...,i_m \in \{1,...,n+m\}$, *such that* $a_{i_1},...,a_{i_m}$ *are linearly independent and*

$$\begin{array}{cccc} a_{i_1}(\mathbf{e})- & b_{i_1} & = & 0 \\ \vdots & \vdots & \vdots & \vdots \\ a_{i_m}(\mathbf{e})- & b_{i_m} & = & 0 \end{array}.$$

The calculation of the vertexes of X proceeds in three steps:

1st step: Determine all linearly independent subsets $\{a_{i_1},...,a_{i_m}\} \subset \{a_1,...,a_{n+m}\}$.

2nd step: Calculate the solutions of the respective systems of linear equations. This yields the points $c_1,\dots,c_k$.

3rd step: Eliminate those points c_i which are not contained in X. In doing so, check whether c_i satisfies the remaining inequations. Choose the numbering such that $c_1,\dots,c_l \in X$ $(l \le k)$ remain.

To calculate the normal vectors of the facets of the polyhedron Y, three more steps must be taken:

4th step: Check which vertexes belong to a certain facet. Each facet is characterized by the vertexes which lie on it.

5th step: Calculate the normal vectors of the facets of X.

6th step: Calculate the set of normal vectors $N_Y = \mathbf{A}(N_X)$ of the facets of Y from the set of normal vectors N_X of the facets of X.

We now turn to the question of how to calculate the vertexes related to a certain facet of the polyhedron X. The notion of neighborhood proves to be useful. We call the vertex c' neighboring the vertex c if the junction-line $\overline{cc'}$ lies on the edge of X (FISCHER 1985: 106). The vertexes which belong to the same facet form a class which we call a neighborhood, i.e. they are all mutually neighboring themselves. If an m-dimensional polyhedron is not degenerated, then its facets are spanned by m vertexes of the respective neighborhood. We will use the "criterion of edges" developed by DANTZIG (1951) to search for neighboring vertexes. The criterion states: If precisely one equation of the system of linear equations which according to Theorem 2 characterizes the vertex c

is replaced, a system of linear equations characterizing a neighboring vertex is obtained. Strictly speaking:

Theorem 3 (criterion of edges by DANTZIG 1951, see e.g. FISCHER 1985: 106) *Let* $X \in \mathrm{R}^m$ *be the set of solutions of the system of linear inequations*

$$\begin{array}{ccccc} a_1(\mathbf{x}) & - & b_1 & \geqq & 0 \\ \vdots & & \vdots & \vdots & \vdots \\ a_{n+m}(\mathbf{x}) & - & b_{n+m} & \geqq & 0 \end{array}.$$

Additionally, choose $a_0, a_1, a_2, ..., a_m$ *such that both* $a_0, a_2, ..., a_m$ *and* $a_1, a_2, ..., a_m$ *are linearly independent. Consider the straight line*

$G = \left\{ \mathbf{x} \in \mathrm{R}^m \,\middle|\, a_2(\mathbf{x}) - b_2 = \cdots = a_m(\mathbf{x}) - b_m = 0 \right\}$. *Then the points*

$c \in G$ *with* $a_0(c) - b_0 = 0$ *and*

$c' \in G$ *with* $a_1(c) - b_1 = 0$

are vertexes of X and the junction-line

$$\overline{cc'} = \left\{ \mathbf{x} \in \mathrm{R}^n \,\middle|\, a_2(\mathbf{x}) - b_2 = \cdots = a_n(\mathbf{x}) - b_n = 0 \right\}$$

is an edge of X. In other words, c *and* c' *are neighboring.*

The normal vectors of a facet are orthogonal to the edges of the facet. This means that if $c_1, ..., c_s$ are the vertexes of a facet, its normal vector **q** is obtained as a solution to the homogenous system of linear equations

$$\begin{array}{ccc} (c_1 - c_2)^T \mathbf{q} & = & 0 \\ \vdots & \vdots & \vdots \\ (c_1 - c_s)^T \mathbf{q} & = & 0 \end{array}.$$

Finally, the normal vectors of the facets of the polyhedron *Y* are obtained by linearly transforming the normal vectors of the facets of the polyhedron *X* by **A**. This completes the calculation of the normal vectors of the facets of *Y*.

6 References

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