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Martin Quaas [*]

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[*] UFZ Centre for Environmental Research Leipzig-Halle GmbH,
Permoserstr. 15, 04318 Leipzig, Germany

Email: Martin.Quaas@ufz.de

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Martin Quaas*

Department of Ecological Modelling, UFZ-Centre for Environmental Research Leipzig-Halle, Leipzig, Germany and

Interdisciplinary Institute for Environmental Economics, University of Heidelberg, Germany

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Abstract Based on a case study on Bombay, we argue that urban infrastructure, like the sewage system and the municipal waste collection, is an important instrument for urban environmental policy. We develop a spatial general equilibrium model of a monocentric city, where infrastructure serves as a public means of abating pollution.

Analyzing the optimal supply of pollution-reducing infrastructure, we conclude that it has to be geographically differentiated, even if pollution is homogenous. In a city with a growing population the provision of infrastructure has to be changed throughout the city, not only in newly inhabited areas.

Urban environmental policies, based on Pigouvian taxes and pollution-reducing infrastructure, are mutually dependent. In two settings of public or private infrastructure, we show that fiscal environmental policies have to be spatially differentiated, and that income transfers are necessary in order to implement the first best allocation as a residential market equilibrium.

JEL-Classification: H54, H23, Q53, Q58, R53

Keywords: urban environmental policy, pollution-reducing infrastructure, spatial environmental economics, unequal treatment of equal

*UFZ Centre for Environmental Research, Department of Ecological Modelling, PO Box 500135, D-04301 Leipzig, Germany. Email: martin.quaas@ufz.de. The author thanks Klaus Conrad, Bouwe Dijkstra, Malte Faber, Frank Jöst and Maximilian Mihm for helpful comments on earlier drafts. Financial support from the Research Training Group "Environmental and Resource Economics" of the Universities Heidelberg and Mannheim, financed by the German Research Foundation DFG and by the Volkswagen Foundation under grant II/79 628 is gratefully acknowledged.

Summary

By studying the case of Bombay, the biggest urban agglomeration in India, we show that providing adequate 'pollution-reducing' infrastructure, e.g. the sewage system or waste collection and disposal, can be a very effective instrument of urban environmental policy. The questions are, how can pollution-reducing infrastructure be supplied efficiently and how has it to be combined with economic instruments of environmental policy (like Pigouvian taxes) in order to reach an efficient outcome?

Our theoretical analysis is based on a general spatial equilibrium model of a monocentric city and its hinterland. The model comprises two private goods (consumption of goods and living space), two public goods (infrastructure and environmental quality), and a continuum of households, who choose their place of residence. Environmental pollution is a by-product of the households' consumption of goods. Infrastructure reduces the polluting emissions per unit of consumption: a part of the pollution is disposed of 'properly' and causes no environmental damage. Infrastructure is a *public good,* since it serves to reduce the polluting emissions of all households residing in the close neighbourhood. The analysis has two parts.

First, the Pareto optimal allocation is determined. We show that the efficient allocation of pollution-reducing infrastructure geographically differentiated, i.e. infrastructure density declines from the centre to the periphery. An adequate provision of pollution-reducing infrastructure is particularly important in the rapidly growing cities of developing countries. We show population growth necessitates improved infrastructure provision throughout the city, not just in newly inhabited areas.

Second, we consider two settings how the Pareto optimal allocation can be implemented in a decentralised economy: in the first setting the urban government provides infrastructure and imposes a Pigouvian tax on consumption. This setting is relevant, if the government can estimate the polluting emissions, but the households cannot. In the second setting pollution-reducing infrastructure is provided privately and subsidised by the urban government, which also imposes a Pigouvian tax on emissions. This setting is applicable, if private households can monitor their polluting emissions.

In the case of public infrastructural provision, the Pigouvian tax on consumption is shown to depend on the location, where the respective unit is consumed. As a result,

the tax burden differs among the locations and income transfers are necessary in order to implement the optimum as a residential equilibrium. In the case of private supply of infrastructure, private households undersupply this public good, unless a subsidy on infrastructure is paid. This subsidy is shown to be geographically differentiated, and a redistribution of incomes is necessary to implement the optimum as a residential equilibrium, too.

Hence, even if pollution is homogenous, geographically differentiated (i) infrastructural provision, (ii) fiscal instruments of environmental policy and (iii) transfer payments are needed to implement the Pareto optimum as a residential market equilibrium. These sophisticated policies result from considering pollution-reducing infrastructure as an instrument of urban environmental policy and jointly determining infrastructure supply and fiscal policy instruments. Given the high relevance of pollution-reducing infrastructure for urban environmental quality, this approach may lead to considerable welfare gains.

1 Introduction

Infrastructure, which generally comprises the stock of physical and social capital owned by the public sector, includes (i) utilities, i.e. water supply, sewage system, electricity, waste collection and disposal etc, (ii) communication infrastructure, (iii) transport infrastructure, i.e. roads, railways, etc, and (iv) land development measures, i.e. drainage improvement, flood control, reforestation projects, etc (Conrad 1994, 2001).[1] In an urban context, a major part of infrastructure has an immediate impact on environmental quality, since it helps either to mitigate pollution or to dispose of waste so that it does less damage to urban environmental quality (Section 2). We call this part "pollution-reducing infrastructure".

In this paper we argue that it is necessary to consider an adequate provision of pollution-reducing infrastructure as a tool of urban environmental policy. We show that the efficient supply of pollution-reducing infrastructure and environmental policy by means of Pigouvian taxes are mutually dependent. Hence, if either infrastructure is taken for granted when determining the Pigouvian tax rates, or if fiscal instruments of environmental policy are treated as fixed when determining the supply of infrastructure, an efficient outcome is missed. In particular, if pollution-reducing infrastructure is supplied publicly, the optimal Pigouvian tax rates are geographically differentiated, even if pollution is spatially homogenous.

An adequate provision of pollution-reducing infrastructure is particularly important in the rapidly growing cities of developing countries. Here, we analyze how the efficient supply of infrastructure has to be adapted to a growing urban population.

The close relation between infrastructure and urban environmental quality has been widely overlooked in both the literature of environmental economics and the economic literature dealing with infrastructure. The comparatively meagre (cf. Verhoef and Nijkamp 2002:159) literature analyzing environmental problems from the perspective of urban economics mainly focuses either on the geographical distribution of polluting firms and households, and considers the trade-off between commuting costs and high damage from pollution in the neighbourhood of the firms (e.g. Henderson 1977; Lucas 2001; Dijkstra and Lange 2003), or analyzes the trade-off between positive Marshallian externalities

[1] In the paper, we focus on physical infrastructure.

promoting growth and the opposing negative externalities of environmental pollution (Verhoef and Nijkamp 2002). None of these papers, however, refers to the role of urban infrastructure.

There are some contributions in urban economics which deal with the spatial distribution of public services within cities (for an overview see Revelle 1998) or the efficient private financing of public infrastructure (e.g. Brueckner 1997; Knaap et al. 2001).[2] The major part of research in related fields, however, considers infrastructure as an exogenously given quantity (Haughwout 2002:406).

Since we are interested in the spatial distribution of pollution-reducing infrastructure, the paper also aims at contributing to the literature on spatial environmental economics. Kolstad (1987) has shown that the spatial differentiation of environmental policies leads to higher welfare than homogenous policies, if marginal costs and marginal damage of pollution differ between the locations of polluting firms.[3] In contrast to Kolstad's setting, we consider homogenous pollution and identical preferences of all households. We show that the optimal environmental policy is nevertheless spatially differentiated, which is the endogenous outcome of jointly determining the public supply of pollution-reducing infrastructure and the Pigouvian tax on polluting consumption.

The paper is organised as follows: The following Section 2 demonstrates the importance of infrastructure for urban environmental quality by referring to the case of the Indian megacity Bombay. Section 3 presents the model of general spatial equilibrium. The optimal allocation is analyzed in the first part of the analysis in Section 4.[4] Here, we are mainly interested in the optimum geographical distribution of infrastructure and in the optimal changes to infrastructure in a city with a growing population.

The second part of the analysis (Section 5) is concerned with the mutual interdependence between a 'classical' environmental economic instrument, i.e. a Pigouvian tax on pollution, and an optimum provision of pollution-reducing infrastructure. Two different

[2] Explicit reference to the environmental impacts of infrastructure is only made in the case of transportation infrastructure (e.g. Lundqvist et al. 1998). The questions posed in that context (e.g. concerning the efficient modal split between public and private transportation) are quite different from ours.

[3] More precisely, the public benefit achieved from spatially differentiated taxation is particularly high, if marginal cost and marginal damage functions are steep.

[4] The relevant literature on the economics of welfare in cities is reviewed briefly in Section 4.

settings are considered: (i) public and (ii) private supply of pollution-reducing infrastructure. Both settings may be adequate to a given problem. (i) A public infrastructual provision (Section 5.2) is required, if households cannot observe their polluting emissions and only the government is able to do so.[5] If, on the other hand, households can monitor their actual emissions, it may be better, if private households provide infrastructure.[6] This setting is analyzed in Section 5.3. Section 6 concludes.

2 Case study: pollution-reducing infrastructure in Bombay

In this section we argue that urban infrastructure contributes to a cleaner urban environment in many respects. We therefore consider the case of Bombay, the largest urban agglomeration in India.[7] In 2001, Bombay officially had a population of 16 million people (Government of India 2001), and population continues to grow rapidly. Bombay's population suffers from a variety of environmental problems, many of which could be reduced by the provision of an adequate infrastructure (Quaas 2004:198):

An inadequate sewage system exposes the population to sewage water contaminated with bacteria and to the pollution of rivers and coastal waters. In Bombay, more than 40% of total population has to rely on public sanitation services, which are often of poor quality (Palnitkar 1998, Government of India 2001).

A better drainage and flood control could also contribute to urban environmental quality. Some areas of Bombay, which are built below sea level, flood repeatedly. This causes severe health problems to the affected population due to the heavily polluted coastal waters (Quaas 2004:189).

As a consequence of insufficient waste collection, much refuse remains at the roadside (Prabhavalkar 2002). This has an immediate impact on the population living there.

[5] For example, individual households may have difficulties to observe their contribution to urban water pollution, but possibly an urban authority may be able to assess this.

[6] This seems quite reasonable in the case of solid waste.

[7] In 1994, Bombay has been renamed *Mumbai*. However, internationally the name Bombay is still common and therefore used here, too.

Waste is frequently burned without any form of protection, which further increases air pollution (Shah and Nagpal 1997, Tondwalkar and Phatak 1997). Similar problems are caused by inadequate disposal sites (Sharma et al. 1997, Tondwalkar and Phatak 1997).

Bad roads contribute to noise pollution, but they also affect air pollution: in Bombay, about a third of the SPM (suspended particulate matter) load of the air, which is one of the most serious health threats, comes from roads dust (Shah and Nagpal 1997, table 2.7).

To summarise, in many cases the adequate provision of infrastructure can be a very effective instrument in urban environmental policy. The question is, how can pollution-reducing infrastructure be supplied efficiently and how has it to be combined with other economic instruments of environmental policy (such as Pigouvian taxes) in order to reach an efficient outcome?

3 The model

The analysis is based on a general spatial equilibrium model of a monocentric city and its hinterland. The model comprises four goods, an aggregate consumption commodity, living space, infrastructure, and environmental quality (which is a public good); and a continuum of identical individuals. It is of the von Thünen-type (see, e.g., Fujita and Thisse 2002, chapter 3), i.e. commuting costs are an important determinant of the city's spatial structure (Anas et al. 1998, Nijkamp 1999:533).

Space has one dimension in the model, represented by $z \in I\!R$, and is symmetric to $z = 0$.[8] The border of the city Z between the city and the hinterland is endogenously determined. Production in the city does not need space and is located in the *Central Industrial District* (CID) at $z = 0$. By symmetry, it is sufficient to consider the half space $z \geq 0$.

Commuting from the place of residence at $z \in [0, Z]$ to the CID takes $t_c = t_c(z)$ units of time, where $t_c(z)$ increases monotonically in the distance commuted, i.e. $t_c'(z) > 0$; and an individual living in the immediate neighbourhood of the CID has no commuting costs. i.e. $t_c(0) = 0$. No further commuting costs arise.

[8]It is straightforward to extend the model to form a symmetric two-dimensional plane and describe space in polar coordinates.

There are $N > 0$ identical individuals living in the city. They have identical preferences on private consumption of goods (amount x), living space (size s), and environmental pollution E. The preferences are represented by the utility function

$$u(x, s, E) = x^{\alpha} \cdot s^{1-\alpha} - d(E), \tag{1}$$

where $\alpha \in (0, 1)$, i.e. $u(x, s, E)$ is increasing and concave in the consumption of goods x and of living space s. Environmental damage $d(E)$ is increasing and convex in the environmental pollution E.

Each individual is endowed with one unit of time, i.e. the gross time being in the city for working and commuting is[9]

$$N = \int_{0}^{Z} n(z)\, dz, \tag{2}$$

where N is the number of individuals living in the city and $n(z)$ is the population density at place $z \in [0, Z]$, i.e. $n(z)$ people live in the area $[z, z + dz]$.

All urban residents are employed in the CID at a competitive wage rate w. Given the commuting time $t_c(z)$ of a worker living at z, her opportunity costs $w \cdot t_c(z)$ of commuting are given by the commuting time $t_c(z)$ valued at wage rate w.

The consumption good is produced by means of labour l alone. The technology is described by the production function $F(\cdot)$, which is assumed to be increasing and concave. The hinterland of the city is big, such that the city may be considered as a small open economy which trades the consumption good at a competitive price p.

Environmental pollution is caused by the consumption of goods, i.e. the polluting emissions $e(z)$ of a household residing at location z are generated as a by-product of consumption.[10] To keep notation simple, we have normalised pollution measurement units so that emissions $e(z)$ equal per capita consumption $x(z)$. Environmental pollution

[9] Leisure is ignored for reasons of simplicity.

[10] One may think of solid waste, wastewater, or air pollutants, e.g., from cooking or heating.

E is a 'public bad' in the city and equals aggregate polluting emissions, i.e.[11]

$$E = \int_0^Z n(z) \cdot e(z) dz \quad \text{for } 0 \leq z \leq Z. \tag{3}$$

Pollution-reducing infrastructure is considered as a public means of abating pollution in the following sense. Emissions $e(z)$ generated by a household residing at location z are assumed to decrease with the density of infrastructure $i(z)$ provided there,

$$e(z) = \gamma(i(z)) \cdot x(z), \tag{4}$$

where $\gamma(i)$ has the properties

$$0 < \gamma(i) \leq 1 \text{ with } \gamma(0) = 1, \ \frac{d\gamma(i)}{di} < 0, \ \frac{d^2\gamma(i)}{di^2} > 0 \text{ and } \frac{d}{di}\left[\frac{\gamma'(i)}{\gamma(i)}\right] \geq 0. \tag{5}$$

The interpretation of modelling infrastructure in this way is as follows: the polluting by-products of each unit of consumption are the same with and without infrastructure. If infrastructure exists with density $i(z) > 0$ at place z, however, only a fraction $\gamma(i(z)) < 1$ of these by-products is actually emitted into the environment, the remainder $1 - \gamma(i(z))$ is disposed of 'properly' by means of the infrastructure and causes no environmental damage.

Infrastructure $i(z)\,dz$ available in the space interval $[z, z+dz]$ is assumed to be a local *public good,* that is, it serves to reduce the polluting emissions of all $n(z)$ households residing in $[z, z+dz]$, so that total emissions there are $n(z)\,e(z) = \gamma(i(z))\,x(z)$.

The curvature properties of $\gamma(\cdot)$ imply that an increased provision of infrastructure lowers the emissions generated by one unit of consumption, but the marginal gain of additional infrastructure decreases with the amount of infrastructure already available.[12]

Building infrastructure comes at two kinds of costs: first, the physical infrastructure has to be bought at a 'world market' price p_i, and second, installing and maintaining one unit of infrastructure requires one unit of labour input.

[11]Since E is a pure public bad it is the same for all urban residents independent of the place of residence. It is assumed that the adjacent neighbourhood is affected by urban pollution to a considerable extent so that there is no incentive for an urban dweller to move into the hinterland just to avoid the environmental damage in the city.

[12]The last property in (5) requires that the curvature of $\gamma(\cdot)$ is not too small. For example, the functions $\gamma(i) = (1+i)^{-\epsilon}$ with $\epsilon > 0$ and the functions $\gamma(i) = \exp(-\mu\, i)$ with $\mu > 0$ fulfil Condition (5). The condition is needed to assure that there is an interior optimum and to derive Propositions 1 and 2.

One point of difference should be noted here. In contrast to much of the literature on infrastructure, it has no direct utility in our model. Rather, it generates a public good in the city by reducing pollution. Environmental pollution, which is given by

$$E = \int_0^Z n(z)\, e(z)\, dz = \int_0^Z n(z)\, \gamma(i(z))\, x(z)\, dz \quad \text{for} \quad 0 \leq z \leq Z, \tag{6}$$

decreases with the density of infrastructure at *any* point in the city, i.e. $dE/di(z) < 0$ for all $z \in [0, Z]$.

All land within the city is owned by an urban government, which buys the land at a given rural land rent $\underline{r}$, and converts it into living space at zero costs in such a way that one unit of land equals one unit of living space, i.e. the 'height' of the buildings is fixed. Except for exchanging land for consumption goods, nothing is transferred between the city and its hinterland.

To complete the model, we assume that moving within the city is costless. Hence, a residential equilibrium is characterised by the 'spatial equilibrium condition': the utility of each individual is the same at all locations in the city. Otherwise, there would be an incentive to move for at least one individual.

4 Optimal allocation

There has been a lot of discussion in the literature about employing welfare functions to determine optimal allocations in an urban context. Using a utilitarian welfare function, Mirrlees (1972) concluded that in the optimum, individuals differing only with respect to their place of residence will have the same marginal utility of consumption of goods and living space, but will differ in the level of utility. Only with a Rawlsian welfare function will all otherwise identical individuals enjoy the same utility in the social optimum (Dixit 1973). Wildasin (1986) has shown that Mirrlees' result of different utilities of identical individuals in the optimum under a utilitarian welfare function ('unequal treatment of equals') is due to differences in marginal utility of income between the individuals living at different places in a city.

Mirrlees' result can also seen from a different point of view: among all Pareto optimal allocations, only one maximises the utilitarian welfare function. At the same time, there is only one Pareto optimal allocation, where all individuals enjoy equal utility, i.e. where the spatial equilibrium condition holds. These two allocations are different, as Mirrlees has shown. The only welfare function, which selects the same allocation out of the Pareto optima as the spatial equilibrium condition, is the Rawlsian welfare function (Dixit 1973).

Rather than to start with a welfare function, we adopt a different approach and determine the Pareto optima. We then concentrate on the Pareto optimal spatial equilibrium, i.e. the Pareto optimal allocation where all individuals enjoy the same utility.

The procedure is to maximise the utility of one individual given a minimum utility level of all other individuals and subject to the constraints, which result from the model specification as described in Section 3.

To derive the conditions for the Pareto optima, we use the Lagrange formalism. Without loss of generality, we maximise the utility of an individual living at $z = 0$ given that all other individuals enjoy at least a level $U(z)$ of utility, which is allowed to differ between different places of residence $z \in [0, Z]$.[13] Formally, this condition reads

$$u(x(z), s(z), E) = U(z) \quad \text{for all} \quad 0 \leq z \leq Z. \tag{7}$$

This equation describes a continuum of constraints, since we require it to hold for each $z \in [0, Z]$. Hence, there is a continuum of Lagrangian multipliers $\lambda(z)$ associated with (7).

We now turn to the economic constraints of the optimisation. Both the size $s(z)$ of living space and the population density $n(z)$ at each place $z \in [0, Z]$ are choice variables. Recalling that one unit of land equals one unit of living space, we have

$$z = \int_0^z n(\tilde{z}) \cdot s(\tilde{z}) \, d\tilde{z} \quad \text{for all} \quad z \in [0, Z]. \tag{8}$$

This condition holds with equality in the optimum, because there are no gaps between buildings (otherwise commuting costs would be unnecessarily high).

The spatial distribution of population in the city determines total labour supply $\hat{L}$, which equals total endowment with time (Equation 2) less total time spent for commuting, i.e.

$$\hat{L} = N - \int_0^Z n(z) \, t_c(z) \, dz. \tag{9}$$

[13] A spatial equilibrium, however, requires $U(z) = U$ for all $z \in [0, Z]$.

Labor supply is divided into labour input L in the production sector in the CID and the amount of labour required to install and maintain the infrastructure.[14] Inserting Conditions (2) and (8) into Equation (9) yields the constraint (Lagrangian multiplier ω)

$$L + \int_0^Z i(z)\, dz = \int_0^Z n(z) \left[1 - t_c \left(\int_0^z n(\tilde{z}) \cdot s(\tilde{z})\, d\tilde{z} \right) \right] dz. \tag{10}$$

The output $F(L)$ of the production sector is used for aggregate consumption of goods and net exports, i.e. exports minus imports, Δ (Lagrangian multiplier π)

$$F(L) = \int_0^Z n(z)\, x(z)\, dz + \Delta. \tag{11}$$

The consumption of goods generates environmental pollution, as described by equation (6). The Lagrangian multiplier for this constraint is η.

We finally require the value of net exports to equal the value of goods acquired from the hinterland, i.e. the value of the land which is rented by the urban government and the value of physical infrastructure bought from abroad (Lagrangian multiplier μ),

$$p \cdot \Delta = \underline{r} \int_0^Z n(z)\, s(z)\, dz + p_i \int_0^Z i(z)\, dz. \tag{12}$$

The Pareto optimal allocation consists of the consumption of goods $x(z)$ and flat size $s(z)$ of all individuals, the supply of infrastructure $i(z)$ and population density $n(z)$ at each place in the city, as well as labour input L in production, pollution E, and net exports Δ. It is found by solving the following problem:

$$\max_{\{x(z),s(z),n(z),i(z)\},\Delta,L,E} u(x(0), s(0), E) \quad \text{subject to} \quad (7), (10), (11), (6), \text{ and } (12). \tag{13}$$

[14] Remember that one unit of labour is required to install and maintain one unit of infrastructure. Obviously it is best to employ people close to the infrastructure in question, because then no commuting costs arise.

The Lagrangian for this problem reads:[15]

$$
\begin{aligned}
\mathcal{L} \;=\; & \int_0^Z \lambda(z)\, n(z)\, \left[u(x(z), s(z)) - d(E) - \bar{U}(z)\right] \, dz \\
& + \omega \left[-L + \int_0^Z n(z) \left[1 - t_c \left(\int_0^z n(\tilde{z}) \cdot s(\tilde{z})\, d\tilde{z} \right) \right] dz - \int_0^Z i(z)\, dz \right] \\
& + \pi \left[F(L) - \int_0^Z n(z) x(z) dz - \Delta \right] \\
& + \eta \left[E - \int_0^Z \gamma(i(z))\, n(z)\, x(z)\, dz \right] \\
& + \mu \left[p \cdot \Delta - \underline{r} \int_0^Z n(z)\, s(z)\, dz - p_i \int_0^Z i(z)\, dz \right]
\end{aligned} \tag{14}
$$

The Pareto optimal allocation is calculated by maximising the Lagrangian (14) with respect to $x(z)$, $s(z)$, $n(z)$, and $i(z)$ for all $0 \leq z \leq Z$, as well as L, E, and Δ. With little rearrangement, the corresponding first order conditions lead – in that sequence – to the following equations.[16]

$$\lambda(z)\, \alpha\, x(z)^{\alpha-1}\, s(z)^{1-\alpha} = \pi + \gamma(i(z))\, \eta \tag{15}$$

$$\lambda(z)\, (1-\alpha)\, x(z)^{\alpha}\, s(z)^{-\alpha} = \mu\, \underline{r} + \omega \int_z^Z n(\tilde{z})\, t_c'(\tilde{z})\, d\tilde{z} \tag{16}$$

$$\omega\, [1 - t_c(z)] = [\pi + \gamma(i(z))\, \eta]\, x(z) + \left[\mu\, \underline{r} + \omega \int_z^Z n(\tilde{z})\, t_c'(\tilde{z})\, d\tilde{z} \right] s(z) \tag{17}$$

$$-\gamma'(i(z))\, n(z)\, x(z)\, \eta = \mu\, p_i + \omega \tag{18}$$

$$\omega = \pi\, F'(L) \tag{19}$$

$$\eta = \int_0^Z \lambda(z)\, n(z)\, d'(E)\, dz \tag{20}$$

$$\mu\, p = \pi \tag{21}$$

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[15] Here, we use the notation $\lambda(0) := 1$.

[16] The curvature properties of the utility function (1) and of the production function $F(L)$ and Assumption 5 about the curvature of $\gamma(i)$ assure that these conditions are also sufficient.

Here, we have already cancelled the common factors $n(z)$, and used constraint (7) to derive Condition (17).[17] Condition (17) may be simplified by inserting (15) and (16). Rearranging and dividing by μ yields:

$$\frac{\lambda(z)}{\mu} = \frac{p\,F'(L)\ [1 - t_c(z)]}{x(z)^\alpha\, s(z)^{1-\alpha}} = \frac{p\,F'(L)\ [1 - t_c(z)]}{U(z) + d(E)}, \tag{22}$$

which is the inverse marginal utility of wealth of the individual residing at z, if his income is $p\,F'(L)\ [1 - t_c(z)]$, i.e. the value of productive labour (see Equation (32) below). In the remainder of the paper, we require that the optimum is a candidate for a spatial equilibrium, i.e. we set $U(z) = U$ for all $z \in [0, Z]$. In this case, $\lambda(z)/\mu$ differs among different places in the city according to differences in commuting time at the different places of residence.[18]

For analyzing the properties of the Pareto optimal allocation, as determined by equations (15) – (21) and the constraints (7), (10), (11), (6), and (12) we introduce two abbreviations. First, consider the damage from pollution. The Lagrangian multiplier η may be interpreted as the social marginal damage from pollution measured in units of utility, and the Lagrangian multiplier μ is the marginal utility of an increase in foreign exchange. The *social marginal damage from pollution* D' in units of foreign exchange is given by the following equation:

$$\begin{aligned} D' \quad &:= \quad \eta/\mu = 1/\mu \int_0^Z \lambda(z)\, n(z)\, d'(E)\, dz \\ &= \quad \frac{p\,F'(L)}{U + d(E)} \int_0^Z (1 - t_c(z))\, n(z)\, d'(E)\, dz \\ &= \quad \frac{\hat{L}\, p\,F'(L)}{N\ [U + d(E)]}\, N\, d'(E). \end{aligned} \tag{23}$$

The factor $N\, d'(E)$ is the aggregate marginal disutility of pollution; the factor $\hat{L}\, p\,F'(L)/\,[N\ [U + d(E)]]$ is an average individual's inverse marginal utility of labour income, as, on average, each urban resident spends $\hat{L}/N$ hours working. Hence, D' is

[17] Observe that condition (17) requires $t_c(z) < 1$. Hence, for the optimal distribution of the population density it follows in particular that $t_c(Z) < 1$, i.e. even residents at the border of the city have some working time remaining after commuting to the CID.

[18] This reproduces Wildasin's (1986) result about the 'unequal treatment of equals' mentioned above in our more general setting.

the aggregate marginal disutility from pollution expressed in terms of average marginal utility of wage income.

The second abbreviation concerns the marginal costs of living space, which are given as the right hand side of condition (16). This expression is in terms of utility. Dividing it by μ yields the marginal costs of living space in units of foreign exchange, which we abbreviate with $r(z)$:

$$r(z) := \underline{r} + p\,F'(A) \int_z^Z n(\tilde{z})\,t_c'(\tilde{z})\,d\tilde{z}. \tag{24}$$

Dividing condition (15) by (16) and inserting the abbreviations (23) and (24) leads to

$$\frac{(1-\alpha)\,x(z)}{(1-\alpha)\,s(z)} = \frac{r(z)}{p + \gamma(i(z))\,D'}. \tag{25}$$

This condition describes the optimal allocation of consumption of goods $x(z)$ and living space $s(z)$.

Equation (18) determines the optimal allocation of pollution-reducing infrastructure. Using Abbreviation (23) and Condition (19), this equation transforms to

$$-\gamma'(i(z))\,n(z)\,x(z)\,D' = \;p_i + p\,F'(L). \tag{26}$$

Differentiating Equation (26) with respect to z and using the conditions (25) and $n(z) = 1/s(z)$, gives the following result:

Proposition 1 (Spatial distribution of infrastructure)
The optimal supply of infrastructure decreases monotonically with the distance from the CID:

$$\frac{di(z)}{dz} = \left[\frac{\gamma'(i(z))\,D'}{p + \gamma(i(z))\,D'} - \frac{\gamma''(i(z))}{\gamma'(i(z))}\right]^{-1} \frac{r'(z)}{r(z)} < 0 \tag{27}$$

for $0 \le z \le Z$.

Proof: See section A.1 in the appendix.

The main idea of this proposition is that the supply of infrastructure has to be spatially differentiated in an adequate manner. More specifically, the optimal density of infrastructure decreases from the CID to the periphery.[19] The underlying reason for this result is that in the neighbourhood of the CID, living space is scarcest: $r(z)$ declines

[19]The exact relationship may change, if, e.g., congestion in commuting or different income classes are considered.

monotonically from the CID to the periphery. As a consequence of Proposition 1, the further outside an individual lives, the more he/she substitutes consumption of goods by consumption of living space in the optimum:

Corollary 1

If the utility level is the same for all urban residents, $U(z) = U$, consumption of goods decreases from the CID to the periphery, while consumption of living space increases. Population density decreases from the CID to the periphery.

Proof: see Appendix A.2.

One question of interest is, how optimal infrastructure supply changes, if the population N of the city grows. It seems obvious that if the city expands, newly inhabited areas should be supplied with infrastructure. But also in other areas infrastructure supply has to be adjusted to the change in population size. In particular, if some additional assumptions are met, it can be shown that the optimal supply of infrastructure increases everywhere in the city. The reason is that under these assumptions, while marginal costs of infrastructure supply are constant, marginal utility of infrastructure increases, since more people suffer from pollution. This argument holds for any place in the city, not just for newly inhabited areas.

Proposition 2 (Adjustment of infrastructure to increasing population)

1. *If the urban population increases, the optimal supply of infrastructure has to be adjusted everywhere in the city, $di(z)/dN \neq 0$ for all $z \in [0, Z]$.*

2. *Assume $t_c(z) = t_c \cdot z$, $F(L) = f \cdot L$, $d(E) = \delta \cdot E$, $\underline{r} = 0$ and $p_i = 0$. Then, the optimal supply of infrastructure increases everywhere in the city, if the urban population increases, $di(z)/dN > 0$ for all $z \in [0, Z]$.*

Proof: see Appendix A.3

5 Urban environmental policy in a decentralised economy

Now we turn to the problem of how to implement the socially optimal allocation in a decentralised economy. We shall proceed in three steps: first, we shall determine the laissez-faire allocation without government intervention (Section 5.1). Second, we shall investigate the situation where the urban government supplies the pollution-reducing infrastructure and imposes a Pigouvian tax on consumption (Section 5.2). Third, we shall consider a setting, in which the government imposes a Pigouvian tax on emissions and households provide the pollution-reducing infrastructure. The government will subsidise infrastructure in order to achieve an optimal provision of infrastructure (Section 5.3).

Whether the setting investigated in Section 5.2 or in Section 5.3 is relevant to a specific context depends on whether the households can observe their consumption of goods only or if they can monitor their polluting emissions: if households cannot monitor their emissions, only the urban government can supply infrastructure efficiently. Since households cannot choose emissions independently of consumption, the Pigouvian tax is on consumption rather than on emissions. If, on the other hand, households can monitor their emissions, infrastructure can be provided by private households. In this case, the Pigouvian tax is on emissions, because households can choose the emission level per unit of consumption by choosing the infrastructure density $i(z)$.

5.1 Laissez-faire

The laissez-faire equilibrium is the allocation in which households maximise utility – given their income $y(z)$ – by choosing the consumption of goods and living space as well as their place of residence, and firms maximise profits. In equilibrium all markets clear, and the 'spatial equilibrium condition' holds, i.e. all individuals enjoy the same utility. In the laissez-faire case, the urban government's role is to rent out living space to urban residents and to redistribute revenues (net of expenditures to rent undeveloped land) equally among the inhabitants of the city. Let w be the wage rate and $r(z)$ be the rent on living space. Then, the income of a household living at place $z \in [0, Z]$ is the sum of wage earnings and the share of redistributed rents,

$$y(z) = w\,(1 - t_c(z)) + \frac{1}{N} \int_0^Z (r(z) - \underline{r})\, dz. \tag{28}$$

Profit maximisation of firms implies that the value of the marginal product of labour equals the wage rate w, i.e. $p\,F'(L) = w$. The optimisation problem of a household living at z is

$$\max_{x(z),\, s(z)} \; x(z)^{\alpha}\, s(z)^{1-\alpha} - d(E) \quad \text{s.t.} \quad y(z) = p\, x(z) + r(z)\, s(z). \tag{29}$$

The Lagrangian is $\mathcal{L}^{\text{lf}} = x(z)^{\alpha}\, s(z)^{1-\alpha} - d(E) + \lambda^{\text{lf}}(z)\,[y(z) - p\, x(z) - r(z)\, s(z)]$, and the first order conditions are

$$\alpha\, x(z)^{\alpha-1}\, s(z)^{1-\alpha} = \lambda^{\text{lf}}(z)\, p \tag{30}$$

$$(1-\alpha)\, x(z)^{\alpha}\, s(z)^{-\alpha} = \lambda^{\text{lf}}(z)\, r(z). \tag{31}$$

Multiplying (30) with $x(z)$ and (31) with $s(z)$ and adding both equations leads to

$$\lambda^{\text{lf}}(z) = \frac{x(z)^{\alpha}\, s(z)^{1-\alpha}}{p\, x(z) + r(z)\, s(z)} = \frac{x(z)^{\alpha}\, s(z)^{1-\alpha}}{y(z)}, \tag{32}$$

which is the marginal utility of income $y(z)$ (Mas-Colell et al. 1995:54f).

The resulting demand functions are $x(z) = \alpha\, y(z)/p$ for goods and $s(z) = (1-\alpha)\, y(z)/r(z)$ for living space. Inserting these demand functions into the utility function (1) yields the indirect utility function

$$v^{\text{lf}}(z) = \alpha^{\alpha}\, (1-\alpha)^{1-\alpha}\, \frac{y(z)}{p^{\alpha}\, r(z)^{1-\alpha}} - d(E). \tag{33}$$

The spatial equilibrium condition requires that the indirect utility is the same at all locations $z \in [0, Z]$, i.e. none of the identical households has an incentive to move. We rewrite this condition in the following way:

$$\begin{aligned} & v^{\text{lf}}(z) = U && \Leftrightarrow \quad v^{\text{lf}\,\prime}(z) = 0 \\ \Leftrightarrow \quad & \frac{y'(z)}{y(z)} = (1-\alpha)\, \frac{r'(z)}{r(z)} && \Leftrightarrow \quad r'(z) = y'(z)\, \frac{r(z)}{(1-\alpha)\, y(z)} = y'(z)\, n(z). \end{aligned} \tag{34}$$

Here, we have inserted the demand for living space and used condition $n(z) = 1/s(z)$. Equation (34) determines the spatial differentiation of rent for living space in the laissez-faire equilibrium. Inserting Equation (28) yields

$$r'(z) = -n(z)\, w\, t_c'(z). \tag{35}$$

Exactly the same condition is derived by differentiating Equation (24), which gives the marginal costs of living space in the Pareto optimum, with respect to space and inserting the condition for the firm's profit maximum, $w = p\, F'(L)$. This observation reflects the fact that the market for living space is undistorted.

5.2 Public supply of infrastructure

Turning to the investigation of urban environmental policy, we start with the case that the urban government provides pollution reducing infrastructure. It is assumed to do this optimally, i.e. according to Equation (26). In addition, the government imposes a Pigouvian tax $\tau(z)$ on polluting consumption, which we allow to be spatially differentiated. Denoting the household's income with $y(z)$ and the rent for living space with $r(z)$, the household's optimisation problem reads

$$\max_{x(z),\, s(z)} \; x(z)^{\alpha}\, s(z)^{1-\alpha} - d(E) \quad \text{subject to} \;\; y(z) \;\; = \;\; (p + \tau(z))\, x(z) + r(z)\, s(z).$$

The first order conditions for this problem yield:

$$\frac{(1-\alpha)\, x(z)}{(1-\alpha)\, s(z)} = \frac{r(z)}{p + \tau(z)} \tag{36}$$

By comparing this equation to (25), we find that the condition for the household's optimum is equal to the condition for the Pareto optimum, if the rent $r(z)$ for living space is as given by Equation (24) and the tax rate $\tau(z)$ on consumption is

$$\tau(z) = \gamma(i(z))\, D', \tag{37}$$

where the social marginal damage from pollution D' is given by Equation (23).

The tax rate $\tau(z)$, Equation (37), depends on the place where the respective unit is consumed because it depends on the amount of infrastructure provided there. Differentiating (37) with respect to space, using $\gamma'(i(z)) < 0$ (Condition 5) and Proposition 1, yields the result that the optimal tax rate increases from the CID to the periphery, i.e. $\tau'(z) > 0$. In other words, the tax rate equals the marginal social damage of one unit of consumption, which depends on the amount of infrastructure available at the place where the unit of the goods is consumed. At places where infrastructure density is high, marginal damage is comparatively low and vice versa. Hence, the tax rate is highest in the city centre, where infrastructure density is highest, and declines towards the periphery, where infrastructure is is provided at a lower level.

To implement the optimum as a spatial equilibrium, we also need to ensure that the spatial equilibrium condition holds, i.e. that the indirect utility of the representative individual everywhere is the same. The indirect utility function reads:

$$v^{\text{pub}}(z) = \alpha^{\alpha}\, (1-\alpha)^{1-\alpha}\, y(z)\, (p + \tau(z))^{-\alpha}\, r(z)^{-(1-\alpha)} - d(E). \tag{38}$$

The spatial equilibrium condition requires

$$
\begin{aligned}
& v^{\mathrm{pub}\,\prime}(z) = 0 \\
\Leftrightarrow \qquad & \frac{y'(z)}{y(z)} = \alpha\,\frac{\tau'(z)}{p+\tau(z)} + (1-\alpha)\,\frac{r'(z)}{r(z)} \\
\Leftrightarrow \qquad & y'(z) = \tau'(z)\,x(z) + r'(z)\,s(z) = \tau'(z)\,x(z) - w\,t_c'(z).
\end{aligned}
\tag{39}
$$

Here, we have inserted the demand functions for goods and living space as well as Condition (35). It can immediately be seen that with the same incomes as in the laissez-faire equilibrium, given by (28), this equation does not hold: in contrast to the laissez-faire equilibrium, the price of the consumption commodity is not independent of space, because $\tau'(z) > 0$. Hence, a redistribution of incomes is necessary. Together with the results derived above, this determines the optimal environmental policy in the case of publicly supplied infrastructure.

Proposition 3 (Environmental policy with public provision of infrastructure) *Three policy instruments are needed to reach a first best in a decentralised economy with a public infrastructural provision:*

1. *a spatially differentiated supply of infrastructure according to* (27),
2. *a spatially differentiated tax on consumption with rate* $\tau(z)$*, where* $\tau(z)$ *increases monotonically with the distance from the CID,* $d\tau(z)/dz > 0$ *for* $0 \leq z \leq Z$*,*
3. *a redistribution of incomes from individuals living near the city centre to individuals living further outside. More precisely, incomes have to be* $y(z) = w\,(1-t_c(z))+\Theta(z)$ *with*

$$
\Theta'(z) = \tau'(z)\,x(z) > 0. \tag{40}
$$

Proof: Only Part 3. remains to be proven. It follows from comparing $y'(z) = -w\,t_c'(z) + \Theta'(z)$ with the spatial equilibrium condition (39). □

Hence, the optimal environmental policy requires three instruments, and all of them have to be spatially differentiated: the density of infrastructure declines from the CID to the periphery, the tax rate increases with the distance from the CID, and there is a redistribution of incomes from the centre to the periphery. All these spatial differentiations

– which are jointly endogenously determined – result from including pollution-reducing infrastructure in the model. If the possibility of a spatially distributed supply of infrastructure is neglected, i.e. if $\gamma(i(z)) \equiv const$, the tax rate τ is uniform all over the city and a spatially differentiated redistribution of incomes would be unnecessary.

The supply of infrastructure $i(z)$ and the redistribution $\Theta(z)$ of incomes are determined by the differential equations (27) and (40), respectively, together with the resource constraints and the condition that the rent $r(Z)$ for living space at the border of the city equals the rural rent $\underline{r}$, $r(Z) = \underline{r}$. Given $i(z)$ and $\Theta(z)$, the tax on consumption may be calculated from (37).[20]

5.3 Private supply of infrastructure

In this section, we shall consider the setting in which infrastructure is provided by private households. We will introduce two policy instruments of the urban government, a tax $\theta(z)$ on polluting emissions and a subsidy $\sigma(z)$ on the private supply of pollution-reducing infrastructure. Both instruments are allowed to be spatially differentiated in the first place. In this setting, the household chooses the amount of polluting emissions $e(z)$ indirectly by choosing the infrastructure supply $i(z)$. The optimisation problem of a household residing at z is

$$\max_{x(z),\, s(z),\, i(z)} \; x(z)^{\alpha}\, s(z)^{1-\alpha} - d(E) \quad \text{subject to} \tag{41}$$

$$y(z) = p\, x(z) + r(z)\, s(z) + (p_i + w - \sigma(z))\, i(z) + \theta(z)\, e(z)$$

$$e(z) = \gamma(i(z))\, x(z)$$

The first order conditions for the households optimum lead to the following equations:

$$\frac{(1-\alpha)\, x(z)}{\alpha\, s(z)} = \frac{r(z)}{p + \theta(z)\, \gamma(i(z))} \tag{42}$$

$$\sigma(z) - \theta(z)\, \gamma'(i(z))\, x(z) = p_i + w \tag{43}$$

The comparison of condition (42) with (25) yields:

$$\theta(z) = \theta = p\, D', \tag{44}$$

[20]However, it is not possible to derive a closed form solution in general.

i.e. $\theta(z)$ equals the social marginal damage of one unit of pollution, which is constant throughout the city. Hence, the Pigouvian tax $\theta(z) = \theta$ is the same for all households.

Inserting Equation (44) into (43) and comparing this equation to the condition for the optimal supply of pollution-reducing infrastructure, Equation (26), yields

$$\sigma(z) = \frac{(n(z) - 1)}{n(z)} (p_i + w). \tag{45}$$

The necessity of subsidising infrastructure results from the fact that infrastructure is a *public* means of abating emissions, i.e. it serves not only one household, but all $n(z)$ households residing in $[z, z + dz]$. To ensure the optimal supply of this local public good, a subsidy is required, as given by Equation (45).

The subsidy (45) varies at different places in the city. As stated in Corollary 1, the optimal population density declines from the city centre to the periphery. Thus, the subsidy declines as well. As a consequence, in the case of privately supplied infrastructure, a similar redistribution of incomes is necessary as in the case of public supply of infrastructure (cf. Proposition 3).

The three instruments of environmental policy in the case of private supply of infrastructure are summarised in the following result.

Proposition 4 (Environmental policy with private supply of infrastructure) *Three policy instruments are needed to achieve a first best in a decentralised economy with a private provision of infrastructure:*

1. *a spatially differentiated subsidy on infrastructure (Equation 45),*

2. *a spatially homogenous tax on emissions (Equation 44), and*

3. *a redistribution of incomes such that incomes are* $y(z) = w\,[1 - t_c(z)] + \Theta(z)$ *with*

$$\Theta'(z) = -\frac{p_i + w}{n(z)}\, i(z)\, \frac{n'(z)}{n(z)} > 0. \tag{46}$$

Proof: see Appendix A.4

6 Conclusions and discussion

Pollution-reducing infrastructure has been incorporated in the model as a public means of pollution abatement. As modelled here, pollution-reducing infrastructure serves to dispose a part of the by-products from consumption of goods properly, i.e. such that only the remainder contributes to the damaging emissions. Thereby, infrastructure is a public means of pollution abatement, as one unit of infrastructure serves all households living in the immediate neighbourhood.

Providing infrastructure in order to improve urban environmental quality is most important in the context of rapidly growing cities in developing countries. We have shown that infrastructural provision has to be changed throughout the city if population increases, not just in newly inhabited areas. In order to investigate this issue in more detail, however, an extension to a dynamic model would be necessary, which would then also be capable of describing infrastructure – more realistically – in terms of capital goods.

We assumed homogenous environmental pollution in the city, in particular, one unit of emissions contributes to the same extent to environmental pollution, irrespective of where it is emitted. Hence, the Pigouvian tax on emissions is the same all over the city. However, how much emissions are generated by one unit of consumption depends on the infrastructure at the place, where the respective unit is consumed. Vice versa, the optimal supply of infrastructure at each place depends on the amount of consumption and the population density there. Given the structure of the city with a CID in the centre and commuting costs increasing with the distance from the CID, infrastructure density should be highest in the CID and decline monotonically towards the periphery. Hence, environmental policy by means of pollution-reducing infrastructure has to be spatially differentiated. There are two possibilities of providing infrastructure.

First, the urban government can build infrastructure. In that case, the household's only opportunity of abating emissions is by reducing consumption. Hence, the tax on emissions is effectively a tax on consumption, which depends on the location at which the respective unit is consumed. As a result, the tax burden differs from one location to another and a redistribution of incomes becomes necessary in order to implement the optimum as a residential equilibrium.

Second, infrastructure may be provided by private households. In that case, households may abate emissions by supplying pollution-reducing infrastructure, i.e. they can choose emissions independently of consumption by choosing the infrastructure density. Since it is a public means of abatement, private households undersupply infrastructure, unless a subsidy on infrastructure is paid.[21] This subsidy has to be spatially differentiated and, hence, a redistribution of incomes is necessary to implement the optimum as a residential equilibrium.[22]

To summarise, if pollution-reducing infrastructure is recognised as a means of urban environmental policy, fiscal policy instruments have to be spatially differentiated, too, and, moreover, adequate transfer payments are needed to implement the Pareto optimum as residential market equilibrium. These sophisticated policies result from considering pollution-reducing infrastructure as a means of urban environmental policy and jointly determining infrastructure supply and fiscal policy instruments. Given the obvious relevance of infrastructure for urban environmental quality, this approach may lead to considerable welfare gains.

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[21]Here, we have not considered the network character of infrastructure. In the presence of network externalities, such a subsidy becomes even more important.

[22]We would like to note that the spatial differentiation of the environmental policy in our model is an endogenous outcome (in contrast to Kolstad 1987), which occurs even though pollution is homogenous.

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A Appendix

A.1 Proof of proposition 1

Inserting $n(z) = 1/s(z)$ into equation (26), we have

$$
\begin{aligned}
& & -\gamma'(i(z))\,\frac{x(z)}{s(z)}\,D' = p_i + p\,F'(L) \\
&\overset{(25)}{\Longleftrightarrow} & -\gamma'(i(z))\,\frac{\alpha}{1-\alpha}\,\frac{r(z)}{p+\gamma(i(z))\,D'}\,D' = p_i + p\,F'(L).
\end{aligned}
$$

Differentiating with respect to z and rearranging yields

$$
\begin{aligned}
&\Rightarrow & -\frac{\gamma''(i(z))}{\gamma'(i(z))}\,i'(z) - \frac{r'(z)}{r(z)} + \frac{\gamma'(i(z))\,i'(z)\,D'}{p+\gamma(i(z))\,D'} = 0 \\
&\Leftrightarrow & \left[\frac{\gamma'(i(z))\,D'}{p+\gamma(i(z))\,D'} - \frac{\gamma''(i(z))}{\gamma'(i(z))}\right] i'(z) = \frac{r'(z)}{r(z)} \qquad (47)
\end{aligned}
$$

The right hand side of this equation is negative, because

$$
r(z) \overset{(24)}{=} \underline{r} + p\,F'(L) \int_{z}^{Z} n(\tilde{z})\,t_c'(\tilde{z})\,d\tilde{z} > 0
$$

and

$$
r'(z) = p\,F'(L)\,n(z)\,t_c'(z) < 0,
$$

as $t_c'(z) < 0$. To prove the proposition, we finally show that

$$
\frac{\gamma'(i(z))\,D'}{p+\gamma(i(z))\,D'} - \frac{\gamma''(i(z))}{\gamma'(i(z))} > 0.
$$

We therefore use Condition (5):

$$
\begin{aligned}
& & \frac{d}{di(z)}\left[\frac{\gamma'(i(z))}{\gamma(i(z))}\right] \geq 0 \qquad (48) \\
&\Leftrightarrow & \left[[\gamma'(i(z))]^2 - \gamma''(i(z))\,\gamma(i(z))\right]\,D' \leq 0 \\
&\Rightarrow & \left[[\gamma'(i(z))]^2 - \gamma''(i(z))\,\gamma(i(z))\right]\,D' < p\,\gamma''(i(z)) \\
&\Leftrightarrow & [\gamma'(i(z))]^2\,D' < \gamma''(i(z))\,\left[p+\gamma(i(z))\,D'\right] \\
&\Leftrightarrow & \frac{\gamma'(i(z))\,D'}{p+\gamma(i(z))\,D'} > \frac{\gamma''(i(z))}{\gamma'(i(z))} \qquad (49)
\end{aligned}
$$

as $\gamma'(i(z)) < 0$ (the last conclusion) and $\gamma''(i(z)) > 0$ (the conclusion from the first to the second line). $\square$

A.2 Proof of Corollary 1

Consumption per unit of space declines from the neighborhood of the CID to the periphery,

$$\frac{d}{dz}\frac{x(z)}{s(z)} \overset{(25)}{=} \frac{d}{dz}\frac{r(z)}{p+\gamma(i(z))\,D'(E)} = \frac{r'(z)\,(p+\gamma(i(z))) - r(z)\,\gamma'(i(z))\,i'(z)}{(p+\gamma(i(z)))^2} < 0, \qquad (50)$$

as $i'(z) < 0$ (Proposition 1).

Given that the utility is the same all over the city, we have

$$\begin{aligned} \frac{dU(z)}{dz} &= x(z)^{\alpha}\, s(z)^{1-\alpha} \left[\alpha\,\frac{x'(z)}{x(z)} + (1-\alpha)\,\frac{s'(z)}{s(z)}\right] = 0. \\ \Leftrightarrow \qquad \frac{x'(z)}{x(z)} &= -\,\frac{1-\alpha}{\alpha}\,\frac{s'(z)}{s(z)}. \end{aligned}$$

On the other hand, (50) yields

$$\frac{d}{dz}\frac{x(z)}{s(z)} = \frac{x(z)}{s(z)}\left[\frac{x'(z)}{x(z)} - \frac{s'(z)}{s(z)}\right] < 0.$$

Combining both equations leads to the result $s'(z)/s(z) > 0$ and $x'(z)/x(z) < 0$. Using the condition $n(z) = 1/s(z)$, we have the last result $n'(z)/n(z) < 0$. □

A.3 Proof of proposition 2

Differentiate (47) with respect to N, considering the endogenous variables as functions of N. This yields:

$$\left[\frac{\gamma'(i(z))\,D'}{p+\gamma(i(z))\,D'} - \frac{\gamma''(i(z))}{\gamma'(i(z))}\right]\frac{di(z)}{dN} - \frac{1}{r(z)}\,\frac{dr(z)}{dN} + \left[\frac{\gamma(i(z))}{p+\gamma(i(z))\,D'} - \frac{1}{D'}\right]\frac{dD'}{dN} = p\,F''(L)\,\frac{dL}{dN}$$

$\Leftrightarrow$

$$\left[\frac{\gamma'(i(z))\,D'}{p+\gamma(i(z))\,D'} - \frac{\gamma''(i(z))}{\gamma'(i(z))}\right]\frac{di(z)}{dN} = \frac{1}{r(z)}\,\frac{dr(z)}{dN} + \frac{p}{p+\gamma(i(z))\,D'}\,\frac{dD'}{dN} + p\,F''(L)\,\frac{dL}{dN}$$

The expression in brackets on the left hand side is positive, as has been shown in the previous section A.1. We will proceed by showing that the right hand side is positive, too, provided $t_c(z) = t_c \cdot z$, $F(L) = f \cdot L$, $d(E) = \delta \cdot E$, $\underline{r} = 0$ and $p_i = 0$.

We start with showing $dr(z)/dN > 0$. From the definition (24), we have

$$\frac{dr(z)}{dN} = \frac{d}{dN}\left[\underline{r} + p\,F'(A)\int_z^Z n(\tilde{z})\,t_c'(\tilde{z})\,d\tilde{z}\right] = p\cdot f\cdot t_c\cdot\frac{d}{dN}\int_z^Z n(\tilde{z})\,d\tilde{z}$$

If the population of the city increases, it is certainly not optimal to move any person further outside. It is neither optimal to locate all the new inhabitants beyond the former border of the city. As a consequence, the population density $n(z)$ is non-decreasing in N everywhere in the city. On the other hand, since it carries opportunity costs to increase the population density, the city will expand, i.e. $dZ/dN > 0$. As a consequence, we have $dr(z)/dN > 0$.

The next step is to prove $dD'/dN > 0$. Using the definition (23), we have:

$$\frac{dD'}{dN} = \frac{d}{dN}\left[\frac{p\,F'(L)\,L}{x(z)^\alpha\,s(z)^{1-\alpha}}\,d'(E)\right] = p\cdot f\cdot\delta\cdot\frac{d}{dN}\left[\frac{L}{x(z)^\alpha\,s(z)^{1-\alpha}}\right].$$

Certainly, the total labor supply L increases, if N increases. It remains to be shown that $x(z)^\alpha\,s(z)^{1-\alpha}$ decreases, if N increases. We therefore show that the average consumption of goods $\bar{x}$ and the average flat size $\bar{s}$ decrease. As a consequence, average utility decreases with increasing urban population N in the optimum.

The average consumption of goods is

$$\bar{x} = \frac{F(L)-\Delta}{N} = \frac{F(L) - \frac{r}{p}Z - \frac{p_i}{p}\int_0^Z i(z)\,dz}{N} = \frac{F(L)}{N}, \tag{51}$$

because $\underline{r} = 0$ and $p_i = 0$ by assumption. Since the average worker lives further away from the city center, and therefore the average commuting time increases if N increases, the average output per worker decreases. Thus, $d\bar{x}/dN < 0$.

The average size of living space is $\bar{s} = \frac{Z}{N}$. We have shown above, that Z increases with N, but since some of the new urban inhabitants move further to the city center, the average lot size will decrease. Thus, $d\bar{s}/dN < 0$.

A.4 Proof of proposition 4

Parts 1. and 2. of the proposition have been proven above. Here, we prove part 3. Given the subsidy (45) on infrastructure, a household residing at z has a net income

$$y^{\text{net}}(z) = y(z) - \frac{p_i + w}{n(z)}\,i(z).$$

Inserting this, the indirect utility is

$$\begin{aligned} v^{\text{priv}}(z) &= \alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\,y^{\text{net}}(z)\,(p+\gamma(i(z))\,p\,D')^{-\alpha}\,r(z)^{-(1-\alpha)} - d(E) \\ &= \alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\left(y(z) - \frac{p_i + w}{n(z)}\,i(z)\right)(p+\gamma(i(z))\,p\,D')^{-\alpha}\,r(z)^{-(1-\alpha)} - d(E). \end{aligned}$$

Hence, the spatial equilibrium condition reads:

$$\frac{y'(z)}{y(z) - \frac{p_i+w}{n(z)}\,i(z)} = \alpha\frac{\gamma'(z)\,i'(z)\,p\,D'}{p+\gamma(i(z))\,p\,D'} + (1-\alpha)\,\frac{r'(z)}{r(z)} + \frac{p_i+w}{n(z)}\,\frac{i'(z) - i(z)\,\frac{n'(z)}{n(z)}}{y(z) - \frac{p_i+w}{n(z)}\,i(z)}$$

Multiplying both sides with the net income leads to

$$\begin{aligned} y'(z) &= \gamma'(i(z))\,i'(z)\,p\,D'\,x(z) + r'(z)\,s(z) + (p_i+w)\,\frac{i'(z)}{n(z)} - (p_i+w)\,\frac{i(z)}{n^2(z)}\,n'(z) \\ &= (\gamma'(i(z))\,n(z)\,p\,D'\,x(z) + p_i + w)\,\frac{i'(z)}{n(z)} + r'(z)\,s(z) - (p_i+w)\,\frac{i(z)}{n^2(z)}\,n'(z) \\ &= r'(z)\,s(z) - \frac{p_i+w}{n(z)}\,i(z)\,\frac{n'(z)}{n(z)} = -w\,t_c'(z) - \frac{p_i+w}{n(z)}\,i(z)\,\frac{n'(z)}{n(z)}. \end{aligned} \tag{52}$$

Here we have inserted Condition (26) for the optimal supply of infrastructure to derive the second equality and used Equation (35) for the last conclusion. By Corollary 1, we have $n'(z)/n(z) < 0$. The comparison of $y'(z) = -w\,t_c'(z) + \Theta'(z)$ with Condition (52) proves

$$\Theta'(z) = -\frac{p_i+w}{n(z)}\,i(z)\,\frac{n'(z)}{n(z)} > 0. \tag{53}$$

□