On the shape of transit time distributions

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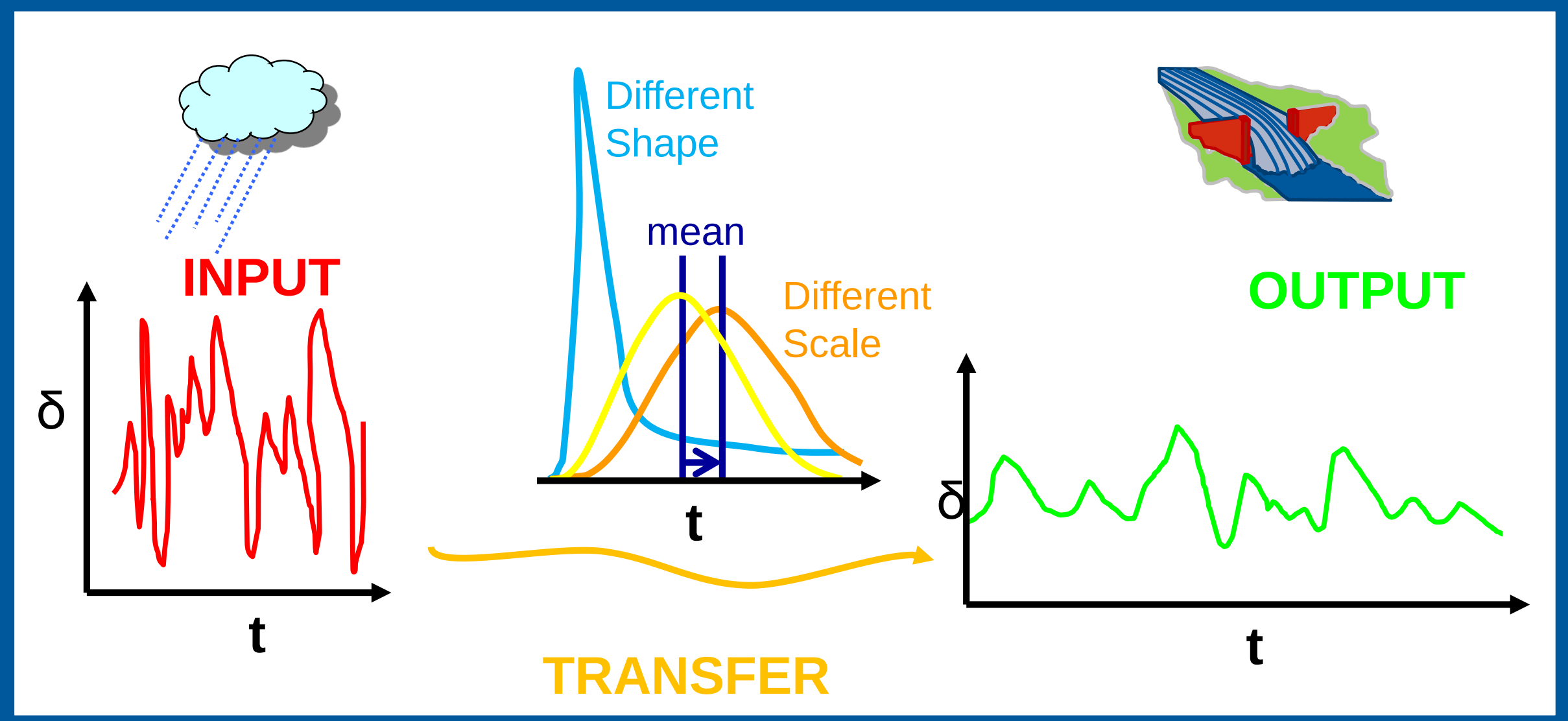
Motivation

Transit time distributions (TTDs) describe catchment behavior unlike any other function.

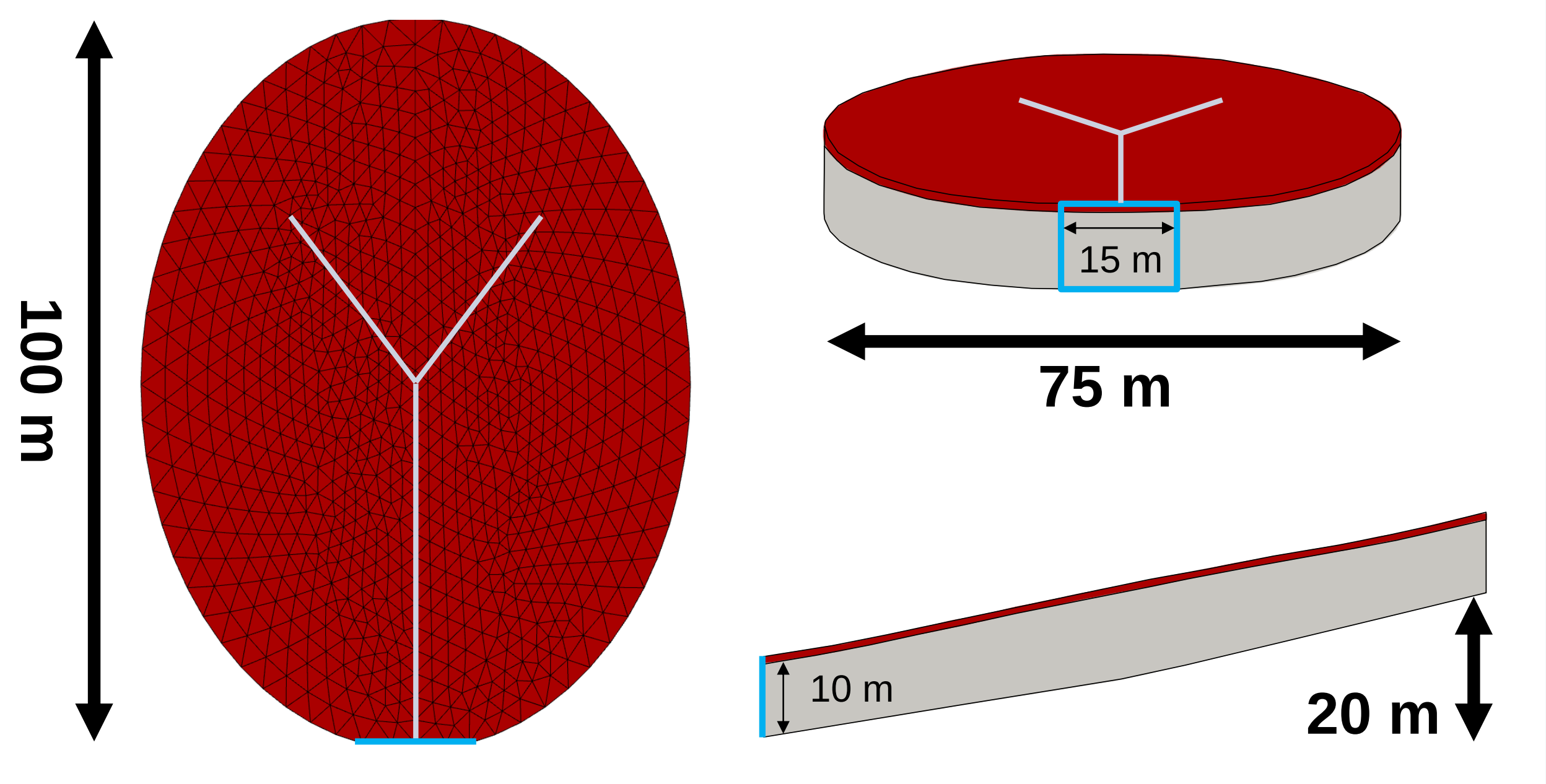
However, time and space variability of the shape (and scale) of TTDs is still poorly understood.

- e.g. causing equifinality problems when using transfer-function convolution models



How do real-world transit time distributions look like?

- How do they change over time with hydrologic conditions?
- How do they change in space with catchment properties?



Approach

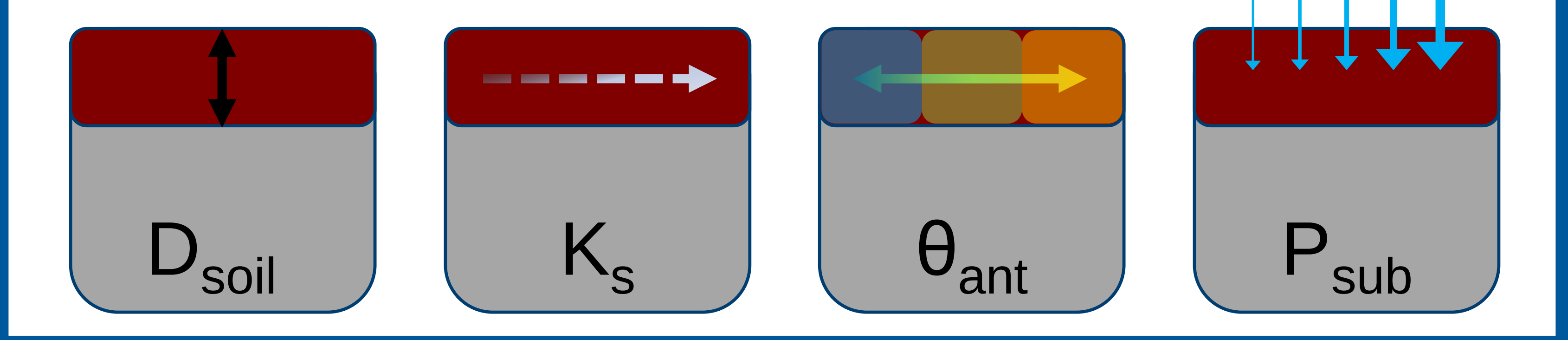
Virtual experiment:

- Simple catchment setup with HydroGeoSphere

- 10 m of bedrock with low hydraulic conductivity
- On top soil layer with higher conductivity

Scenarios

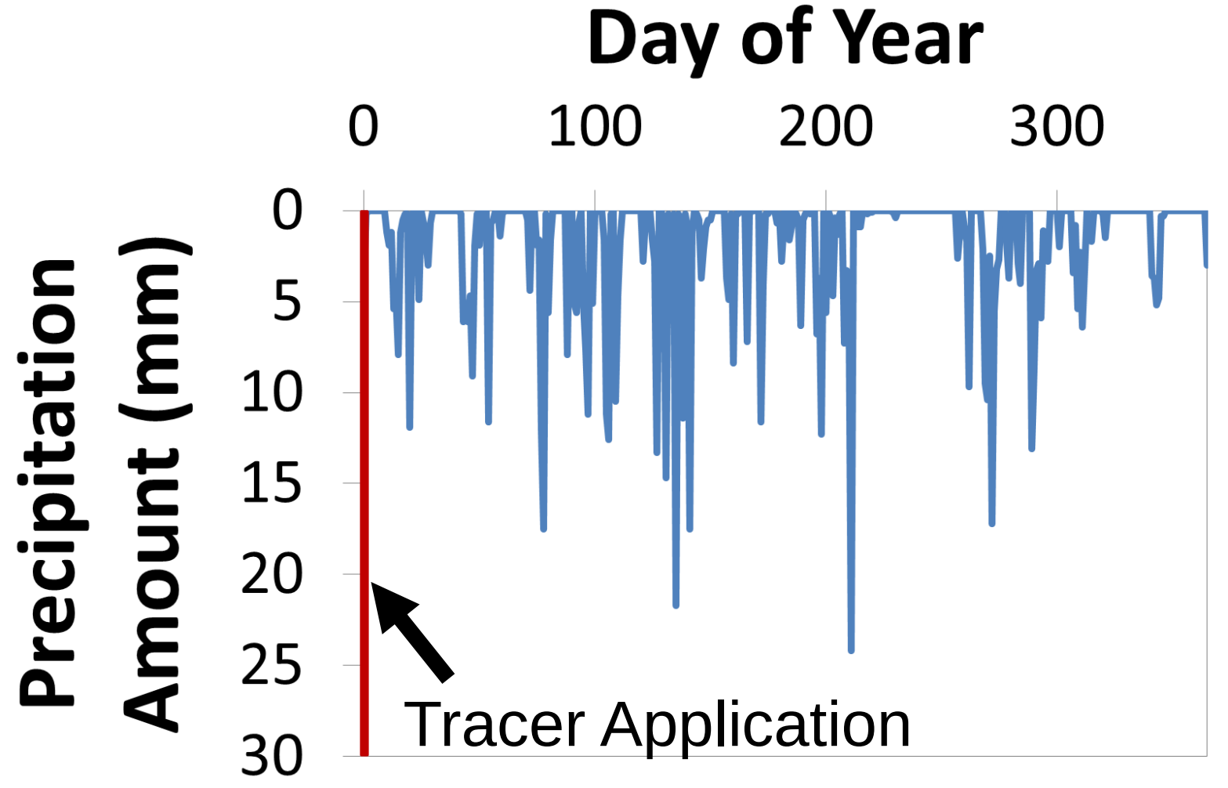
Soil depth	Saturated hydraulic conductivity	Antecedent moisture content	Subsequent precipitation
0.5/ 1.0 m	0.02/ 2.0 m day^{-1}	50/ 70/ 90%	345/ 690/ 1380 mm a^{-1}
Flat/**T**hick	**L**ow/**H**igh	**D**ry/**I**ntermediate/**W**et	**S**mall/**M**edium/**B**ig



- Example of abbreviated name: **F**lat Soil/**H**igh Conductivity/**D**ry Antecedent Moisture Content/**M**edium Subsequent Precipitation Amount = **FHDM**

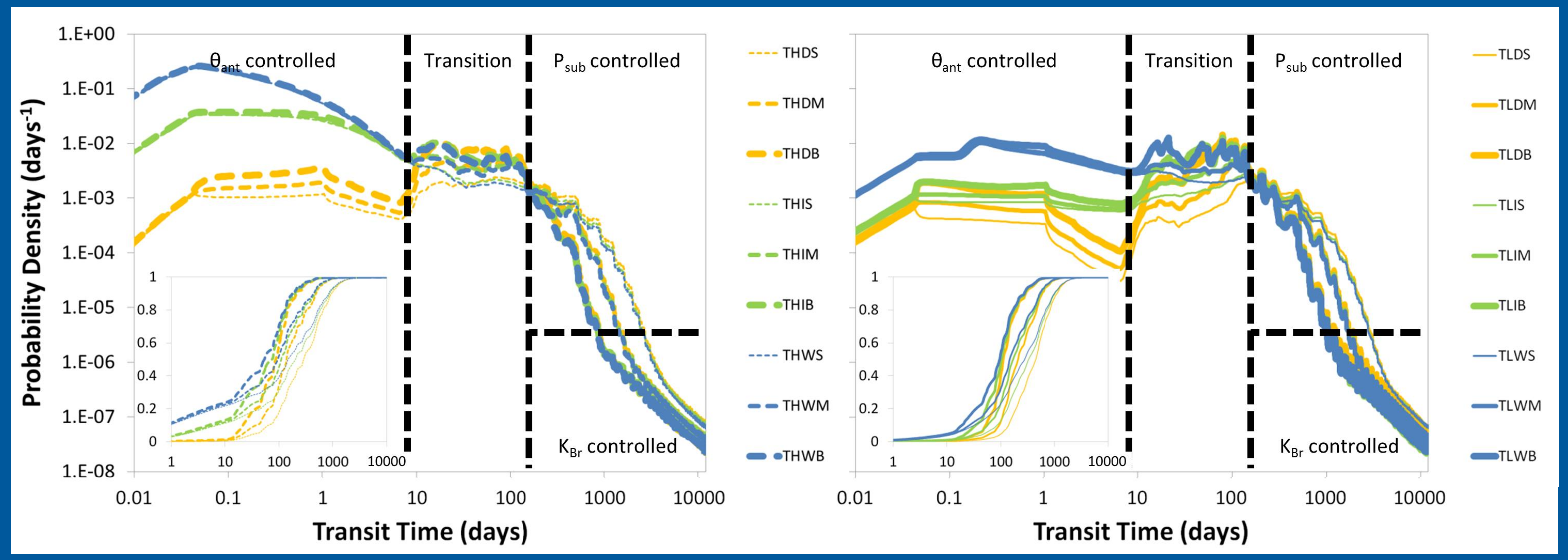
Input

- Tracer application from time t=0 to t=1 h
- Afterwards natural precipitation time series
- One year repeated 32 times



Results

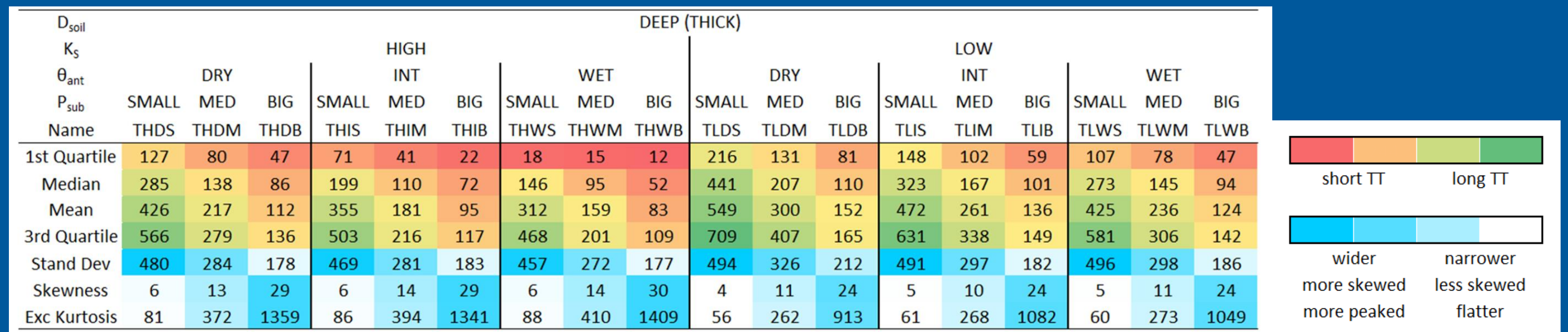
- Transit time distributions can be divided into four parts



High Soil Hydraulic Conductivity

Low Soil Hydraulic Conductivity

- Transit time distribution parameters



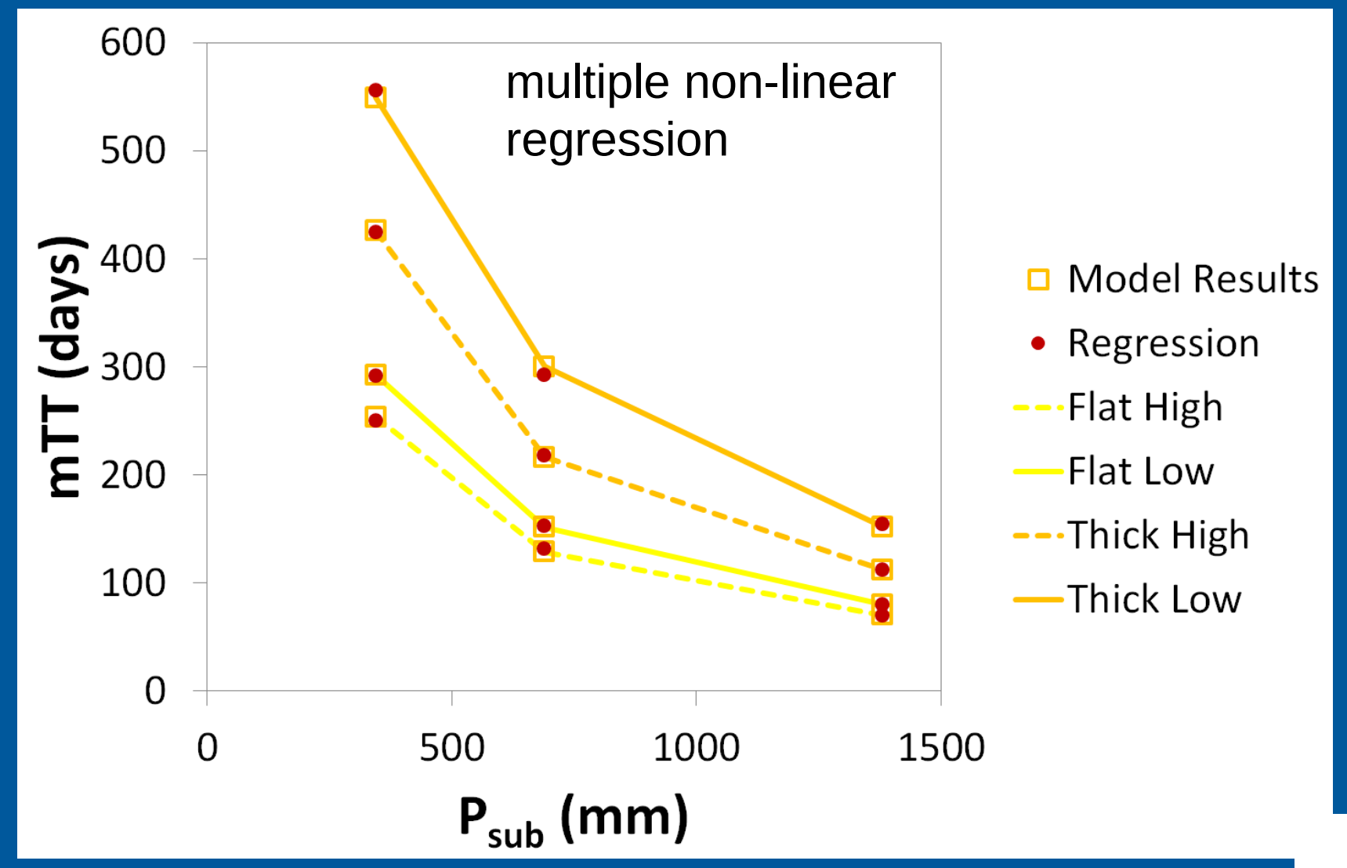
D_{soil}	DEEP (THICK)																	
K_S	HIGH									LOW								
θ_{ant}	DRY			INT			WET			DRY			INT			WET		
P_{sub}	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG
Name	THDS	THDM	THDB	THIS	THIM	THIB	THWS	THWM	THWB	TLDS	TLDM	TLDB	TLIS	TLIM	TLIB	TLWS	TLWM	TLWB
1st Quartile	127	80	47	71	41	22	18	15	12	216	131	81	148	102	59	107	78	47
Median	285	138	86	199	110	72	146	95	52	441	207	110	323	167	101	273	145	94
Mean	426	217	112	355	181	95	312	159	83	549	300	152	472	261	136	425	236	124
3rd Quartile	566	279	136	503	216	117	468	201	109	709	407	165	631	338	149	581	306	142
Stand Dev	480	284	178	469	281	183	457	272	177	494	326	212	491	297	182	496	298	186
Skewness	6	13	29	6	14	29	6	14	30	4	11	24	5	10	24	5	11	24
Exc Kurtosis	81	372	1359	86	394	1341	88	410	1409	56	262	913	61	268	1082	60	273	1049

short TT — long TT

wider, more skewed, more peaked — narrower, less skewed, flatter

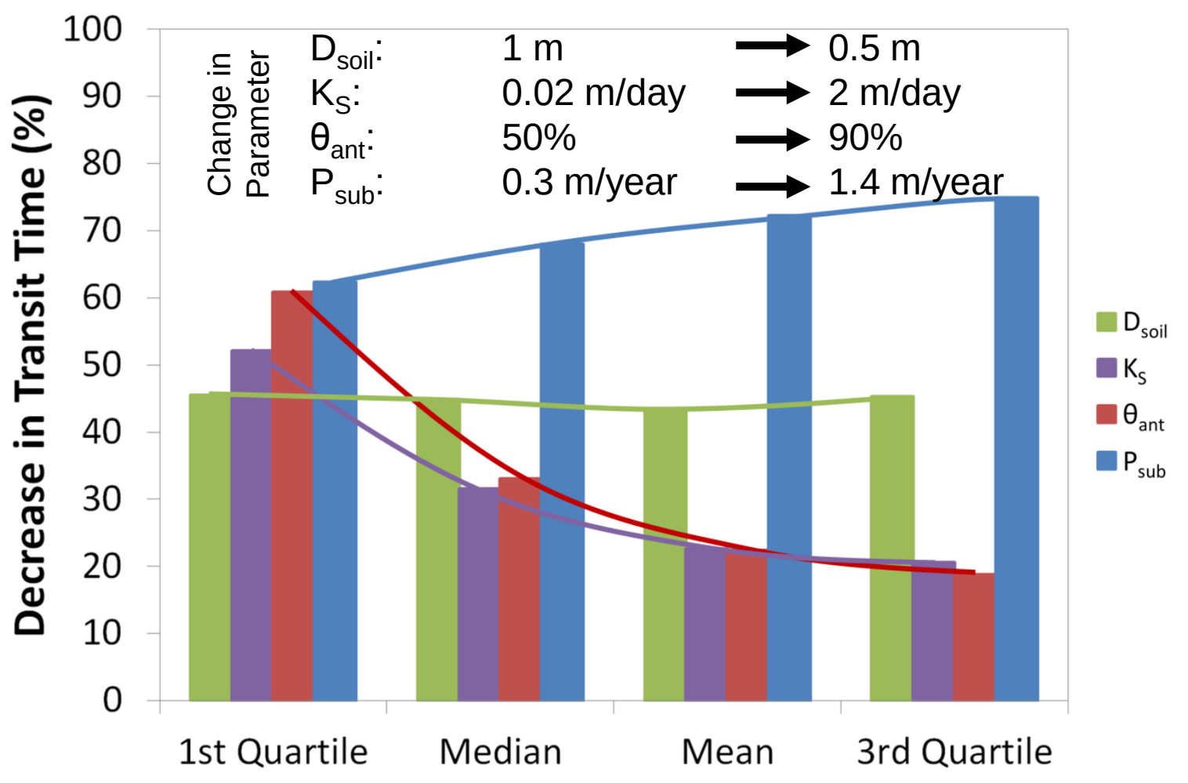
- Mean transit times (mTTs) can be predicted with high accuracy:

$$mTT = [(2648 \ln K_S + 120548)D_{soil} + (-4126 \ln K_S - 2260)]P_{sub}{}^{-[(0.020 \ln K_S + 0.061)D_{soil} + (-0.012 \ln K_S + 0.898)]}$$



- mTTs have a *power-law* correlation with P_{sub}
- mTTs correlate *linearly* with D_{soil}
- mTTs correlate *logarithmically* with K_S

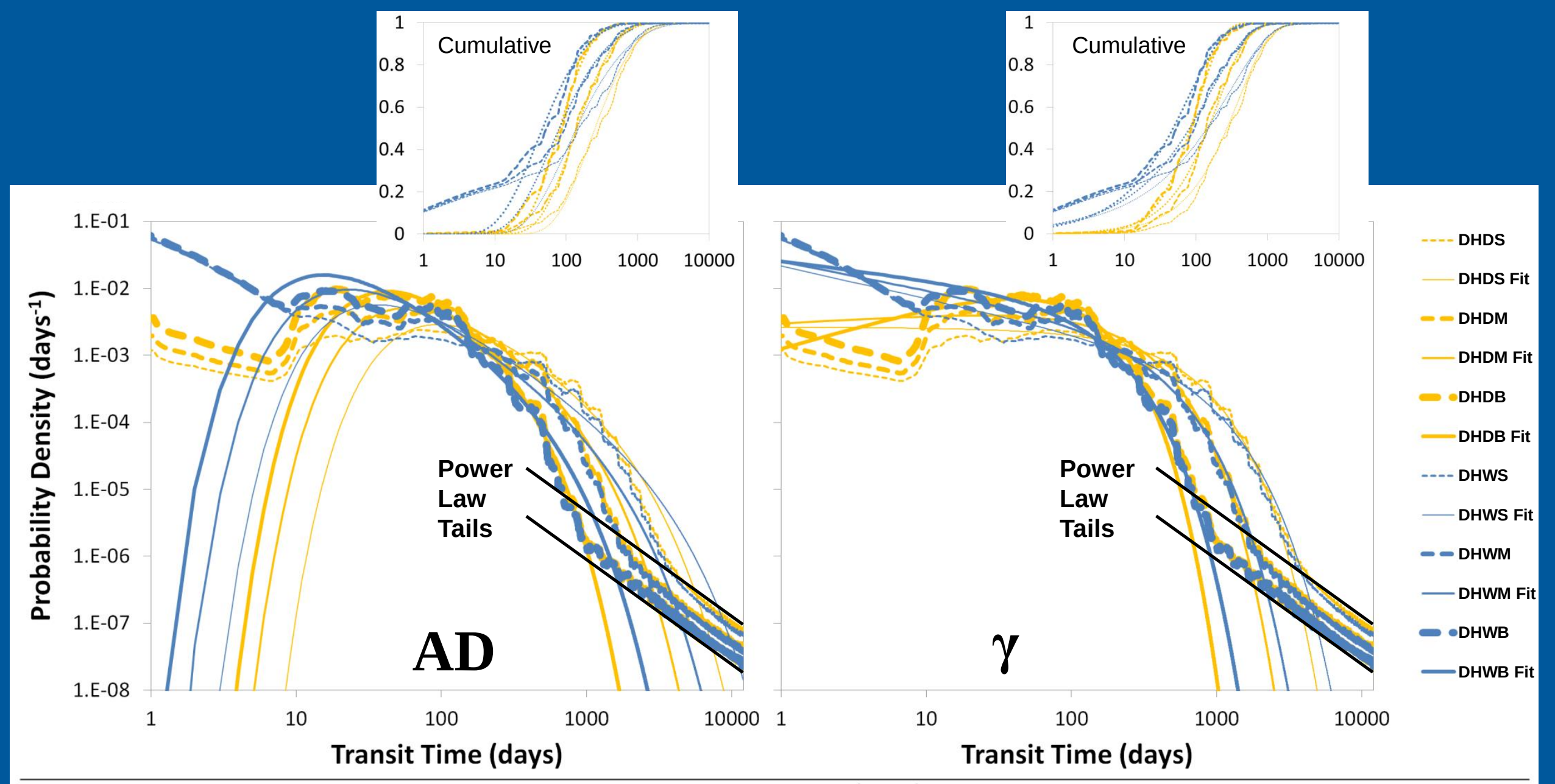
- θ_{ant} and K_S have bigger influence on young TTD fractions
- P_{sub} is affecting the older parts more
- D_{soil} influences all parts equally

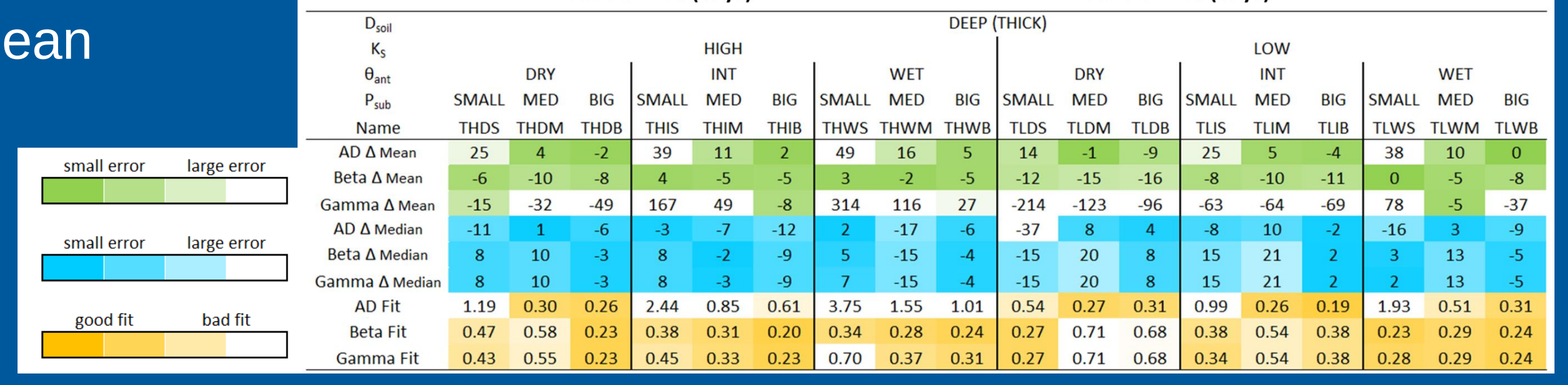


Discussion

Fitting theoretical distributions (advection-dispersion AD, beta β & gamma γ)

- AD works best for dry θ_{ant}
- γ works best for wet θ_{ant}
- Median is more correctly predicted than the mean





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P_{sub}	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG	SMALL	MED	BIG
Name	THDS	THDM	THDB	THIS	THIM	THIB	THWS	THWM	THWB	TLDS	TLDM	TLDB	TLIS	TLIM	TLIB	TLWS	TLWM	TLWB
AD Δ Mean	25	4	-2	39	11	2	49	16	5	14	-1	-9	25	5	-4	38	10	0
Beta Δ Mean	-6	-10	-8	4	-5	-5	3	-2	-5	-12	-15	-16	-8	-10	-11	0	-5	-8
Gamma Δ Mean	-15	-32	-49	167	49	-8	314	116	27	-214	-123	-96	-63	-64	-69	78	-5	-37
AD Δ Median	-11	1	-6	-3	-7	-12	2	-17	-6	-37	8	4	-8	10	-2	-16	3	-9
Beta Δ Median	8	10	-3	8	-2	-9	5	-15	-4	-15	20	8	15	21	2	3	13	-5
Gamma Δ Median	8	10	-3	8	-3	-9	7	-15	-4	-15	20	8	15	21	2	2	13	-5
AD Fit	1.19	0.30	0.26	2.44	0.85	0.61	3.75	1.55	1.01	0.54	0.27	0.31	0.99	0.26	0.19	1.93	0.51	0.31
Beta Fit	0.47	0.58	0.23	0.38	0.31	0.20	0.34	0.28	0.24	0.27	0.71	0.68	0.38	0.54	0.38	0.23	0.29	0.24
Gamma Fit	0.43	0.55	0.23	0.45	0.33	0.23	0.70	0.37	0.31	0.27	0.71	0.68	0.34	0.54	0.38	0.28	0.29	0.24

small error — large error

small error — large error

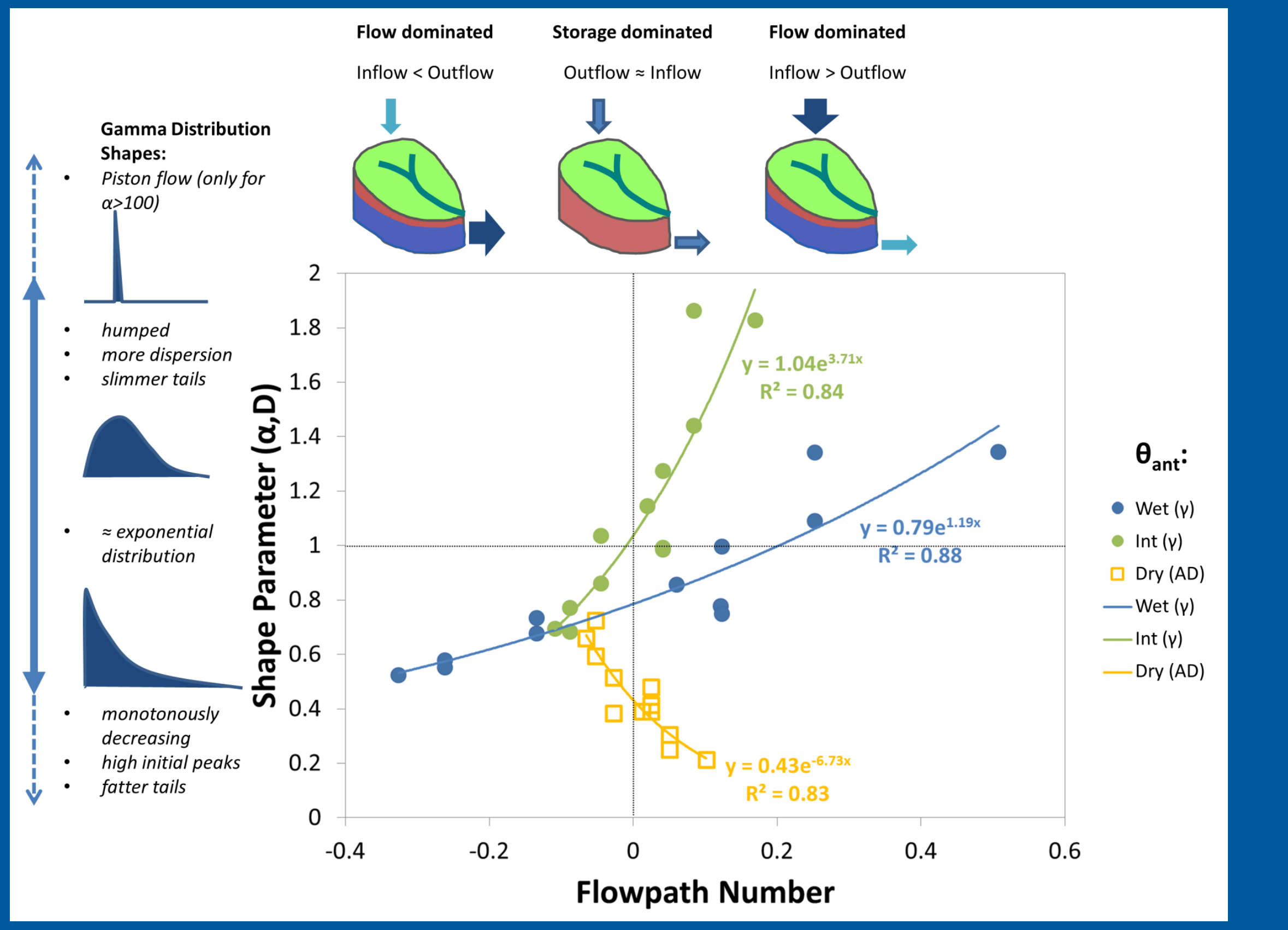
good fit — bad fit

- α, D shape parameters can be predicted with dimensionless flow path number F:

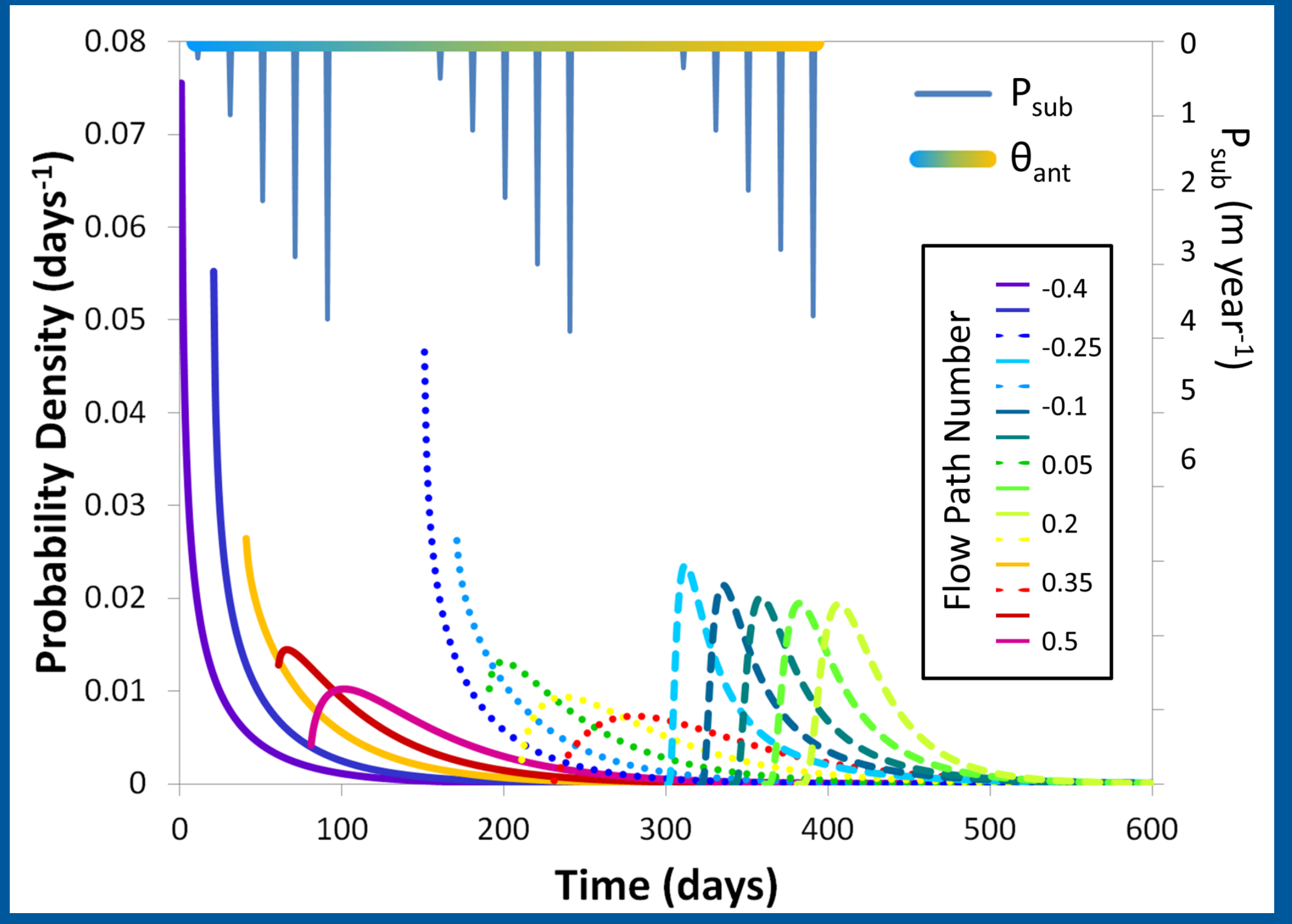
$$F = \frac{P_{sub,i} - K_{rem}}{D_{soil} * (n - \theta_{ant})}$$

$$P_{sub,i} = P_{sub} * \frac{t_i}{365.25}$$

$$K_{rem} = k_{eff} * t_e * \frac{A_{out}}{A_{in}}$$



- n: porosity; t_i: mean inter-event duration; k_{eff}: effective hydraulic conductivity; t_e: mean event duration; A_{out}: outflow area; A_{in}: inflow area



- Shape and scale of TTDs vary systematically with certain environmental parameters
- It is possible to predict the full TTD (shape and scale)

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